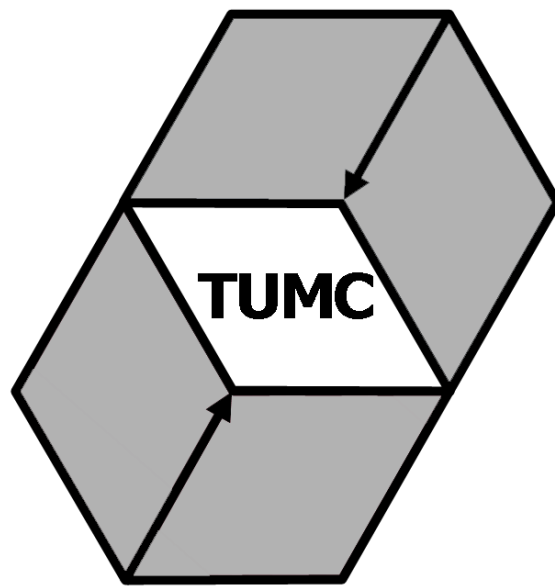


10TH ANNUAL TEXAS UNDERGRADUATE MATHEMATICS CONFERENCE



OCTOBER 17-18, 2014
STEPHEN F. AUSTIN STATE UNIVERSITY
NACOGDOCHES, TEXAS

Funding for TUMC is provided by

- NSF grant DMS-0846477 through the MAA Regional Undergraduate Mathematics Conferences program, www.maa.org/RUMC.
- Stephen F. Austin State University, College of Sciences and Mathematics, Department of Mathematics and Statistics, and STEM Research and Learning Center

Schedule of Events

Friday, October 17, 2014

6:00-7:30 PM	Registration	1st Floor Lobby, Math Building
6:30-7:30 PM	Problem Solving Puzzles	101 Math Building

Saturday, October 18, 2014

8:00-9:00 AM	Registration/Breakfast	1st Floor Lobby, Math Building
9:00-9:55 AM	Contributed Presentations	206, 208, 210, 212 Math Building
10:00-10:55 AM	Panel Discussion: What Can I Do With a Math Degree?	101 Math Building
11:00-12:15 PM	Welcome and Keynote	101 Math Building
12:15-1:30 PM	Lunch	2nd Floor, Math Building
1:30-3:05 PM	Contributed Presentations	206, 208, 210, 212 Math Building
3:15-4:15	Invited Address	101 Math Building
4:15-4:30	Closing Remarks	101 Math Building

Texas NExT: 123 Math Building

Keynote and Invited Addresses

Keynote Speaker: Dr. TJ Hitchman, University of Northern Iowa

Invited Address: Dr. Amy Finkbiner, Vencore

Title: Shorter, Rounder, or more Evenly Bent?

Abstract: Drop a rubber band on a table. Did you get a circle, or did you get a bunch of loops? How much like a circle is your shape? Can you find a way to measure how much your rubber band fails to be circular, and use this to "fix" it? We will discuss a family of questions like these, a process called Curve Shortening Flow, and explore how to make sense of things if we consider polygons instead of smooth loops.

Title: Mathematical Modeling in Healthcare

Abstract: Mathematical epidemiology models the progress of infectious diseases using differential equations. We discuss a basic model that can determine how quickly a disease will spread and whether it will die out over time. We then investigate a real-world question regarding a common disease, rotavirus, for which a more complex model is necessary. As time permits, we will also highlight mathematical applications to other types of healthcare problems.

Discussion Panel Members:

Dr. Matt Beauregard, Stephen F. Austin State University

Dr. Lynn Greenleaf, Stephen F. Austin State University

Dr. Amy Finkbiner, Vencore

Dr. Theresa Martines, University of the Incarnate Word

Schedule of Talks — Morning Session

	Room 206 Algebra	Room 208 Analysis, Differential Equations, Geometry	Room 210 Graph Theory	Room 212 Probability and/or Statistics Topology
9:00-9:15	Brittany Cashi Lamar University Uses of Elliptic Curves in Cryptography	Mathew Gomez University of North Texas at Dallas Maximal Contraction Ratio and Hausdorff Dimension of Sierpinski Snowflakes	Annie Phan El Centro College Yi-Ching and Numerical Systems	Matthew Miller Southwestern University Scoring Cardiac Health: A Model of the Relationship between Diet and the Risk of Cardiovascular Disease
9:20-9:35	Alireza Rafiei Santa Monica College Introductory survey of Quaternions: their algebra and applica- tions	Benjamin Shaw Hardin-Simmons University Exploring the Hyperbolic Trigonometric Functions	Robert Lehr Southwestern University Noisy Clustering Analysis of Landfill Distribution	Audrene Edwards Lamar University Shakin' Things Up: Using the Statistical Approach to Model Natural Disasters
9:40-9:55	Heather Gronewald Southwestern University Computing Cophylogenetic Invariants	Soufiane Abbadi Richland College Newton's method and approximating the solu- tions of the transcenden- tal equations along with the properties of the cha- otic behavior.	Jonathan Hodges Lamar University The G-Graph of a Group	Mark Lugo University of North Texas at Dallas Equivalence Class of Virtual Knot Theory

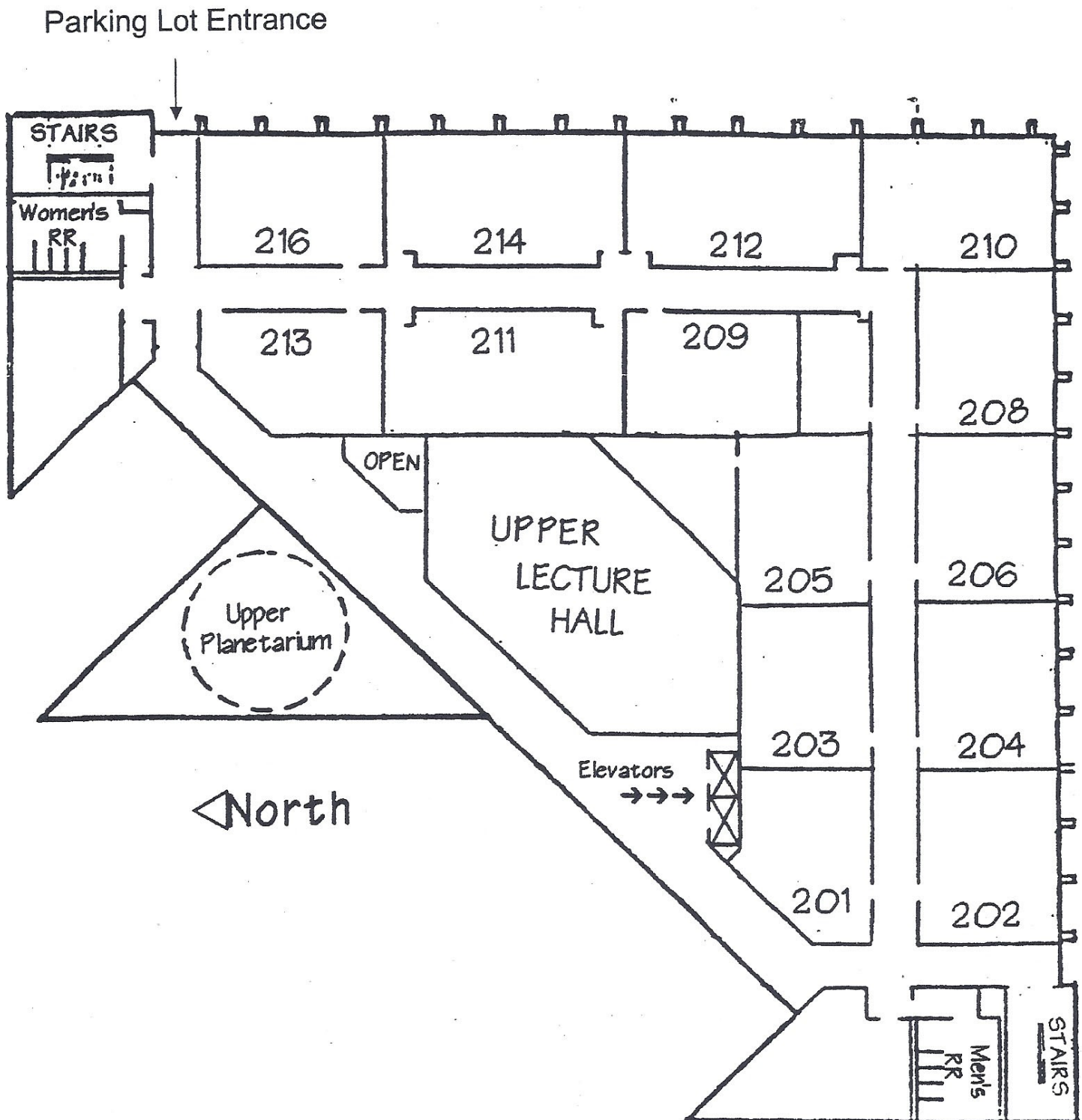
Schedule of Talks — Afternoon Session

	Room 206 Graph Theory Algebra	Room 208 Analysis, Differential Equations, Education	Room 210 Topology	Room 212 Probability and/or Statistics
1:30-1:45	Jillian Parker Sam Houston State University Omega Values of the Generators of Certain Primitive Numerical Monoids	Erica Johnson University of the Incarnate Word Soliton Solutions Associated with the Korteweg de Vries Equation	Thoa Doan & Richard Kassanga Odessa College A Study of Unknotting Number	Matthew Chaltain Southwestern University Earnings Surprise Effect on Stocks
1:50-2:05	Chris York Lamar University Enumerating k th Roots in the Symmetric Inverse Monoid	Jonathan Gutierrez St. Mary's University Modeling Fluid Flow Induced by <i>C. elegans</i> Swimming at Low Reynolds Number	Sarah Renfro Sam Houston State University Pseudoknot Graph Invariants	Emileigh Willems & Jessica Hall Wayland Baptist University Variable Selection on NuVal Scores
2:10-2:25	Cody L Worth Lamar University Introduction to Quantum Computa- tion	Anthony Thomas University of the Incarnate Word Competition Dynamics of Three Strains of the Bacteria That Causes Gonorrhea	Javier Mondragon University of North Texas at Dallas Knot Permutation of Alternative Prime Knot	Megan Myers Southwestern University A Place for All People: Planning for Growth in Summer Camp Enroll- ment
2:30-2:45	Abigail Edgar St. Edward's University Agreement Proportion in Box Societies	Lindsay Hixon Sam Houston State University Statistics Content in Elementary Math Textbooks	Delia Rojas University of North Texas at Dallas Invariance of Virtual Involutory Quadle	
2:50-3:05	Aaron McCoy & Nicolas Nikoloutsos Lamar University Modular Arithmetic Patterns in Pascal's Triangle	Dr. Stephen T Salako Our Lady of the Lake University Image Registration and Applications	Dr. Noreen Khan University of North Texas at Dallas Linking number of Virtual Links	

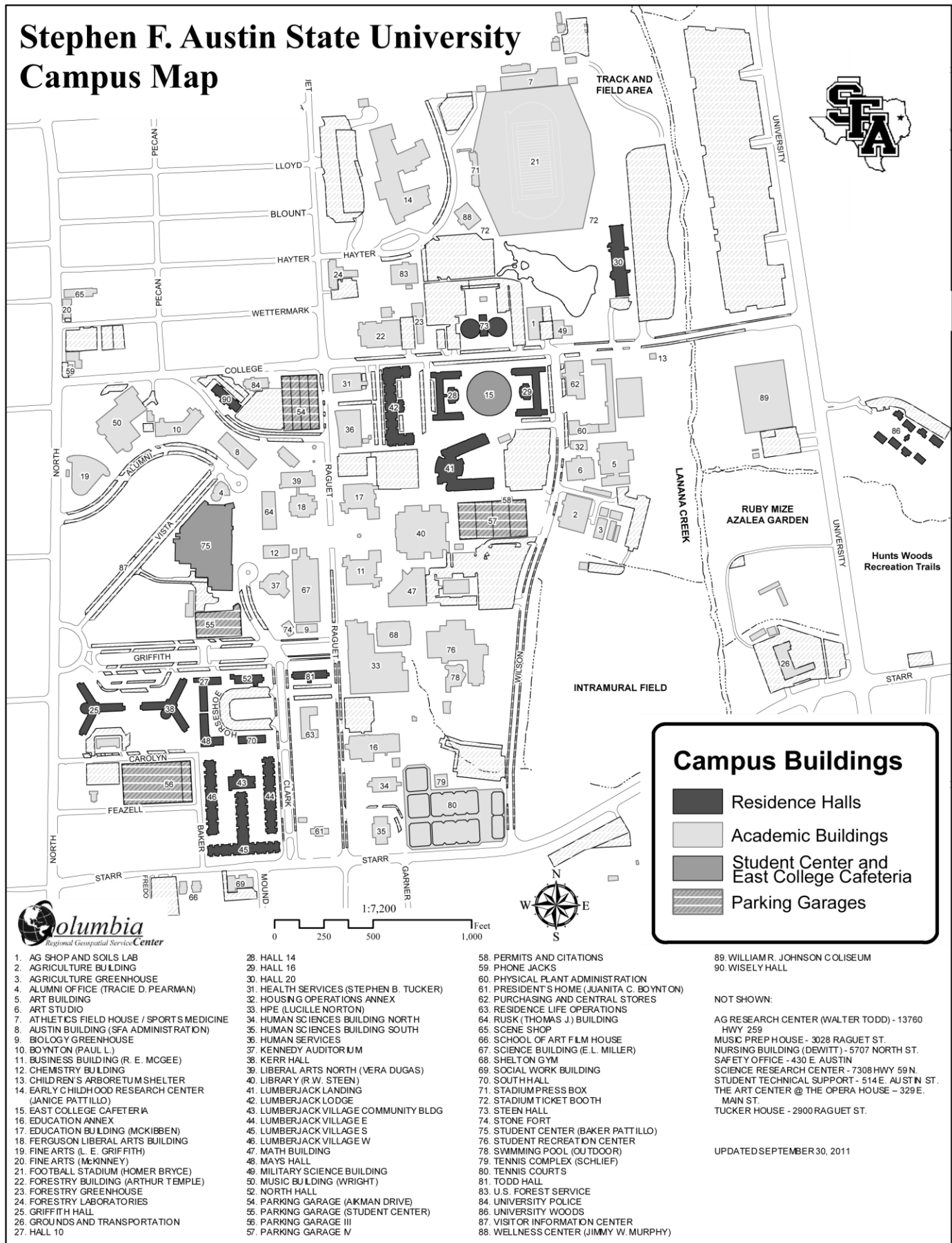
Map — Math Building 1st Floor



Map — Math Building 2nd Floor



Stephen F. Austin State University Campus Map



Abstracts

Algebra

Brittany Cashi, Lamar University

Uses of Elliptic Curves in Cryptography

An elliptic curve is a smooth projective curve of genus 1 with a distinguished point. These curves can help us when looking at Fermat's last theorem, factoring integers, primality proving and most importantly in my research cryptography. The Diffie-Hellman key exchange: Diffie and Hellman proposed a method for two parties to establish a secret key over a public network, based on the discrete log problem. Their method is generic, it works in a cyclic subgroup of any given group. The elliptic curves will help in solving the discrete log problem and finding the best and most secure ways of using public and private keys.

Heather Gronewald, Southwestern University

Computing Cophylogenetic Invariants

Phylogenetics is the study of evolutionary relationships among organisms. One method for reconstructing phylogenetic trees involves establishing phylogenetic invariants (polynomial relationships that vanish when expected pattern frequencies are substituted for variables) and then comparing expected and observed invariants. Cophylogeny is the study of concomitantly evolving organisms (e.g. a host and parasite). Huggins, Owen, and Yoshida suggest that existing methods for reconstructing host and parasite trees can exaggerate true differences between trees. We consider both the independent and pairwise reconstruction of trees, establishing classes of metrics for which independently inferred trees match pairwise inferred trees, and exploring methods for reconstruction that consider the cophylogenetic relationship.

Jillian Parker, Sam Houston State University

Omega Values of the Generators of Certain Primitive Numerical Monoids

Let M be a commutative, cancellative, atomic monoid with units $M^\times$ and atoms (or irreducibles) $\mathbb{A}(M)$. For $x \in M \setminus M^\times$, we define the omega function by $\omega(x) = n$ if n is the smallest positive integer such that if $x \mid a_1 \dots a_t$ with each $a_i \in \mathbb{A}(M)$, then there is a $T \subseteq \{1, \dots, t\}$ with $|T| \leq n$ such that $x \mid \prod_{k \in T} a_k$. Moreover, the ω -function measures how close to prime an element is. We will conjecture simple formulas for determining these omega values of a primitive numerical monoid in any embedding dimension, where the set S is generated by a generalized arithmetic sequence of the form $\langle a, ah + d, ah + 2d, \dots, ah + xd \rangle$ where a, d, h and x are positive integers and $\gcd(a, d) = 1$. We show by applying a theorem by Omidali and Rahmati that these results are valid and enhance the understanding of generators of certain primitive numerical monoids.

Alireza Rafiei, Santa Monica College

Introductory survey of Quaternions: their algebra and applications

$\mathbb{R}$ is not an algebraically closed field, therefore one would be able to write a polynomial equation that does not have a solution in $\mathbb{R}$ e.g. $x^2 = -1$. It is possible to generalize the algebra over $\mathbb{R}$ in a way (i.e. $\mathbb{C}$ that is two-dimensional) for the equation to have a solution and in that sense, there would be no polynomial equation that does not have a solution in $\mathbb{C}$ since $\mathbb{C}$ is algebraically closed. However, the process of generalizing $\mathbb{R}$ may not be dependent to the need of solving polynomial equations and one is able to derive algebras over $\mathbb{R}$ with arbitrary large dimensions. One process to generalize $\mathbb{R}$ that also results in $\mathbb{C}$ is "Cayley-Dickson construction" which, in this talk, will be briefly discussed. Then the focus will be on the four-dimensional generalization of $\mathbb{R}$ (i.e. Quaternions $\mathbb{Q}$

that is one level above the two dimensional $\mathbb{C}$). At the end, one of the applications of Quaternions that is regularly used in Computer Science will be explained (i.e. How they can be used to rotate 3D objects instead of Rotation matrices). The prerequisites for this talk is basic familiarity with complex numbers for the algebra and basic Linear Algebra for the applied part.

Cody L Worth, Lamar University

Introduction to Quantum Computation

In my talk, I am going to discuss the basics of quantum computation. I will introduce qubits, measurement of qubits and the mortality of Schrodinger's cat.

Chris York, Lamar University

Enumerating k th Roots in the Symmetric Inverse Monoid

The symmetric inverse monoid, $SIM(n)$, is the set of all partial one-to-one mappings from the set $\{1, 2, \dots, n\}$ onto itself under the operation of composition. Elements of $SIM(n)$ can be expressed as the product of disjoint paths and cycles, much like elements of the symmetric group. Raising a member of $SIM(n)$ to a power k means applying the mapping onto itself k times. Earlier research on the symmetric inverse monoid delineated the process for determining whether an element of $SIM(n)$ has a k th root, an element such that when raised to the k th power yields the element. The problem of developing formulas for the number of k th roots of a given element of $SIM(n)$ has since been posed, which this research aims to solve. These formulas are developed through heavy use of combinatorics and, interestingly, integer partitions and validated through computer simulations. Through this, it is shown that topics such as $SIM(n)$ can be connected to other mathematical topics such as integer partitions.

Analysis, Calculus, and/or Differential Equations

Soufiane Abbadi, Richland College

Newton's method and approximating the solutions of the transcendental equations along with the properties of the chaotic behavior.

Newton's method is the process which we can find roots of a function whose graphs cross or just touch the x-axis. Newton's method is described in terms of root-finding, but it can also be understood as maximizing a local quadratic approximation to the objective function. In addition, Newton's method can be used to approximate the solutions of the transcendental equations. Finally, Newton's method can exhibit chaotic behavior, and since it's relatively easy to use, it's often used as a way to study the properties of the chaotic behavior.

Jonathan Gutierrez, St. Mary's University

*Modeling Fluid Flow Induced by *C. elegans* Swimming at Low Reynolds Number*

C. elegans have been extensively researched regarding locomotion. However, most mathematical studies have focused on body dynamics rather than the fluid. As the nematodes undulate in a sinusoidal fashion, they cause fluid movement that has been studied experimentally but not modeled computationally on this scale. Utilizing the Navier-Stokes equation, regularized stokeslets, and the method of images, we computed the dynamics of the surrounding fluid. Our results strikingly matched experimental outcomes in various ways, including the distance particles travelled in one period of undulation, as well as qualitatively and quantitatively matching velocity fields. We then implemented this method using video data of swimming *C. elegans* and successfully reproduced the fluid dynamics. This is a novel application of the method of regularized stokeslets that combines theory and experiment. We expect this approach to provide insight in generating hypotheses and informing experimental design.

Erica R. Johnson, University of the Incarnate Word
Soliton Solutions Associated with the Korteweg de Vries Equation

The Korteweg de Vries equation is a nonlinear integrable partial differential equation that models shallow water waves. This presentation focuses on the visualization of soliton waves using a matrix-valued solution to easily identify properties of the individual wave. We consider the motion of a single wave as well as the interaction of multiple waves and discuss how the properties change in these cases. Included is a discussion of the limitations of computer algebra systems associated with the interaction of a large number of waves.

Dr. Stephen T. Salako, Our Lady of the Lake University
Image Registration and Applications

Image registration is the process of finding an optimal geometric transformation between corresponding imaging data. In practice, the concrete type of the geometric transformation as well as the notions of optimal and corresponding depends on the specific application. Image registration has played an increasingly important role in medical imaging in the last 20+ years. Registered images are now used often in a number of different applications such as the treatment verification of pre and post intervention images and time evolution of an injected agent subject to patient motion. Applications include geophysics, computer vision and medicine etc. In this talk we'll focus on medical applications.

Benjamin Shaw, Hardin-Simmons University
Exploring the Hyperbolic Trigonometric Functions

We will introduce the Hyperbolic Trigonometric functions and their interesting relationship to the traditional trigonometric functions. The hyperbolic functions which are combinations of real exponentials are quite useful in Calculus. We will provide derivative and integral formulas as well as their inverse functions. Finally, we will demonstrate a few mathematical applications of the hyperbolic functions.

Anthony Thomas, University of the Incarnate Word
Competition Dynamics of Three Strains of the Bacteria That Causes Gonorrhea

In the United States the Center for Disease Control (CDC) estimates that there are 700,000 new cases of gonorrhea reported every year, making it one of the most common bacterial sexually transmitted diseases (STDs). In this paper we examined the competition dynamics of three strains of the bacteria that causes gonorrhea, *Neisseria gonorrhoeae*. Our model takes into account the effects of varying the levels of antibiotics, rates that bacteria develop antibiotic resistance, immunological responses to the presence of bacteria, and the presence of efflux pumps in the bacteria. We will non-dimensionalize the model, analyze the equilibrium solutions and their stability. Numerical simulations will also be done to examine the role played by each of the parameters.

Geometry

Mathew Gomez, University of North Texas at Dallas

Maximal Contraction Ratio and Hausdorff Dimension of Sierpinski Snowflakes

We derive a formula to find the maximal contraction ratio without overlap for outward invariant fractals of regular n -gons which we called regular Sierpinski Snowflakes. We use infinite geometric series and the quadratic formula to derive the formula for the maximal contraction ratio, and the maximal Hausdorff dimension.

Graph Theory, Combinatorics, or other Discrete Math

Abigail Edgar, St. Edward's University

Agreement Proportion in Box Societies

We study the minimum agreement proportion guaranteed by approval voting in societies where two out of any three voters have overlapping opinions on two issues, i.e., (2,3)-agreeable societies where voters' opinions can be represented by two-dimensional boxes (one dimension for each issue). While the current greatest lower bound on agreement proportion in such societies is $1/4$, the lowest known example exhibits agreement proportion $3/8$. Thus, we investigate whether the lower bound $1/4$ is sharp by attempting to produce such an example or to show such an example is not possible and then trying to improve the lower bound. Our approach includes techniques from graph theory and Ramsey theory applied to the intersection graphs representing (2,3)-agreeable box societies.

Jonathan Hodges, Lamar University

The G -Graph of a Group

A group G with a generating set $S = \{s_1, s_2, \dots, s_n\}$ has a graph whose vertices are the distinct cosets of $\langle s_i \rangle$. This graph is called the G -Graph of G , denoted $\Gamma(G, S)$. Two distinct vertices are joined by an edge when the intersection of the cosets is non-empty. We will be

studying the interaction between the group theoretic properties of G and the graph theoretic properties of $\Gamma(G, S)$.

Robert Lehr, Southwestern University

Noisy Clustering Analysis of Landfill Distribution

Landfills are environmentally hazardous areas and should be placed efficiently. We explored a graph theoretic approach to modeling landfill distributions across the US. We used GIS software to interpolate landfill locations and population density into a map of the US, and then we projected a node-link graph onto the landfills. Clustering analysis was then used to model the optimal distribution of landfills.

Aaron McCoy & Nicolas Nikoloutsos, Lamar University

Modular Arithmetic Patterns in Pascal's Triangle

Pascal's Triangle, the triangular array of binomial coefficients, is a mathematical object with many uses in multiple fields and both pure and applied mathematics. Although it is generated by a very simple rule set, it has been shown that there is enormous complexity within its structure. We will discuss our observations of some interesting patterns with fractal and recursive properties that emerge when we used a C++ program to manipulate the values in Pascal's Triangle via modular arithmetic.

Annie Phan, El Centro College

Yi-Ching and Numerical Systems

This paper shows two rational mathematical approaches to study the Yi Ching. First, we will show how the function of distribution of HEXAGRAMS depend on the numbers of positive YAOS (or Yang YAOS) in hexagrams themselves using the PASCAL TRIANGLE and the ANALYSIS of combination. Second, we will translate the YAOS into the digits in binary systems, then transfer them into decimal numerical system, in order to find the positions of HEXAGRAMS in FOU-Fi ABACUS.

Education

Lindsay Hixon, Sam Houston State University

Statistics Content in Elementary Math Textbooks

This past summer, we investigated the nature and extent of statistical content in U.S. mathematics textbooks for students in grades 1-5 by examining five textbooks series. Using the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, we systematically coded statistics tasks into four phases: Formulate a Question, Collect Data, Analyze Data, and Interpret Results. The Analyze Data phase was further divided into four categories: Read a Display, Perform a Mathematical Calculation, Construct a Display, and Use Other Statistical Reasoning. Our analysis determined the location of the statistics tasks, the distribution of the phases, and the types of displays, such as bar graphs, that appeared in each textbook. The textbooks were then checked to see which Common Core State Standards (CCSS) and Texas Essential Knowledge and Skills (TEKS) standards were met. Our results suggest that the distribution of statistics tasks depended on the series, and we found that our sample of textbooks did not place equal emphasis on the different phases of the statistical process. Textbooks predominantly focus on analyzing data, which may inadvertently restrict opportunities for students to generate and interpret data. Our research was supervised by Dr. Dusty Jones (Sam Houston State University) and funded by NSF grant DMS-1262897.

Probability and/or Statistics

Matthew Chaltain, Southwestern University

Earnings Surprise Effect on Stocks

A mathematical model will determine the effect quarterly earnings surprises for selected publicly traded companies will have on stock prices. The variable of interest will be the percentage difference of forecasted earnings from actual earnings, known as the earnings surprise, to view the effect market expectations have on the stock market. Statistical regression will be used to model how the magnitude of an earnings surprise affects stock prices.

Audrene Edwards, Lamar University

Shakin' Things Up: Using the Statistical Approach to Model Natural Disasters

The extreme value theory (EVT), is becoming a well known discipline in statistics. It has been developed in order to model and analyze rare but extreme events. This study focuses on using EVT to model extreme earthquakes within data of selected earthquakes of general historic interest, provided by the U.S. Geological Survey. For this study, the data will be taken and processed in the statistical package R. The Peak Over Threshold approach will determine those earthquakes that have a higher than normal magnitude - those causing catastrophic damage. Next, the estimation techniques: maximum likelihood, method of moments, and probability-weighted moments, will be utilized to find the unbiased estimators needed for the generalized Pareto distribution.

Matthew Miller, Southwestern University

Scoring Cardiac Health: A Model of the Relationship between Diet and the Risk of Cardiovascular Disease

Like many other types of disease, cardiovascular diseases can be treated with a greater success rate the earlier that they are detected. For this reason medical researchers have long been developing means of detecting cardiovascular diseases earlier and earlier. Our goal is to develop a “scorecard” to measure cardiac health in the hopes of identifying cases of serious cardiovascular disease earlier. To this end we compiled the diet and health profiles of patients who came to an emergency room complaining of chest pains and have begun to study them for relationships between diet and other factors and the patients’ outcomes. Those who chose to participate were asked to complete a food frequency questionnaire, and their doctors completed surveys about the health profiles of the patients. From the results of the questionnaire, we measured the subjects’ diet compositions in terms of five nutrients: protein, carbohydrates, fat, fiber, and total caloric intake. We will apply regression analysis to measure the relationships between those nutrients and the presence of certain biomarkers associated with cardiac health—an established method of diagnosing cardiovascular disease—and between the nutrients and the occurrence or non-occurrence of acute myocardial infarction and heart failure. We intend for the scorecard to also be used to measure general cardiac wellness.

Megan Myers, Southwestern University

A Place for All People: Planning for Growth in Summer Camp Enrollment

Summer camps, often non-profit entities, must balance the financial demands of today’s housing, food, and staffing requirements with investment in tomorrow’s potential growth. Many do so based on little more than intuition, without access to mathematical resources or models. In this project, we will develop a mathematical model for enrollment trends at a summer camp based on longitudinal data. Trends including those related to overall summer enrollment, program enrollment, and

weekly enrollment will be considered. Models will include regression analysis as well as curve and surface mapping.

Emileigh Willems & Jessica Hall, Wayland Baptist University

Variable Selection on NuVal Scores

The NuVal Nutrition Scoring System was developed to provide a single value to help shoppers make healthier choices in their food purchases. This project attempts to use nutritional information available on product packaging to derive the algorithm used to generate the NuVal Scores. A sample of products was taken from the United Supermarkets in Plainview, Texas, and the nutritional facts for each product was recorded. Backward Elimination and Stepwise Regression techniques were used to determine the best model to approximate the algorithm.

Topology

Thoa Doan & Richard Kassanga, Odessa College

A Study of Unknotting Number

The goal of this project was to put an upper bound on the unknotting number of a specific family of knots. An equation was derived that calculates this bound based on some characteristics of the family. The equation was found to be applicable to a different family of knots as well.

Noureen Khan, University of North Texas at Dallas

Linking number of Virtual Links

Linking number is well known invariant of classical knots and links, we extend the notion and its invariance for virtual knot and link (VKL) diagrams. Moreover, we establish its relation to other invariants, like unknotting number and ascending number of VKL.

Mark Lugo, University of North Texas at Dallas*Equivalence Class of Virtual Knot Theory*

We introduce virtual 4-plats and formulate table of virtual 4-plats and their Conway symbols, $(c_1, c_2, \dots, c_n)$ up to 8 crossings. We conjecture that the minimal number of classical crossings of any virtual 4-plat is greater than or equal to its minimal number of virtual crossings. We compute and classify the rational number β/α with $0 < \beta < \alpha$ to formulate equivalence classes of virtual 4-plat modulo Reidemeister moves

Javier Mondragon, University of North Texas at Dallas*Knot Permutation of Alternative Prime Knot*

Knot theory is a promising field that can possibly answer many important scientific open problems. In our research, we introduce new invariants, called knot permutations to distinguish alternative prime knots. We show that the knot permutation is unique for alternative prime knots up to nine crossings. Furthermore, we develop a new notion called the knot set and show that alternative prime knots are uniquely regenerated by using the knot set.

Sarah Renfro, Sam Houston State University*Pseudoknot Graph Invariants*

In mathematics, a knot is an embedding of a circle in 3-dimensional Euclidean space. However, the study of knots is primarily on 2-dimensional representations of knots, called knot diagrams. In a knot diagram, a space is placed in a strand at crossings to indicate that the strand passes under the other strand. Two knot diagrams are equivalent if and only if they are related by a finite sequence of Reidemeister moves. Pseudoknot diagrams, conventionally called pseudodiagrams, are generalizations of knot diagrams where, at a crossing, the over and under strands may be undetermined. Two pseudoknot diagrams are defined to be equivalent if and only if they are related by a finite sequence of Reidemeister moves and pseudo-Reidemeister moves. Equivalent pseudoknot diagrams are said to be diagrams of the same pseudoknot. In my presentation, I will introduce a graph theory inspired invariant for pseudoknots that was the result of a collaboration with Ryan Coopergard and Dr. Jeff Boerner. Using the invariant, I will answer the following question posed by Allison Henrich et al. Does the pseudoflype preserve pseudoknot type?

Delia Rojas, University of North Texas at Dallas*Invariance of Virtual Involutory Quandle*

In this paper we extend the notion of virtual operator T as an invariant of virtual knots and links. We use the operator T to calculate virtual involuntary quandle (VIQ) for knots and links. Moreover, we compare VIQ with the fundamental groups to show the invariance of operator T for virtual knots and links.

Notes

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