Fall Meeting of The Southern California-Nevada Section of The Mathematical Association of America

Program and Abstracts



California State University Channel Islands October 7, 2023

Acknowledgements

The Southern California-Nevada Section of the MAA welcomes participants to our 2023 Fall Meeting at CSU Channel Islands. We are excited that you have joined us!

We would like to thank our invited speakers and our contributed paper session presenters for their participation in the meeting.

Our student volunteers pay a vital role in putting on the meeting. Many thanks to our volunteers from CSU Channel Islands for this meeting.

Finally, the Section thanks CSU Channel Island, and especially Cindy Wyels, for their hospitality in hosting the meeting. We would like to especially thank our program committee who handled the arrangements for the meeting program.

Section Officers, 2023-2024

MAA Representative: Edray Goins Section Chair: Shanna Dobson Section Vice-Chair: Kim Ayers Past Section Chair: Brian Katz Program Chair: Brian Katz Program Chair: Bahar Acu 2nd Program Vice-Chair: Maribel Bueno Secretary: Mary Legner Treasurer: Natalia Postrigan Meetings Coordinator: Youngsu Kim Newsletter Editor: Shanna Dobson Student Chapters Coordinator: Ryan DeMoss Section NExT Liaison: position open Las Vegas Liasion: Zhijian Wu Web Page Editor: Karrolyne Fogel

Looking for a way to engage with and support our math community? Our officer positions rotate frequently. Each spring we elect a new Section Vice-Chair and 2nd Program Vice-Chair, who serve on the board for three years. Other upcoming openings are the MAA Representative (elected) and the Section NExT Liaison to support early career faculty (appointed). If you are interested in serving in any of the Section Officer positions, but especially one of these that will be open soon, please contact the Past Section Chair (Brian Katz).

Schedule

9:00-9:05	Welcome Remarks
	Shanna Dobson, CSU Los Angeles
9:05-10:05	Invited Address
	Cynthia Wyels, CSU Channel Islands
	Data Science for (\mathfrak{G} by) Pure Mathematicians
10:05-10:35	Section Business Meeting
10:35-10:45	Break
10:45-12:30	Faculty and Student Contributed Paper Session
	Organized by Konrad Aguilar, Pomona College
12:30-1:45	Lunch
1:45-2:45	Data Sciene Panel
	Moderated by Maribel Bueno , UC Santa Barbara
	Panelists:
	• Treena Basu, Occidental College
	• Volodymyr Minin, UC Irvine
	• Kyle Mylonakis, Protopia AI
	• Uma Ravat, UC Santa Barbara
	• Evan T.R. Rosenman, Claremont McKenna College
2:45-3:00	Break
3:00-4:00	Invited Address
	Elizabeth Thoren, Pepperdine University
	Leveraging Student Thinking
4:00-4:15	AIM introduction by David Crombecque , University of Southern California
	and Closing Remarks by Shanna Dobson , CSU Los Angeles

Contributed Paper Sessions

10:45-12:30	Contributed Paper Session (Discrete math)
	Del Norte 1535
10:45-11:05	Joshua P. Bowman, Pepperdine University
	Duality for cyclic n-color compositions
11:05-11:25	Mithra Karamchedu, Harvey Mudd College
	Generating Spanning Trees and Their Orbits up to Graph Automorphism
11:25-11:45	Peter Kagey, Harvey Mudd College
	A complete classification of tilings of the $n \times m$ grid, cylinder, and torus
11:45-12:05	Runze Li, UC Santa Barbara
	Graphic 3-uniform hypergraph degree sequences
12:05-12:25	Katherine Ortiz, Timothy Harris, Maria Schmidt, CSU Los Angeles
	Matrix Methods for Determining Chromatic Numbers of Cayley Graphs and
	Their Applications

10:45-12:30	Contributed Paper Session (Algebra and Math Education)
	Del Norte 1545
10:45-11:05	Brianna Huynh, Pomona College
	What's in a Grade? Exploring Faculty Rationale for Grading in Math and Physics
11:05-11:25	Karina Behera, Pomona College, and Rachael Combes, Biola University
	On the t-elasticity of numerical semigroups
11:25-11:45	Sogol Cyrusian, UC Santa Barbara.
	t-Delta Sets of Numerical Semigroups
11:45-12:05	Ruilin Zhu
	Expanding on An Equation-solving Problem from Gazeta Matematică
12:05-12:25	Megan Ichinose, CSU Fullerton
	Making the Perfect Waffle: The Transposition Number for Permutations with Repeated Letters

10:45-12:30	Contributed Paper Session (Geometry/Topology and Dynamical
	Systems)
	Del Norte 1555
10:45-11:05	David Weed, California State University Fullerton
	A Characterization of the Archimedean Solids
11:05-11:25	Bogdan Suceavă, CSU Fullerton
	New Relations Between Intrinsic and Extrinsic Geometric Quantities
11:25-11:45	Cole Twogood, California State University San Bernardino
	Crushtaceans: A Graph Theoretic Representation of Fully Augmented Links
11:45-12:05	Amy Somers, UC Santa Barbara
	(S,w)-Gap Shifts and Their Entropy
12:05-12:25	_

10:45-12:30	Contributed Paper Session (Applied Math and Analysis)
	Del Norte 1500 (Main Room)
10:45-11:05	Maria van der Walt, Westmont College
	A function approximation approach to the prediction of blood glucose levels
11:05-11:25	Nathan Schroeder, Claremont Graduate University
	Steklov EigenSteklov Eigenvalue Problems on Nearly Spherical and Nearly Annular Domains
11:25-11:45	Jaden Segovia, CSU Fullerton
	A Natural Characterization of Uniform Continuity
11:45-12:05	Brian Sittinger, CSU Channel Islands
	An Euleresque derivation of $\zeta(2m)$
12:05-12:25	Evelyne Knight, Esteban Ayala, Chloe Marple, Pomona College
	Contractivity of Quantum Channels with respect to Quantum State Induced Norms

Invited Addresses

Invited Address: "Leveraging Student Thinking"

Elizabeth Thoren, Pepperdine University

Our job as educators is to support our students in moving the mathematical discourse forward. But rather than expecting students to come to our way of thinking, we must be prepared for this discourse to broaden our own understanding of mathematics. My part as an educator in this work has reshaped how I understand fundamental mathematical ideas. In this talk I will give some examples of how my students' thinking has helped me see mathematical ideas and mathematics differently, and how these new perspectives have impacted my teaching.

Invited Address: "Data Science for (& by) Pure Mathematicians"

Cynthia Wyels, CSU Channel Islands

Consider the skills and habits of mind developed through studying pure mathematics. These – and some basic statistical techniques – are enough to fruitfully address some questions of interest given a small data set. With a larger investment of time for individual learning, a healthy dose of humility, and perhaps some collaborators, those with preparation focused in pure mathematics can produce data-based studies of interest to wide audiences. Join me for a story involving a years-long transition, a cast of dozens, some alluring marine megafauna and lots of serendipity as I argue for the value of all types of research for and by all types of researchers.

Invited Address Biographies

Elizabeth Theron, Pepperdine University

Dr. Elizabeth Thoren of Pepperdine University is the winner of the 2023 Section Award for Distinguished College or University Teaching of Mathematics and our Section's nominee for the national MAA's Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching of Mathematics.

Dr. Thoren has over 15 years of experience teaching with inquiry and has been part of the AIBL Workshop Leader Team since 2018 where she has co-developed and co-facilitated both virtual and in-person IBL Workshops. Elizabeth also served as a guest Associate Editor for the PRIMUS special issue on Teaching Inquiry and is currently part of the leadership council for COMMIT-CaN (COMmunities for Mathematics Inquiry in Teaching in California and Nevada). The MAA Southern California-Nevada Section recognizes Elizabeth as a skilled and effective educator who is deeply committed to building classroom communities, as someone who helps students find joy and creativity in mathematics that they may not have experienced before, as a leader in our regional mathematics communities, and as a passionate and impactful professional developer who loves thinking with other educators about improving our practice. We are honored to present her with the Section Award for Distinguished College or University Teaching of Mathematics.

Cynthia Wyels, CSU Channel Islands

Dr. Cynthia Wyels came to CSU Channel Islands in Fall 2005 after several years at California Lutheran University. At CSUCI, she co-authored three \$6 million HSI-STEM grants and multiple NSF REU grants, directed the campus' LSAMP program, and served as chair of the Faculty Senate. Dr. Wyels has received SACNAS' Distinguished Mentor Award for work mentoring students and faculty from historically underserved groups as well as the MAA's Haimo Award for Distinguished University Teaching of Mathematics. Her mathematical research interests began in combinatorial mathematics and linear algebra. She now applies data analysis tools to study the effectiveness of educational interventions and to collaborate on environmental issues.

Contributed Paper Session Abstracts

Del Norte 1535(Discrete Math)

10:45-11:05

Presenter(s): Joshua P. Bowman, Pepperdine University **Title:** *Duality for cyclic n-color compositions*

Abstract: An *n*-color composition of a natural number k is an ordered sum of positive integers whose total equals k, in which each part of size n is allowed to have one of n different "colors," or subscripts. In a cyclic *n*-color composition, the order of the terms may be permuted cyclically. A cyclic *n*-color composition can be represented using paths packed into a cycle graph in which each path has one marked vertex. This representation leads to a notion of "duality" for cyclic *n*-color compositions, which is closely related to compositions into odd parts. This talk will show how to count classes of dual cyclic *n*-color compositions and how to characterize the "self-dual" cyclic *n*-color compositions. *Discrete Mathematics*

11:05-11:25

Presenter(s): Mithra Karamchedu, Harvey Mudd College

Other Author(s): Research advisor: Lucas Bang, Harvey Mudd College

Title: Generating Spanning Trees and Their Orbits up to Graph Automorphism

Abstract: We study the problem of generating a graph's spanning trees up to symmetries ("automorphisms") of the graph. Specifically, given a graph G, we seek to generate the different spanning trees of G up to the automorphisms of G (where two spanning trees T_1 and T_2 are equivalent if $\sigma T_1 = T_2$ for some $\sigma \in Aut(G)$). Further, we seek to determine the orbits of these spanning trees. However, solving this problem can be computationally expensive, growing superexponentially for many input graphs. Our work tries to find a more efficient solution to this problem by using *modular decomposition*. A modular decomposition is a representation of G that lets us take advantage of G's substructures that have inherent symmetry. With this technique, we construct a recursive algorithm that will likely provide superexponential improvements over brute force, for many classes of graphs. Additionally, we have fully characterized the spanning tree orbits for some classes of graphs, in particular the $2 \times n$ ladder graphs. Our approach may have potential applications in network theory and designing fault-tolerant network topologies. *Theoretical Computer Science* (Discrete Math and Algebra)

Audience: Although not essential, knowledge of Discrete Math and Abstract Algebra (as well as any theoretical computer science) will help!

11:25-11:45

Presenter(s): Peter Kagey, Harvey Mudd College

Title: A complete classification of tilings of the $n \times m$ grid, cylinder, and torus.

Abstract: We count all tilings of the $n \times m$ rectangular grid, cylinder, and torus with arbitrary tile sets up to arbitrary subsets of the dihedral group of the square D_8 . These results build on work by Ethier and Lee counting tilings of the torus by tiles of two colors. This is joint work with Bill Keehn via the Prison Mathematics Project (PMP). Discrete Mathematics

Del Norte 1535(Discrete Math)

11:45-12:05

Presenter(s): Runze Li, UC Santa Barbara

Title: Graphic 3-uniform hypergraph degree sequences

Abstract: A 3-uniform hypergraph is a generalization of simple graphs where each hyperedge has 3 vertices. The degree of a vertex in a hypergraph is the number of hyperedges incident to it. The degree sequence of a hypergraph is the sequence of the degrees of its vertices. The degree sequence problem for 3-uniform hypergraphs is to decide if a 3-uniform hypergraph, called a realization, exists with a prescribed degree sequence. Recently, Deza *et al.* proved that the degree sequence problem for 3-uniform hypergraphs is NP-complete. However, some special cases are easy. So far, polynomial algorithms can be used only for some very restricted degree sequences to construct their realizations. Our research mainly shows that if all degrees in a degree sequence D are between $\frac{n^2}{27} + O(n)$ and $\frac{2n^2}{27} - O(n)$, further, the number of vertices is at least 38, and the degree sum is divisible by 3, then D has a 3-uniform hypergraph realization. Our proof is constructive and it runs in polynomial time. This is the first polynomial algorithm to construct a 3-uniform hypergraph realization of a highly irregular and dense degree sequence. D iscrete Mathematics

12:05-12:25

Presenter(s): Maria Schmidt, Katherine Ortiz, and Timothy Harris, CSU Los Angeles **Other Author(s):** Research advisor: Dr. Michael Krebs, CSU Los Angeles **Title:** Matrix Methods for Determining Chromatic Numbers of Cayley Graphs and Their Applications

Abstract: Cayley graphs are a special type of graph, having vertices from a group G and a special subset of G normally called S, which is closed under inverses. Adjacency between two vertices $g, h \in G$ is equivalent to the condition that $gh^{-1} \in S$. We will give a presentation on a specific class of Cayley graph, where $G = \mathbb{Z}^n/H$, for some finitely generated subgroup H of \mathbb{Z}^n , and $S = \{\pm e_1, ..., \pm e_n\}$, the "standard basis" elements of \mathbb{Z}^n/H and their inverses. The generators of this subgroup H may be arranged in a matrix, giving us a succinct representation of any Cayley graph of this form. We will discuss especially chromatic numbers and their applications, more general graph theoretic strategies for finding them, known chromatic numbers of classes of graphs we are currently studying, and classes which are soon to be determined. *Graph Theory/Combinatorics* Audience: Algebra, Number Theory, and Graph Theory

DEL NORTE 1545 (ALGEBRA AND MATH EDUCATION)

10:45-11:05

Presenter(s): Brianna Huynh, Pomona College

Other Author(s): Research advisor: Dr. Warren Christensen, North Dakota State University Title: What's in a Grade? Exploring Faculty Rationale for Grading in Math and Physics Abstract: A course grade is typically composed of multiple graded categories chosen by faculty. The multidimensional nature of grading thus leads to variations in grading and assessment systems. This study uses interviews with math and physics faculty to determine how they select these systems for their courses. Semi-structured interviews were conducted to present math and physics faculty with questions regarding enrollment size, length of implementation, and feedback processes. Preliminary findings indicated two central student learning goals and the emergence of four dominant motivations: (a) grades reflect learning, (b) assumed maturity, (c) clear feedback, and (d) ease of grading. This study found that faculty across the disciplines of math and physics use similar rationale when selecting systems. We hope by identifying common motivations among faculty, we can provide insight into the current landscape of collegiate teaching and improve strategies for implementing alternative grading practices. Education/Pedagogy

Audience: No Prior Math Experience Needed

11:05-11:25

Presenter(s): Karina Behera, Pomona College, and Rachael Combes, Biola University **Other Author(s):** James Howard, San Diego State University, Shawn Perry, Saint Joseph's College of Maine, and Brianna Worms, James Madison University; Research advisors: Christopher O'Neill, San Diego State University, and Vadim Ponomarenko, San Diego State University **Title:** On the t-elasticity of numerical semigroups

Abstract: A numerical semigroup S is an additive semigroup of non-negative integers that contains zero and has a finite complement. We can associate with every S a unique vector \mathbf{a} of atoms of S, so that every factorization of $n \in S$ is a vector \mathbf{x} satisfying $\mathbf{a} \cdot \mathbf{x} = n$. The length of the factorization \mathbf{x} is the sum of the components of \mathbf{x} , or $\|\mathbf{x}\|_1$. We may extend our definition of length to t-length for arbitrary t by using t-norms instead of only 1-norms. Using any t-length, we are able to determine the elasticity of S and state if that elasticity is accepted. Moreover, viewing the elasticity of a semigroup or element as a function of t, we give an analysis of elasticity functions for arbitrary semigroups as well as elements of two-generated semigroups. Number Theory

Audience: elementary knowledge of Linear Algebra

DEL NORTE 1545 (ALGEBRA AND MATH EDUCATION)

11:25-11:45

Presenter(s): Sogol Cyrusian, UC Santa Barbara

Other Author(s): Alex Domat, Eric Ren, Mayla Ward; Research advisors: Christopher O'Neill and Vadim Ponomarenko, San Diego State University

Title: t-Delta Sets of Numerical Semigroups

Abstract: A numerical semigroup is a cofinite submonoid of $(\mathbb{N}_0, +)$ containing all linear combinations of a finite number of coprime integer generators. These semigroups allow for non-unique factorizations, meaning that elements can often be expressed as sums of the generators in multiple ways. Traditionally, the length of these factorizations has been measured using the 1-norm. The Delta set of an element consists of the differences between its consecutive factorization lengths when in ascending order. We introduce a method of computing lengths using *t*-norms for various *t*, and identify properties of the associated *t*-Delta sets for different families of numerical semigroups. In particular, the Δ_0 and Δ_{∞} sets of multiple families of semigroups are explicitly given. The periodicity of the Δ_0 and Δ_{∞} sets of individual semigroup elements is also proven, along with general results for *t*-lengths between 1 and ∞ . We also relate semigroup trade structure, *t*-catenary degree, and Δ_t sets. Algebra

Audience: No specific background knowledge required, some prior exposure to proofs will help.

11:45-12:05

Presenter(s): Ruilin Zhu

Title: Expanding on An Equation-solving Problem from Gazeta Matematică

Abstract: Problem 28550 (written by MaThinkers, Constanța) from Gazeta Matematică (No. 3/2023) is an inequality problem disguised as an equation problem. My solution to the problem involves using the AM-GM inequality in a creative way. By analyzing and experimenting with the problem, I was able to delve into its other variations and construct a new type of minimum-finding problems, which take a more generalized form and have broader implications than the original problem. In my presentation I will discuss this fun mathematical development. Algebra

12:05-12:25

Presenter(s): Megan Ichinose, CSU Fullerton

Title: Making the Perfect Waffle: The Transposition Number for Permutations with Repeated Letters

Abstract: This past year, I studied a game called Waffle. In the case of Waffle, each player is given six five letter words to unscramble in the form of a crossword and 15 swaps to unscramble the whole puzzle. The minimum number of swaps for success is 10 swaps, so how do we algorithmically find a sequence of ten swaps that completes the puzzle? With the use of permutations, our research resulted in a strategy that guarantees success in solving the puzzle every time. *Algebra*

Del Norte 1555(Geometry/Topology and Dynamical Systems)

10:45-11:05

Presenter(s): David Weed, California State University Fullerton

Title: A Characterization of the Archimedean Solids

Abstract: In studying any family of mathematical objects, a fundamental issue is to understand how one object can "sit inside" another object in the family, preserving the mathematical structure. We are concerned with convex uniform polyhedra. Two famous families of polyhedra live in this class: the Platonic and Archimedean solids, as well as the prisms and antiprisms. Our main result geometrically characterizes the famed Archimedean solids among the convex uniform polyhedra by studying how they sit inside a regular tetrahedron. *Geometry/Topology*

11:05-11:25

Presenter(s): Bogdan Suceavă, CSU Fullerton

Title: New Relations Between Intrinsic and Extrinsic Geometric Quantities

Abstract: In a work on the geometry of minimal submanifolds written in 1968, S.-S.Chern invited more efforts and reflections to identify relationships between intrinsic and extrinsic curvature invariants of submanifolds in various ambient spaces. After 1993, when Bang-Yen Chen introduced the first of his curvature invariants, a lot of work has been done to explore this avenue, which represents an active research area. We will survey some of these results obtained in the last three decades, and conclude our talk with new relationships between intrinsic and extrinsic curvature invariants. *Geometry/Topology*

11:25-11:45

Presenter(s): Cole Twogood, California State University San Bernardino

Title: Crushtaceans: A Graph Theoretic Representation of Fully Augmented Links

Abstract: Distinguishing between various links is a difficult task in knot theory, including for Fully Augmented Links. For a long time there has existed a graph called a crushtacean correlated with Fully Augmented Links. My research seeked to create a more strict powerful relationship between crushtaceans and fully augmented links. We were successful in proving various things about crushtaceans including a way to accurately use them to distinguish between any two fully augmented links. This improved relationship has the potential to translate many problems about fully augmented links and other knot theory problems into purely graph theoretic problems. Geometry/Topology

Del Norte 1555(Geometry/Topology and Dynamical Systems)

11:45 - 12:05

Presenter(s): Amy Somers, UC Santa Barbara

Other Author(s): Cristian Ramirez, UC Berkeley, Research advisor: Dr. Daniel Thompson, Ohio State University

Title: (S, w)-Gap Shifts and Their Entropy

Abstract: Given an $S \subset \mathbb{Z}_{\geq 0}$, an S-gap shift is defined to be the shift space consisting of all sequences in $\{0,1\}^{\mathbb{Z}}$ such that any two 1's are separated by a word 0^n for some $n \in S$. The S-gap shifts have a dynamically and combinatorially rich structure. It is a well known result that the entropy of the S-gap shift is given by $h(X) = \log(\lambda)$, where $\lambda > 0$ is the unique solution to the equation $\sum_{n \in S} \lambda^{-(n+1)} = 1$. Fix a point w of the full shift $\{0,1\}^{\mathbb{Z}}$. We introduce a generalization of the S-gap shift, known as an (S, w)-gap shift, consisting of sequences in $\{0, 1, 2\}^{\mathbb{Z}}$ in which any two 2's are separated by a word u appearing in w such that $|u| \in S$. We extend the formula for the entropy of the S-gap shift to a formula describing the entropy of this new class of shift spaces. Additionally we investigate dynamical properties of the (S, w)-gap shifts including irreducibly and mixing. Dynamical Systems

Audience: (some measure theory knowledge will help)

12:05-12:25

There is no talk scheduled for this session at this time.

Del Norte 1500 (Main Room) (Applied Math and Analysis)

10:45-11:05

Presenter(s): Maria van der Walt, Westmont College

Title: A function approximation approach to the prediction of blood glucose levels

Abstract: The problem of real time prediction of blood glucose (BG) levels based on the readings from a continuous glucose monitoring device is a problem of great importance in diabetes care, and therefore, has attracted a lot of research in recent years, especially based on machine learning. An accurate prediction with a 30, 60, or 90 minute prediction horizon has the potential of saving millions of dollars in emergency care costs. In this paper, we treat the problem as one of function approximation, where the value of the BG level at time t + h (where h the prediction horizon) is considered to be an unknown function of d readings prior to the time t. This unknown function may be supported in particular on some unknown submanifold of the *d*-dimensional Euclidean space. While manifold learning is classically done in a semi-supervised setting, where the entire data has to be known in advance, we use recent ideas to achieve an accurate function approximation in a supervised setting; i.e., construct a model for the target function. We use the state-of-the-art clinically relevant PRED-EGA grid to evaluate our results, and demonstrate that for a real life dataset, our method performs better than a standard deep network, especially in hypoglycemic and hyperglycemic regimes. One noteworthy aspect of this work is that the training data and test data may come from different distributions. This is joint work with Hrushikesh Mhaskar and Sergei Pereverzyev. Applied Mathematics

Audience: Multivariable Calculus (some data science knowledge will help)

11:05-11:25

Presenter(s): Nathan Schroeder, Claremont Graduate University

Title: Steklov EigenSteklov Eigenvalue Problems on Nearly Spherical and Nearly Annular Domains **Abstract:** We consider Steklov eigenvalues on nearly spherical and nearly annular regions in *d*dimensions. By using the Green-Beltrami identity of spherical harmonic functions, the derivatives of Steklov eigenvalues with respect to the domain perturbation parameter can be determined by the eigenvalue of a matrix involving the integral of product of three spherical harmonic functions. The trace of this matrix can then be used to determine whether spherical and annular regions are critical points when we maximize Steklov eigenvalues subjects to a volume constraint. The obtained numerical results in two and three dimensions are in agreement with our analytical results. *Analysis*

11:25-11:45

Presenter(s): Jaden Segovia, CSU Fullerton

Title: A Natural Characterization of Uniform Continuity

Abstract: In this talk we introduce a natural characterization of uniform continuity in terms of infima of suprema for functions over certain subsets of the real line. We present example proofs and disproofs of the uniform continuity of certain functions using this equivalent criterion. *Analysis*

Del Norte 1500 (Main Room) (Applied Math and Analysis)

11:45-12:05

Presenter(s): Brian Sittinger, CSU Channel Islands

Title: An Euleresque derivation of $\zeta(2m)$

Abstract: Euler's derivation of $\zeta(2) = \frac{\pi^2}{6}$ is a justly well-known proof relying on both the infinite series and infinite product representation of the sine function. One natural question is how this can be extended to find the values of $\zeta(2m)$ for any positive integer m. By using suitable roots of unity, we can extend the scope of aforementioned derivation to find the value of each of the even Riemann zeta constants. *Analysis*

12:05-12:25

Presenter(s): Esteban Ayala, Evelyne Knight, and Chloe Marple, Pomona College

Other Author(s): Research advisor: Dr. Konrad Aguilar, Pomona College

Title: Contractivity of Quantum Channels with respect to Quantum State Induced Norms

Abstract: Quantum Channels (QCs) are used to send quantum information in the form of quantum states and are major objects of interest in Quantum Information Theory and Quantum Computing, which has been a popular field for several years due to its ability to improve the transmission and storage of data. The most desirable QCs are non-expansive, meaning they don't add superfluous information, or "noise," to the quantum information. This noise can be detected using various metrics, functions which quantify a notion of distance. One specific metric we use is the Bures metric. While it is known that a given QC is Bures-nonexpansive, our study seeks to classify Bures-contractive QCs and estimate their Lipschitz constants. Another way to measure distance between matrices is using norms. We have classified several specific QCs as contractive with respect to the well-established and valuable 1-,2-, and infinity-operator norms. We also generated matrix norms using quantum states and found sharp two-sided norm equivalence constants for them, which were computationally tested. A key application of our work is the ability to detect contractivity of QCs with respect to new norms of interest by utilizing our work with these well-established norms and our new found equivalence constants. *Analysis*

Audience: Linear Algebra (some analysis knowledge will help)