

Mathematical Association of America
 Wisconsin Section
 Mathematics Contest Examination
 December 4, 2008

1. Do not open this booklet until you are directed to do so.
2. This is a multiple choice test. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

EXAMPLE:

If x is 3 and y is 4 then $2x - y$ is

- (a) -1 (b) 0 (c) 1 (d) 2 (e) none of these.

3. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.
4. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.
5. Fill in the following blank and wait for the signal to start the examination.

PRINT _____
 First Name Last Name

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____ x 4 = ____	____ x 1 = ____
2	8	____ x 8 = ____	____ x 2 = ____
3	2	____ x 12 = ____	____ x 3 = ____
Total	18	Sub-Total ____	Sub-Total ____

Score (Sum of both sub-totals) _____

Part I:

1. Let s be a perfect square with the tens digit being 9. What is the units digit?
a) 6 b) 1 or 6 c) 2 or 6 d) 4 or 6 e) 9 or 6
2. How many times must a standard die be thrown to have a better than $\frac{3}{4}$ chance of rolling at least one 6?
a) 6 b) 7 c) 8 d) 9 e) 10
3. How many integer coordinate points are there between (and including) the two endpoints of the line segment joining the origin to the point (150, 90)?
a) 29 b) 30 c) 89 d) 31 e) 90
4. Find the remainder when 2008^{2008} is divided by 5.
a) 0 b) 1 c) 2 d) 3 e) 4
5. Consider the sequence of numbers 2, 2, 4, 4, 4, 4, 6, 6, 6 ... where each even number appears its value number of times. Find the 2008th number in this sequence?
a) 45 b) 90 c) 96 d) 180 e) 88
6. Estimate the sum: $\ln(1 + \frac{1}{2}) + \ln(1 + \frac{1}{3}) + \ln(1 + \frac{1}{4}) + \ln(1 + \frac{1}{5}) + \dots + \ln(1 + \frac{1}{10000})$
a) 8.50 b) 8.51 c) 8.52 d) 8.53 e) 8.54
7. Consider the set of all numbers of the form $\frac{a}{|a|} + \frac{b}{|b|} + \frac{ab}{|ab|}$ that can be formed from two real numbers a and b . The set of all such numbers is
a) $\{-3, -1, 1, 3\}$ b) $\{-3, -1, 3\}$ c) $\{-1, 3\}$ d) $\{1, 3\}$ e) all real numbers

8. Let $u_0 = 2008$. For each integer $n \geq 0$ define u_{n+1} as

$$u_{n+1} = \begin{cases} \frac{1}{2}u_n & u_n \text{ is even} \\ u_n + 1 & u_n \text{ is odd} \end{cases} \quad \text{Find } u_{2009}.$$

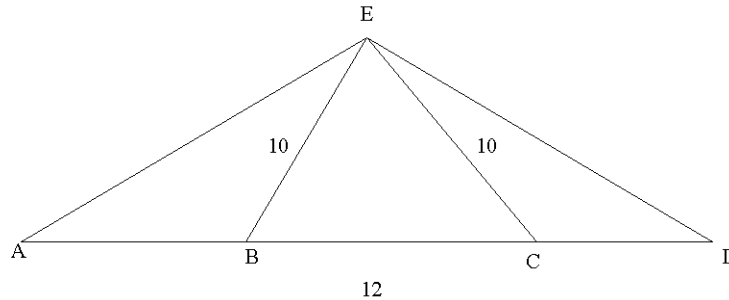
- a) 1 b) 2009 c) 2008 d) 2 e) 4

Part II:

9. If n is a positive integer, how many values can n have so that $n^2 + 231$ is a perfect square?

- a) 0 b) 1 c) 3 d) 4 e) 5

10. Points A, B, C, and D lie on a line, in that order, with $AB=CD$ and $BC=12$, $BE=CE=10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.



- a) 10 b) 12 c) 9 d) 8 e) 11

11. A pool is to be filled. It would take the first pipe 8 hours to fill the pool. It would take the second pipe 26 hours to fill the pool. The pool also has a drain that would take 13 hours to drain the pool completely. The first pipe is turned on and then, one hour later, the second pipe is turned on. An hour after that the drain is opened. How long will it take to fill the pool completely from when this process is started? (Round to the nearest minute)

- a) 8 hours 13 minutes b) 2 hours 51 minutes c) 11 hours 33 minutes
 d) 4 hours 57 minutes e) 10 hours 13 minutes

12. Let $0 < x < y < 1$. Let a and b be such that $x = \sin(a)$ and $y = \sin(b)$. Let $z = \sin(a+b)$ and assume $z > 0$. How many distinct positive ways can z be expressed in terms of x and y ?

- a) 1 b) 2 c) 3 d) 4 e) Impossible to tell.

13. How many integers bigger than 10 and not divisible by 10 have the property that if the final digit is deleted, the integer is divisible by the new number?

- a) 9 b) 17 c) 18 d) 23 e) 24

14. Let $0 < a < b$. The equation $\frac{1}{x} + \frac{1}{x+a} + \frac{1}{x-b} = 0$ has two solutions, where are they located on a number line?

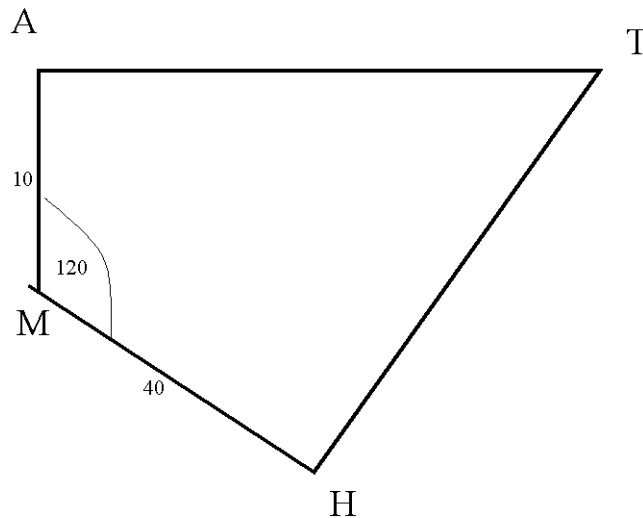
a) $\left(\frac{-2a}{3}, 0\right) \cup \left(0, \frac{2b}{3}\right)$ b) $\left(-a, \frac{-2a}{3}\right) \cup \left(\frac{2b}{3}, b\right)$ c) $\left(\frac{-a}{3}, 0\right) \cup \left(\frac{2b}{3}, b\right)$

d) $\left(-a, \frac{-2a}{3}\right) \cup \left(0, \frac{2b}{3}\right)$ e) not enough information given

15. What is the center digit of $20082008200820082008200820082008 \times 1111$?

- a) 0 b) 1 c) 2 d) 8 e) 3

16. Quadrilateral MATH contains right angles at vertices A and H. If angle AMH measures 120 degrees, MA=10 and MH=40, find the length of TH.



- a) 40 b) $20 + \frac{40\sqrt{3}}{3}$ c) $40\sqrt{3}$ d) $40 + \frac{20\sqrt{3}}{3}$ e) $20\sqrt{3}$

Part III:

17. Compute $\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \dots + \sqrt{1 + \frac{1}{2007^2} + \frac{1}{2008^2}}$

- a) 2008 b) $\frac{2007 \times 2009}{2008}$ c) $\frac{2007^2}{2008}$ d) $\frac{2007 \times 2008}{2009}$ e) none of the above

18. Three numbers form a geometric progression. If one subtracts 4 from the third number, then the sequence becomes an arithmetic sequence. From the resultant arithmetic sequence, if 1 is subtracted from the second and third terms of the new sequence, then the new progression is a geometric sequence. How many triples of numbers exist that satisfy this progression?

- a) 0 b) 1 c) 2 d) 3 e) infinitely many