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SURPRISING EFFECTS OF INQUIRY BASED LEARNING

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ABSTRACT. This paper will compare and contrast the performance to students exposed to two different methods of teaching a vector calculus class. One calculus class was taught using traditional lecture methods, while the other was taught via a modified Moore method. While the sample sizes were small, the differences between the two populations will be discussed, as will differences between the performance of students who continued into the "Introduction to Proofs" course from the Calculus III courses mentioned above.

1. INTRODUCTION AND DEMOGRAPHICS

This paper will compare two sections of Calculus III taught by the author. The first section was taught in a fairly traditional manner during the Fall 2003 semester. The second section was taught in a Modified Moore Method (MMM) approach during the Fall 2004 semester. According to Peter Renz [4] the Moore method is described in the following way: "Motivate what is to be done. Let the students Discover how to do it. Have the students Present their results in good order before a critical but friendly audience." While both of the classes were fairly small, making a comparison difficult, we can discuss the lack of both quantitative and qualitative differences in the two groups as demonstrated by answers to similar questions on the final exams.

The Calculus III class cover topics including vectors in two- and three-dimensions and vector-valued functions. The topics in the course are largely procedural although students are starting to see more proofs than they have in the first two calculus courses. The goal of the class is to get students to an investigation of line integrals culminating with the statements of Green's theorem. This goal was not reached with the Fall 2003 section, but Green's theorem was covered on the last class-day in the Fall 2004 section.

In order to be enrolled in Calculus III, students must have completed Calculus II, which includes the study of integral calculus and convergence of series, with a grade of C or better. Of the sixteen students enrolled in the course in the Fall of 2003, four students had retaken Calculus II to raise their grade to meet the prerequisite. After having done that, three had earned A's in Calculus II, seven had earned B's and six had earned C's. The Fall 2004 section had three students who had retaken Calculus II to meet the prerequisites and one student who was exempt from the prerequisites. After having done that, one student had earned an A in Calculus II, seven had earned B's and six had earned C's.

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FIGURE 1. Demographics of the Two Classes

	Fall 2003 Traditional	Fall 2004 MMM
Gender		
Male	4	6
Female	12	9
Grade Distribution from Calc II		
Retakes	4	3
A	3	1
B	7	7
C	6	6
Class Standing		
Freshman	1	1
Sophomore	5	5
Junior	5	4
Senior	4	3

Of the sixteen students enrolled during the Fall 2003 semester, twelve were women and four were men. Of those enrolled in the Fall 2003 section, 15 completed course evaluations. Of these fifteen, one was a freshman, five were sophomores, five were juniors, and four were seniors. Of the fifteen students enrolled during the Fall 2004 semester, six were men and nine were women. Of those enrolled in the Fall 2004 section, thirteen completed course evaluations. Of these thirteen, one was a a high school student, five were sophomores, four were juniors, and three were seniors. A summary of this information is available in Figure 1.

The students enrolled in the two sections are very similar in all demographic categories. Therefore it is not expected for there to be innate differences between the two sections.

2. Design of the Fall 2003 Course

This section was run primarily as a traditional lecture-oriented class. The course met five hours per week, on Monday through Thursday for a fifteen week semester. At least three days per week were spent on professor lecture. Most weeks, one day was devoted to allowing students to present their solutions to more challenging homework problems. Quizzes were given twice per week to allow students to test their mastery of material covered and provide them with feedback often.

Student presentations were described in the course syllabus in the following way: "Students will be expected to solve problems at the board on Mondays. This will be on a volunteer basis, with preference going to students who have presented the fewest solutions. In the event of a tie, the professor will randomly select the student to present. During student presentations, the rest of the class is encouraged to ask questions, and to think critically about the solution presented by the classmate."

There were three exams given during the semester, and a cumulative final exam. The course grade was weighted 10% presentations, 15% quiz average, 15% for each of the three mid-term exams, and 30% for the final. A comparison of the grade weighting between this and the Fall 2004 course is available in Figure 2.

FIGURE 2. Weight of Assignments Between the Two Classes

Fall 2003 Traditional		Fall 2004 Modified Moore Method	
Presentations	10%	Presentations	30%
Quizzes	15%	Quizzes	10%
Exam 1	15%	Exam 1	15%
Exam 2	15%	Exam 2	15%
Exam 3	15%		
Final Exam	30%	Final Exam	30%

3. DESIGN OF THE FALL 2004 COURSE

For this section, quizzes were also given twice per week, on Tuesday and Thursday. There were only two exams given during the semester, plus the cumulative final exam. The course grade was weighted as follows: quizzes 10%, presentations 30%, 15% for each of the two mid-term exams, and 30% for the final exam.

The primary difference from the previously described course is that most material in the course was presented by students. The students were allowed access to a textbook, which was the same book as used during the section described above. However, students were not allowed to talk to anyone except for the instructor about exercises which had not yet been presented in class.

Student presentations were infrequently supplemented by "mini-lectures," usually given when there was a new topic to be covered, or when many of the students expressed difficulties with the next part of the problem set. For example, a mini-lecture was given to introduce vectors at the beginning of the semester, and a mini-lecture was given on polar coordinates and polar equations. As the semester continued, mini-lectures were given less often.

Students were expected to present accurate solutions to the problem; be able to defend their work; and when they were not presenting, they were to provide constructive criticism of their classmates' work. They were awarded points in each of the following categories: a "Presentation" point (worth five times as much as a point in any other category) for presenting the solution to a problem, a "Question" point for asking a good question of the presenter, a "Contribution" point for any oral contribution other than the two categories above, and an "Insight" point for contributing any comment which demonstrates mathematical insight into the presentation or question asked.

During this process, problems were presented in order, and presenters were selected on a volunteer basis. To deal with the fact that students were frequently not prepared to present anything, students were required to e-mail (or announce before class) when they had solved a problem. Presenters were then selected as mentioned above.

The primary job of the professor during these sessions was to clarify and restate important points presented by the students, and to moderate comments from the students in the audience. It was up to the professor to decide when the presenter was fundamentally stuck and offer the student the chance of forfeiting the problem or continuing the presentation the next day.

Frequently as the semester progressed, a student would ask a question after a presentation, which would be added as a "Prove or Disprove" exercise to be

FIGURE 3. A Comparison of Calculus III Performance

Quizzes		**Presentations**		**Final Exam**		**Course Grade**	
Fall 2003	Fall 2004	Fall 2003	Fall 2004	Fall 2003	Fall 2004	Fall 2003	Fall 2004
32	18	0	12	7	32	27	15
52	27	4	22	38	43	57	38
56	44	21	26	53	52	59	48
64	49	23	33	56	53	60	56
65	49	30	41	59	53	60	57
65	50	32	50	59	55	61	60
66	53	32	53	60	59	65	61
67	66	39	54	62	62	65	61
72	74	39	57	71	68	66	64
73	76	43	71	72	80	68	67
73	85	45	80	78	86	68	88
76	89	52	83	80	90	74	94
76	89	52	83	82	93	77	97
81	91	58	92	82	100	77	97
83	96	95	100	82	did not take	79	106
84		100			85	82	
P-value	0.4325		0.0548		0.4483		0.3783

discussed the next class day. Some of the most interesting ideas of the course developed this way, as students began to ask questions of themselves and each other.

4. Comparative Statistics

The performance of the two groups was compared using non-parametric hypothesis testing, considering the students in the traditional group to be the control group, and students in the MMM group to be the experimental group. The test was run with with null hypothesis being that the students in the MMM group would perform at the same levels as the students in the traditional course, with alternate hypothesis being that the MMM students would perform at a higher level. The quiz averages, final exam scores, presentation scores (where the students were awarded grades based on the total number of points that they earned as a percentage of the maximum points earned by any student), and final course average were compared. There were no statistically significant results. See Figure 3.

Since the mathematics taught in Calculus III is largely procedural, it is not surprising that the MMM group did not show a greater mastery of the skills. The Moore method was designed originally to be used in upper and graduate level mathematics courses. These courses are more conceptually based, and the Moore method has been seen to be a valuable asset in these courses.

We also note that because of the small sample sizes, it is more interesting to examine qualitative differences between the traditional class and the MMM class, both in terms of their performance on the final exam and their reactions to the class, as demonstrated by their comments on student evaluations.

5. Performance on the Final Exam

While there are differences between the topics emphasized on the two final exams, there were some common topics, and it is worthwhile to compare performance on these pairs of questions between the traditional section and the MMM section. Each exam contained the following types of questions

- state the requested definition or theorem
- a application problem about projectile motion,
- a problem about absolute (local) maxima, minima and saddle points,

FIGURE 4. A comparison of final exam questions

Topic	Average percentage of points earned		P-value
	Fall 2003	**Fall 2004**	
Definitions	48.75	79.78	0.0024
Projectile motion	70.31	72.85	0.4286
Maxima and Minima	69.79	66.43	0.2783
Equation of a plane	64.84	68.57	0.4207
Jacobian	50	42.85	0.1762
Volumes	68.75	72.14	0.3557
Limit	78.125	89.28	0.1314

- a problem asking for the equation of a plane satisfying certain criteria,
- a problem utilizing the Jacobian,
- problems about three-dimensional volumes, and
- a calculation of a limit in a function of more than one variable.

Notice that all of these problems are procedural in scope. The application problem about projectile motion required the students to gather the necessary information from a word problem, but then to apply the appropriate formula to arrive at an answer. The question about maxima and minima and saddle points asked the students to take partial derivatives and use the second derivative test to analyze the extrema that they located. The problems asking for the equation of a plane required using given information to arrive at the requested equation. The Jacobian problems required that the students know what the Jacobian of a transformation was, and use that definition to translate perform a change of coordinates. Clearly, calculating a limit is a procedural exercise. As mentioned above, the Moore method was not developed to encourage procedural learning, so it would be surprising to find differences in performance on these items.

There were sixteen students who completed the final exam from the Fall 2003 section, and fourteen from the Fall 2004 section. The average percentage of points earned on these problems is summarized in Figure 4. Again, using non-parametric statistical methods, the only statistically significant difference was the MMM students scored significantly better on the definition portion of the exam, with $P = 0.0024$.

It is also informative to consider the types of mistakes made by students on the final exam questions. A careful examination of the final exams allow the answers to the common questions to be evaluated according to the following rubric:

(1) All points earned or minor error made - the student demonstrated the conceptual knowledge required for the problem, but may have made a procedural mistake
(2) Did not complete the problem (no mistakes made, but stopped work on that problem), major error made (indicating a lack of conceptual or procedural knowledge), or skipped (the student did not even attempt the problem)

The first category contains the students who demonstrated an understanding of conceptual and procedural knowledge being tested by that item. The second category indicates that the student lacked conceptual and/or procedural knowledge of the material being examined by the particular question. The number of students from each class making each type of error on comparable final exam questions is

FIGURE 5. Breakdown of Types of Errors Made on Final Exams

Error	Proj.	Limit	Plane	Max/Min	Vol.	Jacobian
Fall 2003						
All points earned or Minor error	10	13	10	10	8	8
Did not complete or Major error or Skipped	6	3	6	6	8	8
Fall 2004						
Error	**Proj.**	**Limit**	**Plane**	**Max/Min**	**Vol.**	**Jacobian**
All points earned or Minor error	11	12	9	4	9	5
Did not complete or Major error or Skipped	3	2	5	10	5	9
χ^2	1.5429	0.1831	0.9714	6.876	1.1429	1.1429

summarized in Figure 5. A statistical comparison of the numbers of students in each category, using the χ^2-statistic, could not detect any differences between the type of mistakes made the traditional students and the MMM students. In particular, if given an exam at random, it seems unlikely to be able to guess whether the paper was from a student in the traditional group or the MMM group.

Therefore, in addition to there being no quantitative differences between the two groups, there is also no qualitative difference between the groups on the procedural tasks required of students on the final exam.

6. Students' Evaluation of the Course

As mentioned above, during the Fall 2003 semester, fifteen students completed course evaluations. In the Fall 2004 semester, thirteen students completed course evaluations. The tone of the two sets of evaluations is extremely different, with the students from the Fall 2003 semester commenting primarily on the professor. A sample of the comments are the following:

"Enjoy your class and like your concern for us as students. Hard material but you explain it well. Thanks for caring."

"... is a wonderful professor in + out of the classroom. She teaches well, but this course is more difficult than average."

"I really enjoy [this] class...She is willing to work w/ the students & her teaching style presents the info in a way that is understandable."

"She explains things clearly..."

Notice that the Fall 2003 evaluations do not mention the presentations at all. As a matter of fact, they do not mention any aspect of the course format. In contrast, none of the course evaluations from the Fall 2004 course referred to the professor. All written comments were about the format of the course. As G.E. Parker [3] noted

"Unless a teacher works in a department where most of the courses are given by the Moore method..., the teacher should be aware that there is likely to be considerable student reaction to the differences between the Moore-method classroom and what the student has likely experience before." A sample of the comments received from the Fall 2004 students includes the following:

"The Moore method was hard to deal with...I think everyone would have appreciated a few more lectures, but I loved this class."

"The Moore method is very hard to do when you have a full load of classes. I think the class would benefit with having lecture on two days + student presentations on the other two days. I believe she would be a very good prof. if she actually taught."

"The problem that I have with this class is that so much work is needed outside of class that it is difficult for me to get my work done for other classes. Also, I do not feel that the work I am doing at home on my own is having much, if any, effect on how well I understand the material. Basically, I love this class, I just do no the think that the set-up of the course allows me to have as great of a return for the work that I am doing."

"I learn a lot more when you teach us..."

"Enjoy learning the material but often is frustrating. Course format requires visiting professor frequently, but that is hard based my schedule"

"I think this course was made harder than it had to be because of the Moore Method. I am learning a lot but don't feel that it is helping me learn more than if I had been lectured to..."

The students did not seem to feel that their mastery of the material was improved by the format, which is backed up by the statistical data. The format, therefore, seemed to be more challenging than the format in a typical mathematics course. However, one student commented that they would be bored if the course had developed into a lecture-style course as the semester continued, and were glad to have the interactive class format.

7. Performance in Future Courses

Many students who take Calculus III in the fall semester enroll in "Introduction to Mathematical Thought", the first course on proofs, in the spring semester. Nine students progressed from the traditional Calculus III section to this transition course, and eight students who were in the MMM section enrolled in "Introduction to Mathematical Thought" in Spring 2005. During both spring semesters, the author taught this first course in proof techniques. In Figure 6, you see how the students preformed in the "Mathematical Thought" course after completing the standard Calculus III course, and after completing the MMM Calculus III course. During Spring 2004 the students handed in homework, which was graded. During the Spring 2005 semester, this requirement was replaced by weekly quizzes. Students were allowed to rewrite quizzes, for which students could earn back up to

FIGURE 6. Performance in a "Introduction to Mathematical Thought" (grades listed in descending order)

Traditional Calculus, Intro. to Math Thought Spring 2004					
Grade in Calc III	**Grades in Intro. Math. Thought**				
	HW	**Exam 1**	**Exam 2**	**Final Exam**	**Course Grade**
B	75	79	56	49	59
C	73	77	48	47	57
C	69	69	46	45	47
C	32	59	40	27	37
C	30	55	40	22	32
C	13	52	35	no exam	17.25
C	11	39	12	Quit	Quit
C	7.5	Quit	Quit	Quit	Quit
F	6	Quit	Quit	Quit	Quit
Average	35	61	39	38	49

MMM Calculus, Intro. to Math Thought Spring 2005					
Grade in Calc III	**Grades in Intro. Math. Thought**				
	Quiz	**Exam 1**	**Exam 2**	**Final Exam**	**Course Grade**
A	99	96	92	95	94
A	97	94	90	87	90
A	96	91	89	84	89
A	95	91	79	83	89
C	90	80	79	79	73
C	79	65	60	75	71
C	75	62	57	59	71
C	63	59	43	55	60
Average	86.75	79	85	77	79
$P-$value	0.0192	0.0162	0.0009	0.0018	0.0015

half of the points they lost when they took the quiz during class. This did increase the scores on these assignments. The homework and quiz averages are, however, considered to be comparable, since the students had access to the professor for help on homework assignments.

Again using a non-parametric statistical comparison of these two groups with null and alternate hypotheses as above, there is a statistically significant difference between these two groups in each of the categories examined. Though there does not seem to be a large difference in the performance of the two groups in Calculus III, there is a strong indication that the students who have taken the MMM Calculus III class are better prepared for "Introduction to Mathematical Thought". The students in the MMM Calculus III course learned to evaluate their explanations, and critique solutions proposed by other students. Since "Introduction to Mathematical Thought" is a conceptual course, where each problem needs to be freshly analyzed and evaluated, it would be expected that students in this course would react strongly to the Moore method. It seems that having exposure to the Moore method in a previous course has made the students' proof-writing ability stronger and their critical thinking skills more developed than other students who have never been exposed to that teaching strategy.

The MMM students, as they attested in their course evaluations, are also prepared to put more time into the course than students who are used to traditional mathematics courses. It seems that this extra work-ethic, instilled by making them responsible for the material has prepared them more strongly for the next level of mathematics.

8. Conclusions and Open Questions

The MMM students seem to have a better grasp of definitions, and have developed an ability to memorize and understand definitions in a different way from non-Moore method students. This is evident from their final exams. That seems to serve them well in higher mathematics courses. It would be interesting to see what other skills become evident as they enroll in more advanced mathematics courses.

It also seems clear that there should be some additional differences in the populations, and further study is necessary to find the areas affected by the change in classroom style. Will the MMM students excel beyond others who have more traditional lecture-style backgrounds? Have they learned proof-writing and critical thinking in a different way? Is there a better long-term retention of the material in the MMM course? All of these are questions that should be addressed in future projects.

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