

Mathematical Association of America  
 Wisconsin Section  
 Mathematics Contest Examination  
 December 6, 2007

1. Do not open this booklet until you are directed to do so.
2. This is a multiple choice test except for #18. Each multiple choice question has five possible answers, exactly one of which is correct. You are to circle the letter corresponding to the correct response on the answer sheet for as many problems as you can do in the 75 minutes allowed.

EXAMPLE:

If  $x$  is 3 and  $y$  is 4 then  $2x - y$  is

- (a) -1   (b) 0   (c) 1   **(d) 2**   (e) none of these.

3. Question #18 is a fill in the blank question. You are to write your answer on the line provided.
4. Use pencil or pen. A sheet of paper will be provided for your scratch work. Calculators may be used. Tables, books, notes, etc. may not be used.
5. The scoring system has been set up to give more credit in the long run for leaving a question unanswered than guessing rashly. On the other hand, whenever you can eliminate three possibilities, it is better to guess between the remaining two possibilities than to leave the question unanswered.
6. Fill in the following blank and wait for the signal to start the examination.

PRINT \_\_\_\_\_

First Name \_\_\_\_\_

Last Name \_\_\_\_\_

Your teacher will fill in the following blanks:

Part	Number of Questions	Number Right	Number Not Answered
1	8	____ x 4 = ____	____ x 1 = ____
2	8	____ x 8 = ____	____ x 2 = ____
3	2	____ x 12 = ____	____ x 3 = ____
Total	18	Sub-Total _____	Sub-Total _____

Score (Sum of both sub-totals) \_\_\_\_\_

## **Part I:**

1. In a certain town, exactly one-fifth of the population is dark-haired women and the same number is light-haired men. If exactly four-sevenths of the women have light colored hair and women have only light or dark colored hair, then which is a possible population of the town?

- a) 700      b) 800      c) 900      d) 1000      e) 1100

2. A watermelon weighs 100 pounds and is 99% water. After sitting in the sun for a few days, some of the water has evaporated leaving the melon 98% water. How much did the watermelon weigh after the evaporation?

- a) 90 lb      b) 50 lb      c) 95 lb      d) 99 lb      e) 60 lb

3. One day Matt started out in his new fuel-efficient car. For the first five minutes of his trip his average speed was 30 miles per hour and his car averaged 25 miles per gallon. For the next five minutes things improved; his average speed was 40 miles per hour and the car's fuel economy was 50 miles per gallon. What was the cars average miles per gallon over the entire 10-minute trip?

- a) 35 mpg      b)  $35\frac{1}{2}$  mpg      c)  $35\frac{5}{7}$  mpg      d)  $39\frac{2}{7}$  mpg      e) 36 mpg

4. The interior angles of a regular polygon are 12 times the (supplementary) exterior angles. How many sides does the polygon have?

- a) 20      b) 24      c) 26      d) 28      e) 30

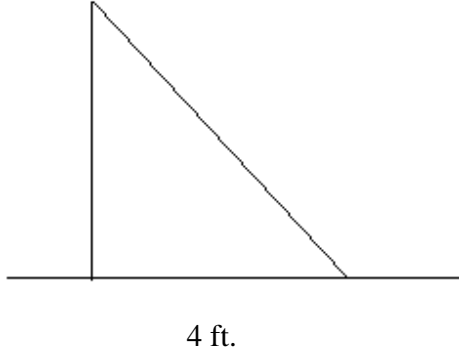
5. How many digits are in the base seven representation of  $2007$  to the  $2007^{\text{th}}$  power?

- a) 8447      b) 8474      c) 7484      d) 7844      e) none of the above

6. Three standard dice are rolled. The resulting numbers are all distinct, i.e., no two are matching. Find the probability that the sum of the numbers on the three dice is 8 or less.

- a)  $1/20$       b)  $1/12$       c)  $5/36$       d)  $24/196$       e)  $1/5$

7. A 12 foot stem of bamboo was growing perpendicular to the ground until it snapped and fell over. Its tip is now 4 feet from the base of the stem (see picture). How high up the stem did it break?



- a)  $16/3$  ft    b)  $4\sqrt{2}$  ft    c)  $3\sqrt{3}$  ft    d) 5.6569 ft    e) 5.1962 ft

8. How many real numbers have the property that the sum of the number and its multiplicative inverse equals the product of the number and its additive inverse?

- a) none    b) one    c) two    d) three    e) four

**Part II:**

9. Determine the ratio  $L/P$  where  $L$  is the least common multiple of the integers from one to one hundred and  $P$  is the product of all primes less than one hundred.

- a) 210    b) 6340    c) 30240    d) 103840    e) 6350400

10. Two cyclists, Jane and Jim, 12 miles apart start pedaling towards each other at 5mph each. At the same moment a fly sitting on Jim's bike takes off at 8mph towards Jane's bike. Once it reaches Jane, it turns around and flies towards Jim's bike, etc. How many miles will the fly cover before Jim and Jane meet?

- a) 19.2 miles    b) 7.38 miles    c) 3.75 miles    d) 9.6 miles    e) none of these

11. What is the smallest number of people, chosen at random, that you would need to gather together to guarantee that 3 of them know each other or 3 of them are strangers?

- a) 3    b) 4    c) 5    d) 6    e) 7

12. An isosceles trapezoid has perpendicular diagonals and parallel sides of lengths 1 and 2. Find the area of the trapezoid.

- a) 2.25      b) 2.2498      c)  $2\sqrt{2}$       d)  $\sqrt{2}$       e) 2.2499

13. Suppose  $f(x) = x/(x + 1)$ . What is the domain of real numbers for the following composition of functions:  $f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f$

- a) all real numbers      b) all nonzero real numbers      c) all real numbers except -1  
d) all real numbers except -1/8      e) none of these

14. A sequence is formed by the sequence of primes with each prime appearing in order the same number of times as the prime - i.e.  $a_1 = a_2 = 2$ ,  $a_3 = a_4 = a_5 = 3$ ,  $a_6 = \dots = a_{10} = 5$ , etc. Find the term number (subscript) of the tenth appearance of the first prime  $\geq 50$ .

- a) 338      b) 319      c) 53      d) 63      e) 1441

15. A sequence is formed in the following way:  $a_1=7$  and for  $n \geq 1$ ,  $a_{n+1}$  is the greatest prime factor of  $1+a_n$ . What is the least common multiple of the terms in the sequence?

- a) 42      b) 70      c) 210      d) 105      e) impossible to calculate

16. Three hoses are filling a swimming pool. Hose B alone fills the pool in half the time it takes hose A alone. Hose C alone takes  $12\frac{3}{4}$  hours longer than hose B alone. Together all three hoses A, B, and C take 10 hours. How many hours would it take hoses A and C together? (Round to two decimal places)

- a) 10.69      b) 18.89      c) 50.25      d) 2.5      e) 20.25

### **Part III:**

17. A quadrilateral with sides 2, 3, 5, 6 is inscribed in a circle. Let  $x$  denote the diagonal that makes triangles with sides 2, 3,  $x$  and  $x$ , 5, 6. Find the value of  $x^2$ .

- a) 21      b) 18      c) 19      d) 20      e) 22

18. An integer has 5 as its last digit. If we multiply it by 5, the last digit moves up to the front of the number with the rest of the digits shifting without change. Find the last 6 digits of the original number.

Answer: \_\_\_\_\_