Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 5, 2013

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

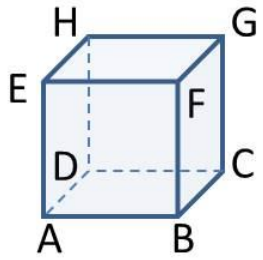
Score (Sum of both sub-totals)_____

Part I:

1. 99 consecutive integers are added together. Which of the following numbers can be the sum of these integers?

   (a) 12176 (b) 12177 (c) 12179 (d) 12181 (e) any of (a)-(d) is possible

2. A spider is crawling on the surface of cube ABCDEFGH, where AB=5cm. Currently it is sitting at a point K on edge HG, 1cm away from G. The spider wants to visit his friend, who resides at point L, 1cm from B on edge AB. What is the length of the shortest path from K to L?



   (a) 10 cm (b) $\sqrt{72}$ cm (c) $\sqrt{74}$ cm (d) $5\sqrt{2}$ cm (e) none of these

3. How many different vertices can you choose in a regular 13-gon so that the distance between any two of them is different?

   (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

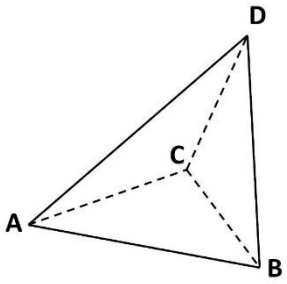
4. Pebble Pete and Rocky Joe take turns taking away rocks from a pile. A person can take at least one, and at most 10 rocks at a time. When there are no more rocks in the pile, Pete and Joe announce the total number of their rocks. If the two numbers are relatively prime (their greatest common divisor is 1) then Pete wins, otherwise Joe is declared the winner. How many rocks can be in the pile if Pete wins every game no matter how he and Joe choose the number of rocks to take in each turn?

   (a) 1001 (b) 437 (c) 247 (d) 324 (e) 1019

5. A regular fair six-sided die is thrown and the numbers shown on the die are added together until their sum, S, is greater than 100. Which value of S has the largest probability?

   (a) 101 (b) 102 (c) 103 (d) 105 (e) 106

6. The edges of a tetrahedron have length 7, 13, 18, 27, 36 and 41 cm. If AB is 41 cm, what is the length of CD?

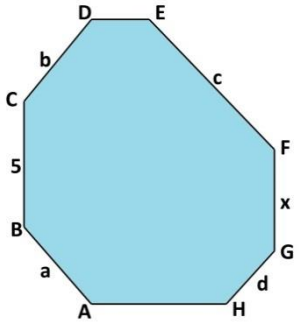


(a) 13 cm (b) 18 cm (c) 27 cm (d) 36 cm (e) 7 cm

7. Mrs Armstrong's class had an activity building solid cubes by gluing together unit cubes. Next, the students painted some of the faces of their big cube red. It turned out that the glue wasn't strong enough to hold the unit cubes together, so the next day the students realized that they can take their big cube apart, and have unit cubes again. When they did this, they counted up the cubes that had no paint on them. One group found that they had 45 unit cubes with no paint. How many unit cubes this group used to build their big cube?

(a) 125 (b) 64 (c) 216 (d) 343 (e) cannot be determined

8. Assume that all angles in the octagon ABCDEFGH are equal. Additionally, $a+b=6\sqrt{2}$ cm, and $c+d=7\sqrt{2}$ cm . What is the length of side x?



(a) 3 cm (b) 2 cm (c) 4 cm (d) $\sqrt{2}$ cm (e) 2.5 cm

Part II:

9. Let f(x) and g(x) be two different quadratic polynomials with leading coefficient 1. Given that f(1)+f(10)+f(100)=g(1)+g(10)+g(100), for what values of x will f(x) equal g(x)?

   (a) x cannot be determined (b) 0 (c) 1 (d) 111 (e) 37

10. Given six non-collinear points in the plane, draw the perpendicular bisector of the line segments determined by every possible pair of these points. What is the largest number of intersection points that these perpendicular bisectors can have?

    (a) 15 (b) 105 (c) 85 (d) 75 (e) none of these

11. Let S_n be the sum $1^2 + 2^2 + 3^2 + \ldots + n^2$, where n is a three digit positive integer. Consider the remainder of S_n when divided by 4 as n runs through all three-digit positive numbers starting at n=100 ending with n=999. Which remainder is the most frequent?

    (a) 3 (b) 2 (c) 1 (d) 0 (e) each remainder has the same frequency

12. The function f is defined in the following way:

$$f(x) = \begin{cases} x - 2013 & if\ x > 2020 \\ f\big(f(x + 2014)\big) & if\ x \leq 2020 \end{cases}$$

    What is the value of $f(0)$?

    (a) 8 (b) 7 (c) 2014 (d) 0 (e) none of these

13. Fifty numbers are chosen from the integers 1,2, …, 99. No two of the chosen numbers sum up to 99 or 100. Let N be the smallest number that is chosen.

    (a) N is 1 (b) N is 2 (c) N is 50 (d) N is 49 (e) N cannot be determined

14. Let $f(x)=x^2+12x+30$. How many real numbers satisfy the equation f(f(f(f(f(x)))))=0 ?

    (a) 0 (b) 32 (c) 16 (d) 4 (e) 2

15. What is the sum of the digits of the largest number that cannot be written as a sum of exactly 2013 composite numbers? (0 or 1 are not composite numbers)

(a) 16 (b) 14 (c) 17 (d) 18 (e) none of these

16. Call a positive integer *one-full* if it satisfies the following criteria:
(A) Every digit is either 0, 1, or 2.
(B) Out of every two consecutive digits, at least one of them is a 1.
(Numbers beginning with the digit 0 are not allowed.) The number of different 10 digit one-full numbers is

(a) 511 (b) 512 (c) 1024 (d) 256 (e) none of these

Part III:

17. In the city of *Egalite* the clock in the tower on Main Square has a minute hand and an hour hand that are identical. How many times a day is it impossible to tell the time by the clock, provided the people always know whether it is a.m. or p.m.?

(a) 2 (b) 22 (c) 11 (d) 132 (e) 264

18. The area of a convex quadrilateral is 32 cm^2. The sum of the length of one of its diagonals and the two opposite sides connected to this diagonal is 16 cm. The other diagonal of the quadrilateral has length

(a) 8 cm (b) $6\sqrt{3}$ cm (c) $6\sqrt{2}$ cm (d) $8\sqrt{3}$ cm (e) $8\sqrt{2}$ cm