Know Your Wisconsin Mathematician

Interview with Professor Richard Askey, UW-Madison, by J. Sriskandarajah, MATC

This is our sixth interview in this series. Professor Askey retired recently after a very successful career at the UW-Madison, Department of Mathematics (1963-2003). To learn more about his accomplishments, please visit: http://www.math.wisc.edu/~askey/

Q. Tell us something about your education, starting with elementary school.

A. Here is one story from grade school. I was in third grade, which was on the first floor, and walked by the room on the second floor in which math was taught to fifth and sixth grade students. I saw a symbol I did not know so asked my teacher what it was. We had done addition, subtraction, multiplication and division, but not square roots. She explained it was the symbol for taking a square root, explained what square roots were and how to calculate them. Later when I was in fifth and sixth grades, the teacher who taught math to all of the fifth and sixth grade students was still teaching. She and all of the teachers I had in grade school were good. This was in a suburb just outside of the St. Louis City line. It was a neighborhood with some professionals and some manual workers. Two doors one direction was an electrical engineer who worked for the telephone company and on the other side was a bus driver. We lived in a small six room house, with four children and our parents, with a grandmother living with us when there were only three children.

There are also two related stories from later years which are worth mentioning. In eighth grade, the teacher we had was just out of the Army. This was 1946-47. He finished the course with about six weeks left, so told us he was going to teach algebra. We did not have to listen, but we had to be quiet so others could learn. We moved at the end of that year, to Baltimore. There was an accelerated program where you could start high school in grade 9 and the last year was college level courses. I did not know about this, so finished junior high school and started high school in tenth grade, as was normal then. I found out about the accelerated program, and the 12th grade the math course was mostly analytic geometry and some calculus. I wanted to take this course and it was possible by taking intermediate algebra and solid geometry the first semester and trigonometry and advanced algebra the second semester in 11th grade. My first class was one of the math classes, and I went to the room for this class. The teacher started by saying that we would be learning how to solve some equations we did not know how to solve, and gave an example something like $x^2 + 4x + 5 = 0$. I raised my hand and said the solutions were $x = -2 \pm i$. He looked surprised and asked me where I had learned this. My answer was a good illustration of how to teach, tell the truth but not the whole truth. I said I had learned it in a different school system. I did not say that it was an eighth grade class, which it was. Later in the day I looked carefully at my schedule and found out I had gone to the wrong room. I should have been in the next room taking solid geometry. The class I had attended was advanced algebra. In the afternoon I went to the correct room for the intermediate algebra class. When I walked in, the teacher looked at me and asked what I was doing there. I said I had gone to the wrong room that morning, and this was where I should be. I have left out the story of when another student and I were taken to the principal's office in seventh grade for something I had written and my friend had put on the teacher's desk, and how I got out of the last half of trigonometry, the grubby solving triangles part with tables, so I could tutor a student who had to pass trigonometry to graduate. By and large, school was good, and I did what I could to get by without having to work too hard on things I was not interested in.

In the Honor Society in 11th grade, I helped set up a before school tutoring program. I left home at 7 to get to school by 8 to work with students who were in need of help. There was a Math Club in the high school. I went to one meeting in tenth grade, but when I found out they were playing math games, I did not go back. I was immature and did not appreciate the fact that math games could be useful. The student running the Math Club that year was a senior, Solomon Golomb. After he graduated, I went back to the math club and for most of the next two years helped teach some calculus. It was the most exciting mathematics I had seen, and I wanted to share some of what I was learning.

Q. What was the influence of your family on your education?

A. What mathematical talent I have was inherited from my mother. She only had one year of college due to family responsibilities, taking care of two younger half brothers, and family finances. A few years ago my wife and one of my sisters found a solution my mother gave to a homework problem given in analytic

geometry in her year in college. A particular triangle was given with specific vertices and the problem was to prove that the symmetric medians meet at a point. Her solution was about three pages long of detailed calculations with trigonometric functions. There are elegant ways to prove this theorem in general, but her solution was one I would like students to be able to do, since she appropriately used a lot of trigonometry and facts about equations of lines. She was a wise woman. She told me many times that the grades I got were not important, what was important was what I learned. The only pushing I got was to finish a few projects which I was not interested in. She strongly suggested I take typing in high school. This was useful when I typed my thesis, and now in the internet age, it is again useful to be able to touch type.

Q. What about your work in college and graduate school?

A. My undergraduate work was at Washington University in St. Louis. This was a good place for me. They had some very good faculty and a few good students, some undergrads and some graduate students. I.I. Hirschman gave me a problem to work on when I was a senior, and we eventually jointly published a solution of it. This is what got me involved in special functions. I was hired as a freshman to grade homework in some math classes. I got sick and tired of grading, but it is a good idea and I have been disappointed a number of times when it was not possible to hire undergraduates at UW Madison to grade classes. Lack of money was not always the problem, sometimes it was someone in the Dean's Office who felt this was a bad idea. That was not always the case. Back about 1967, I was allowed to hire an undergraduate to grade the homework in a graduate course which he had not taken. He had taken the year analysis course which Walter Rudin had developed, which was enough for him to grade a course on Hilbert Spaces. This undergraduate is now a Professor of Geophysics and when he was an Associate Dean, it was possible to hire qualified undergraduates to grade papers. I would likely not have pushed as hard on this if I had not graded so many papers, including grading advanced calculus papers one year when I was taking the course.

For graduate school, I spent a year at Harvard, decided I had made a mistake in going there rather than Princeton, so wrote and mentioned this, and was accepted at Princeton. Princeton got good graduate students, and most of them had taken the courses which elsewhere were usually taken by first year graduate students, so they did not give these courses regularly, and encouraged students to help each other fill in gaps of knowledge which were not being taught then. I worked at Bell Labs in Murray Hill for two summers, and got to know two very important mathematicians who visited for part of a summer. Mark Kac was one, and Arne Beurling was the other. Beurling gave me a couple of problems as possible thesis topics, but I was never able to solve them. After two years at Princeton, I had passed the exams but not written a thesis. I got married and decided to take an instructorship at Washington University and try to write a thesis there. I made some progress and went back to Princeton for a summer to finish the work. I saw Bochner once a week, and each time started from scratch since the topic was one he probably did not care that much about. I agree with this now, but then it was what I could do. He suggested two other problems to work on after I got my degree, both of which were more interesting and important than what was in my thesis. That is an ideal gift from a major professor, something harder to work on next.

Q. You went to the University of Chicago and then came to Madison. How did this influence your development?

A. The University of Chicago is one of the great universities in the world. I learned a lot from attending the Calderon-Zygmund seminar, but the most important thing which happened to me was finding out that Steve Wainger and I had complementary knowledge so were able to solve some problems which had vexed us separately. In particular, we solved both of the problems Bochner had suggested. This took some time and to make it easier for us to work together Wainger visited the University of Wisconsin for a year, and then was hired. In the course of solving one of the problems, we came across some gaps in the knowledge of some classes of orthogonal polynomials. At this point Wainger and I went different ways. I worked on the orthogonal polynomial problems and Wainger worked in a number of different areas. My work was initially on norm inequalities in L_p spaces. The polynomial problems led to work on positive operators, which in some sense can be thought of as sharp inequalities. This led to certain identities which were not known and should be found. George Gasper, who spent time in Madison as a postdoc, Tom Koornwinder, whom I met in Amsterdam where we spent a year on leave in 1969-70, and others were the people who solved the harder problems. Three of the others were Ph.D. students in Madison, Charles Dunkl who worked with Rudin and Dennis Stanton and James Wilson who were my students. George Andrews spent a year on leave in Madison, and I caught the q-disease that year. Mourad Ismail also spent a post-doc year in Madison, and has solved many interesting problems.

Hypergeometric series are series whose ratio of the n+1st term to the nth term is a rational function of n. Basic hypergeometric series have a term ratio which is a rational function of q^n. There was an important paper on the classical type orthogonal polynomials which are represented as basic hypergeometric series by Wolfgang Hahn. This was published in the late 1940s, and 30 years later no one had worked out the orthogonality conditions which he left open. Andrews and I both wanted to work this out, which was the main reason he came to Madison. Half of his support came from the University Research Committee, WARF money, and the other half from the Mathematics Research Center. We worked out the orthogonality relation of the most general polynomials Hahn found, but more importantly, polynomials at one higher level were discovered and their orthogonality worked out. There are called the q-Racah polynomials when the orthogonality is a finite sum and the Askey- Wilson polynomials when the orthogonality has an absolutely continuous measure. Polynomials discovered by L.J. Rogers in the middle 1890s are a special case, but the orthogonality relation for them was first found as a special case of the general polynomials Wilson and I found. These polynomials and special cases are showing up in many different contexts. There was one other thing which came from George Andrews's year in Madison. In the spring he and his family went to Europe, and in the Wren Library at Trinity College Cambridge he found a bit over 100 pages in Ramanujan's handwriting, which he called the "Lost Notebook". This is not the place to write about the importance of this find, but it led to at least one important result which can be explained to non-mathematicians. There was an interview with Andrews in "The Hindu", and at the bottom of the full page story was an interview with Ramanujan's widow. She said that they had promised to make a statue of her husband, and lamented: "Where is the statue?" After reading this, a bust was commissioned, to be made by Paul Granlund. Initially four castings were made, and one was given to Janaki Ammal, Ramanujan's widow. You can see a photograph of this bust on my website. www.math.wisc.edu/~askey We visited her in December, 1987, and she said that when the bust arrived it was like the return of Ramanujan's spirit to her home.

Q. You have spent a lot of time on mathematics education. Why, and what do you hope to accomplish?

A. The "why" has a simple answer. Over 20 years ago I was finding it harder to teach calculus than before, and I wanted to find out why and see what could be done to reverse this trend. The "what" is more complicated. I would like teachers to have more content knowledge. Twenty years ago almost no one was saying this. One person who was saying this was Lee Shulman. In his presidential address to the American Education Research Association, Shulman called content "the forgotten part of education", and called for teachers to develop pedagogical content knowledge. He started with some questions for teachers in California in 1875. You can see questions like those he used by going to www.google.com and search for “askey mad lit talk”. This is a talk I gave to the Madison Literary Club.

There are quite a few textbooks which define lines to be perpendicular when the product of their slopes is –1. That should be a theorem, not a definition. That is but one example of what I would like to see in school mathematics, a distinction between a theorem and a definition. I dislike reading a program which uses the law of sines and the law of cosines to prove similarity theorems for triangles when similarity is needed for at least right triangles to even define the trig functions.

There has been some progress. Curriculum Focal Points from NCTM is a significant improvement over their 1989 Standards and their 2000 version. The National Mathematics Advisory Panel has a good report on preparation for algebra and a good description of what school algebra should be. To understand that their listing of topics is not just a list, you have to read the Task Group Report on Conceptual Knowledge and Skills. It is on the web and worth reading. It should influence courses given for prospective teachers.

Q. Do you have any other comments?

A. The work I have been doing in mathematics education is something which needs to be done. Without firm knowledge of content, mathematics education can take some unfortunate detours. Every so often something is done which can help change unfortunate directions. Liping Ma's book "Knowing and Teaching Elementary Mathematics" is one example. If you have not read it, do! A more recent book which I highly recommend is "Arithmetic for Parents" by Ron Aharoni. He is an Israeli mathematician who has been teaching math in elementary school since about 2000. This book is published by a private firm set up by a mathematician at Berkeley. The name of the publishing firm is "Sumizdat", one of the most interesting names for a publisher I have seen. Ask a Russian if you do not understand why this is so interesting. This mathematician, Alexander Givental, has also published two geometry books, translations with some changes of books originally written by Kiselev in the 19th century. I recommend all three of these books.