Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 9, 2010

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

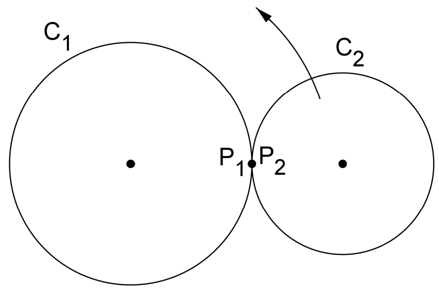
Score (Sum of both sub-totals)_____

Part I:

1. Calculate $49^{1-\frac{1}{4}\log_7 25}$

a) 49/5 b) 10 c) 25 d) 7 e) 49

2. Two circles C_1 and C_2 whose radii are 8 and 6 respectively are adjacent to each other at the points P_1 on C_1 and P_2 on C_2. If circle C_2 rolls around C_1, how many times does C_2 go around C_1 until P_1 and P_2 meet again?

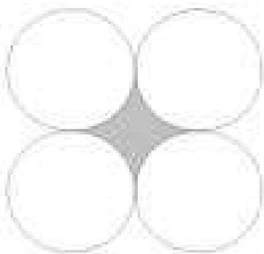


a) 2 b) 3 c) 4 d) 6 e) 8

3. Let a and b be positive integers less than 10. What is the probability that $\log_2 a + \log_3 b$ is a positive integer?

a) 4/27 b) 4/81 c) 2/27 d) 10/81 e) 11/81

4. If the radius of the circles is 1, find the area of the shaded region.



a) π -4 b) 4-π c) 4+π d) π/4 e) 2π-4

5. A single die is tossed repeatedly and the results observed. Find the probability that the second appearance of a 5 happens on the 5^{th} toss.

a) 125/7776 b) 5/18 c) 125/1944 d) 25/108 e) 1/18

6. A rectangular solid with dimensions 4x5x6 is constructed using 1x1x1 cubes. The outside of the solid is painted packer green and then the solid is disassembled. Find the probability that one of the original small cubes chosen randomly will have paint on at least one side?

a) 9/10 b) 4/5 c) 11/15 d) 5/6 e) none of above

7. Three math teachers Mr. A, Ms. B, and Mr. C talked at a math conference. Mr. A said, “I have 12 more students than does Ms. B, and 15 fewer students than does Mr. C.” Ms. B replied, “The three of us have 270 students altogether.” The number of students Mr. C has is

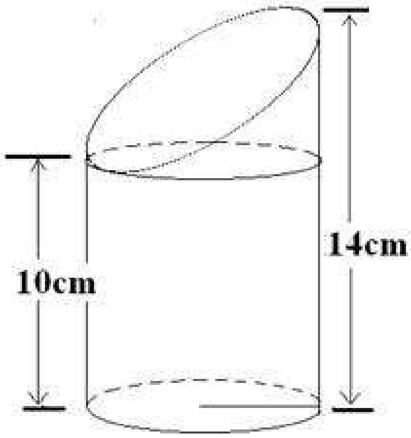
a) 76 b) 86 c) 94 d) 104 e) none of these

8. Towns A, B and C are at the vertices of an equilateral triangle and are each 100 miles from the other. On a recent trip, Jim drove from A to B at 70 mph, and from B to C at 50 mph. Approximately how fast did he drive from C back to A if his average speed for the trip was 60 mph?

a) 56 mph b) 58 mph c) 60 mph d) 62 mph e) 64 mph

Part II:

9. A right circular cylinder is cut off at an angle of 30 degrees with respect to the horizontal so that the remaining piece has a shorter height of 10 cm and the longer height of 14 cm long as shown in the diagram below:



Find the volume of the remaining piece.

a) 60π b) 600π c) 144π d) 256π e) 192π

10. The polynomials

$$x^2 + ax + b \text{ and } x^3 + 3abx + 2a \text{ (} a \text{ and } b \text{ are positive constants)}$$

are both divisible by $(x+1)$. Find $a+b$.

a) 1 b) 2/3 c) 5/3 d) $\frac{\sqrt{13}}{2}$ e) $\frac{2+\sqrt{13}}{3}$

11. Find the value of D in the following system of linear equations.

$$A + D + G = 2010$$
$$C + D + E = 2010$$
$$F + D + B = 2010$$
$$A + C + F = 2010$$
$$G + E + B = 2010$$

a) 680 b) 670 c) 710 d) 720 e) 730

12. $abcd$ is a four digit positive integer such that $bd = 10$ and $a + c = 20 - 2b$. The smallest value that the sum of the digits can take is

a) 23 b) 14 c) 17 d) 32 e) 27

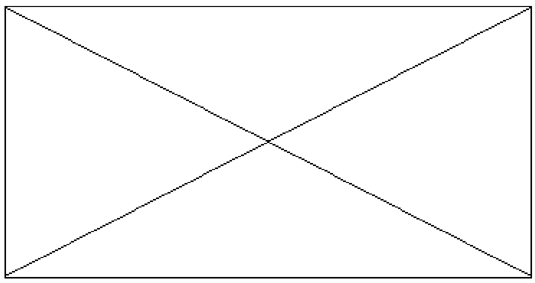
13. Determine the largest x-value of all points whose distance from (0, 1) is twice its distance from (3/2, 5/2).

a) $2 + \sqrt{2}$ b) $2 - \sqrt{2}$ c) $3 + \sqrt{2}$ d) $3 - \sqrt{2}$ e) no largest value exists

14. How many six digit numbers with strictly decreasing digits are there (e.g., 987621)?

a) 200 b) 210 c) 225 d) 240 e) 256

15. A gardener is to put fencing around a rectangular plot of land. She is going to divide the plot into four subplots by installing fencing on the diagonals.



If the area of the rectangular plot is 336 ft^2 and the total amount of fencing (sides and the diagonals) is $112\sqrt{2}$ ft, then what is the perimeter of the rectangular plot?

a) $56\sqrt{2}$ ft b) $58\sqrt{2}$ ft c) $60\sqrt{2}$ ft d) $62\sqrt{2}$ ft e) $64\sqrt{2}$ ft

16. Define the operation "$*$" on the real numbers by the equation $a * b = ab^2(a - 2b)$, for any real numbers a and b. If $2a * 2b = k(a * b)$, then the value of k is

a) 2 b) 4 c) 8 d) 16 e) 32

Part III:

17. Consider the two sets

$$A = \{\ x:\ |x-1| < 3\} \text{ and } B = \{x:\ x^2 - 2ax + a^2 - 4 \ge 0\}$$

Find the constant a so that $A \cap B = \{x:\ -2 < x \le 1\}$.

a) -1 b) 3 c) 0 d) -4 e) 1

18. Which of the following *cannot* be an integer factor of $3^{2010} - 2^{2010}$?
[Hint: For any positive integer n, $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1})$]

a) 5 b) 7 c) 19 d) 211 e) All of these are factors