Mathematical Association of America
Wisconsin Section
Mathematics Contest Examination
December 3, 2009

1. Do not open this booklet until you are directed to do so.

2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

   EXAMPLE:
   If x is 3 and y is 4 then $2x - y$ is

   (a) –1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.

4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.

5. Fill in the following blank and wait for the signal to start the examination.

PRINT_____
First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____x 4 =____	____x 1 = ____
2	8	____x 8 = ____	____x 2 =____
3	2	____x 12 = ____	____x 3 = ____
Total	18	Sub-Total _____	Sub-Total ____

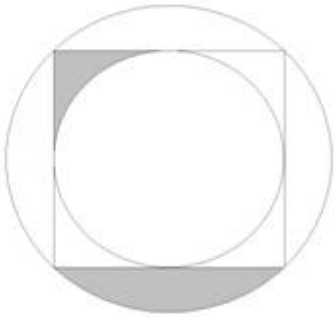
Score (Sum of both sub-totals)_____

Part I:

1. A motorist travels the first 10 miles of a journey at 30mph. How fast would he have to drive for the next 10 miles if the total journey had an average speed of 50mph?

(a) 70mph (b) 90mph (c) 120mph (d) 150 mph (e) It is impossible

2. The sides of the square measure 4cm.



What is the combined shaded area?

(a) π (b) $4\pi/3$ (c) $3\pi/4$ (d) $2\pi/3$ (e) $3\pi/2$

3. A student tosses a coin until a head first shows. What is the probability that a head first appears after an even number of tosses?

(a) ½ (b) 1/3 (c) ¼ (d) 2/3 (e) 3/4

4. The Fibonacci Sequence starts 1,1,2,3,5,8,13,... where, after the first two numbers, 1,1, every number is the sum of the previous two numbers. If the 36^{th} number in the sequence is 14,930,352, and the 38th number is 39,088,169, what is the 40^{th} number?

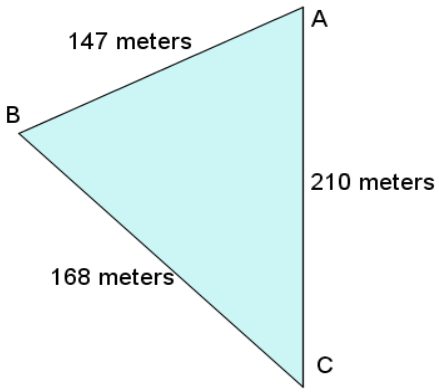
(a) 102,334,151 (b) 102,334,152 (c) 102,334,153 (d) 102,334,154 (e) 102,334,155

5. Determine the minimum value of

$|x - 1| + |x - 2| + |x - 3| + |x - 4| + |x - 5| + \ldots + |x - 10|$

(a) 23 (b) 25 (c) 27 (d) 29 (e) 31

6. The figure shows the dimensions of a triangular lake. The city development team wants to plant trees at each of the corners A, B, C, and around the lake shore with equal distance between neighboring trees. Determine the minimum number of trees required.



(a) 22 (b) 25 (c) 44 (d) 47 (e) 50

7. If the diagonals of a parallelogram are perpendicular then the parallelogram must be a

(a) square (b) rectangle, but not necessarily a square (c) trapezoid
(d) rhombus, but not necessarily a square (e) none of the above

8. Determine the approximate area of a regular hexagon each of whose sides have length one inch.

(a) 2.53 in^2 (b) 2.57 in^2 (c) 2.60 in^2 (d) 2.63 in^2 (e) 2.67 in^2

Part II:

9. A student rolled a fair die until the sum of her scores was a prime number. She had to roll three times. What is the probability that her last throw was a 6?

(a) 0 (b) 1/24 (c) 2/24 (d) 6/24 (e) 7/24

10. John and Jean take turns rolling an unfair (loaded) die, with Jean rolling first. If the probability of a result of K appearing on one roll of the die is given by K/21 (with K = 1, 2, 3, 4, 5, or 6) find the probability that Jean's roll is less than John's.

(a) ½ (b) 220/441 (c) 5/12 (d) 211/441 (e) 175/441

11. Let a piece-wise function be defined in the following manner:

$$f(x,y) = \begin{cases} 2y \text{ if } x = 0 \\ 0 \text{ if } x \geq 1 \text{ and } y = 0 \\ 2 \text{ if } x \geq 1 \text{ and } y = 1 \\ f(x-1, f(x,y-1)) \text{ if } x \geq 1 \text{ and } y \geq 2 \end{cases}$$

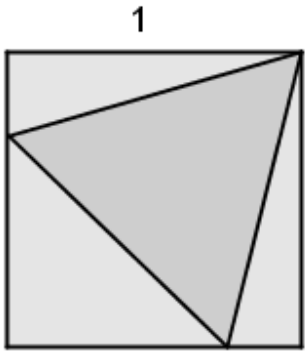
Find $f(1,3)$.

(a) 5 (b) 13 (c) 8 (d) 0 (e) 6

12. Eleven non-negative numbers have a mean of 30. What is the largest possible value for the median of this collection?

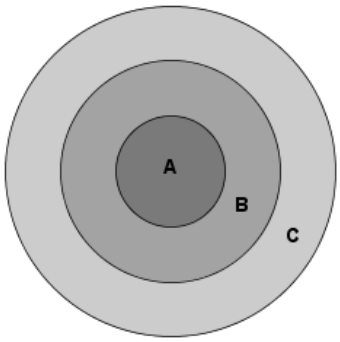
(a) 22 (b) 33 (c) 44 (d) 55 (e) 66

13. An equilateral triangle is to be inscribed inside a square with side length 1 inch. If one corner of the triangle coincides with a corner of the square, find the exact value of the length of one side of the triangle.



(a) $\sqrt{7-4\sqrt{2}}$ (b) $\sqrt{8-4\sqrt{3}}$ (c) $\sqrt{7-4\sqrt{3}}$ (d) $\sqrt{8-4\sqrt{2}}$ (e) $\sqrt{7-3\sqrt{3}}$

14. On a target (see the figure), points are assigned for hitting each of the areas A, B and C. If a player hits areas A and B, he gets 15 points. Similarly hitting A and C, and B and C the player gets 13, and 10 points respectively. Dave hit the target five times and got 31 points. How many times did Dave hit B?



(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

15. We write all positive integers on a single line, which starts as 123456789101112… We can see that number 11 first appears starting at the 12th symbol. At which earliest symbol in the above sequence would we be able to see number 435?

(a) at 435^{th} symbol (b) at 59^{th} symbol (c) at 42^{nd} symbol
(d) at 1194^{th} symbol (e) never

16. Simplify [n!+ (n-1)!][(n+1)! – n!]

(a) n! (b) (n-1)!(n+1)! (c) n!(n-1)! (d) $[n!]^2$ (e) n!(n+1)!

Part III:

17. Consider the alphabetic sequence made by repeating the 26 letters from A to Z. We group the letters starting from the first letter in such a way that the n^{th} group contains n many letters, that is,

(A), (BC), (DEF), (GHIJ), (KLMNO), (PQRSTU), (VWXYZAB), ····

Find the first letter in the 23^{rd} group.

(a) Q (b) R (c) S (d) T (e) U

18. A bowl contains ten red, six blue, and eight white balls. All of the balls are withdrawn one at a time without replacement. What is the probability that the color on the first ball withdrawn is the same as the color on the last ball?

(a) 22/69 (b) 1/3 (c) 5/16 (d) 179/552 (e) 7/24