4. The number of divisions required to find the g.c.d. of two numbers is never greater than five times the number of digits in the smaller number, (this depends on the denary scale).
5. There are just five complex fields $F(\sqrt{m})$ for which there is a euclidean algorithm, viz., $m=-1, -2, -3, -7, -11$.
6. There are just five known Fermat primes $2^{2^t}+1$, $(t=0, 1, 2, 3, 4)$.
7. Every map on the sphere can be properly colored if no two regions having a whole segment of their boundaries in common, receive the same color.
8. Kuratowski's Theorem on nonplanar graphs.
9. Heawood's Theorem that every planar graph is 5-chromatic.

7. *Computation of elliptic integrals using Gauss' transformation*, by H. E. Fettis, Applied Mathematics Research Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base.

By means of Gauss' transformation, the computation of the three kinds of elliptic integral may be reduced to routine operations involving only elementary functions, without any further restrictions on the modulus and parameter. The resulting formulae are easily programmed to provide subroutines for a digital computer.

8. *Projective invariants of a curvilinear element*, by Rodney Angotti, University of Akron.

The projective invariants of certain configurations associated with a regular third order differential element, i.e., expansions including the third degree terms in a projective three space are discussed; in particular, a construction of one such invariant is exhibited.

9. *The construction of the real numbers*, by L. D. Rodabaugh, Ohio Northern University.

An extension and refinement of the author's earlier work on this subject as reported to the Illinois Section in May 1951, (see this MONTHLY, 59 (1952) 286).

10. *Some theorems about simple semigroups*, by C. E. Aull, Kent State University.

The following are proved: A semigroup S is a simple semigroup iff for a, $b\in S$, the equation $xay=b$ has at least one solution x, $y\in S$. A simple semigroup S, with identity is a group if any of the following conditions is satisfied: (a) S is commutative, (b) S is left (right) cancellative, (c) S is finite.

11. *Statistical hypothesis modification—a new point of view for statistical inference*, by Thaddeus Dillon, Youngstown University.

Instead of accepting or rejecting the hypothesis $\theta=\theta'$, it is suggested that the hypothesis be modified to $\theta=(Q+\lambda\theta')/(1+\lambda)$, where Q is a statistic and λ is a nonnegative real function of three variables: (1) sample size, (2) population size, and (3) the probability used for comparison to decide whether to accept or reject the hypothesis. Under rather general conditions on the power function such procedures are self-correcting in the sense that the worse of two theories is likely to receive less weight and a really bad theory θ' is likely to receive negligible weight.

FOSTER BROOKS, *Secretary*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-seventh annual meeting of the Rocky Mountain Section of the MAA was held at Colorado College, Colorado Springs, Colorado, on Friday and Saturday, May 1 and 2, 1964.

The following officers were elected for 1964–65: Chairman, F. M. Carpenter, Colorado School of Mines; Vice-Chairman, F. M. Stein, Colorado State University; and Secretary-Treasurer, W. N. Smith, University of Wyoming. E. R. Deal, continues, in his second year of a three-year term, as coordinator of High School Mathematics Contests.

The 1965 spring meeting will be held at the Colorado School of Mines, Golden, Colorado.

Changes in the By-Laws for the section were considered and discussed and a new draft approved for presentation to the Board of Governors for approval.

The Friday evening guest speaker was Professor W. J. LeVeque, visiting professor at the University of Colorado; his topic was *Probability and Number Theory.*

The following papers were presented:

1. *Idempotent matrices* (mod p^a), by J. H. Hodges, University of Colorado.

For positive integers m, a and prime p, the number $N(m, p, a)$ of idempotent matrices (mod p^a) of order m is determined. First, the number for $a=1$ is determined by using canonical forms, involving elementary divisors, for matrices under similarity. Then it is shown that $N(m, p, a+1) = N(m, p, 1)$ for all $a \geqq 1$. The method employed in the second step is the standard recursive one in number theory of using solutions mod p^a to generate solutions mod p^{a+1}. $N(m, p, 1)$ can be expressed as a simple sum involving the number g_r of nonsingular matrices of order r (mod p).

2. *An application of symmetric functions to statistics*, by P. W. Mielke, Colorado State University.

It is well known that symmetric functions have desirable statistical estimation properties. Methodology for treating the two-way classification finite model with disproportionate population subcell sizes is discussed. In particular some symmetric function variance component estimators are introduced which can be applied to this present model even if the sample subcell sizes are disproportionate. An immediate consequence of the use of symmetric functions is the unbiased estimation of the sampling variance for these variance component estimators.

3. *Estimation with some prior information*, by M. M. Siddiqui, Colorado State University.

4. *Chebyshev lines*, by B. L. Foster, Denver Research Center, Marathon Oil Company.

The best fitting line for a set of data points depends on what is meant by best. According to Chebyshev, that line is best which minimizes the worst data deviation. The x-Chebyshev line is the one minimizing the worst x-deviation; the y-Chebyshev line minimizes the worst y-deviation. With uninteresting exceptions, these lines are the same. Using the over-under-over theorem discussed by Scheid (this MONTHLY, 68 (1961) 862), this can be proved by a simple geometrical argument that extends to oblique coordinate systems. A different proof was announced at this meeting of the Association by Professor M. M. Siddiqui.

5. *Some results on T-fractions*, by B. W. Jones and W. J. Thron, University of Colorado.

A T-fraction is a continued fraction of the form $(1+d_0z)+z/(1+d_1z)+z/(1+d_2z)+\cdots$, where z is a complex variable and the d_n are complex numbers. Among convergence criteria for T-fractions given by W. J. Thron (Bull. Amer. Math. Soc., 54 (1948) 206–218) is the following: if $d_n>0$ for $n\geqq 0$, the T-fraction converges for all z, not on the negative real axis, to a function $f(z)$ which is holomorphic in the interior of this region. In the present work the authors show that if $d_n > 0$ for $n \geqq 0$, there exists a bounded, nondecreasing function $\psi(t)$ such that $f(z) = 1 + d_0z + z\int_{-\infty}^{0} d\psi(t)/(z-t)$. If t_0 is the largest point of increase of $\psi(t)$ then $z=t_0$ is a singular point of $f(z)$ or if $\{d_n\}$ is unbounded then $f(z)$ has a singularity at $z=0$.

6. *Hankel transforms and entire functions II*, by K. R. Unni, Utah State University.

7. *The value of a coalition in applied games*, by W. C. White, Cadet, USAFA.

8. *The University of Colorado Computer Center for secondary schools*, by R. L. Albrecht, Control Data Corporation.

A center for exploring methods of secondary school computer education has been established by the University of Colorado, College of Engineering, Denver Center, with the cooperation of the Control Data Corporation and the Denver Chamber of Commerce. During the 1963–64 school year, 144 high school students and 26 high school teachers were enrolled in an experimental program. The main objective is the development of methods for using a computer to reinforce classroom training in secondary school mathematics and science. The computer is regarded as a "mathematics laboratory" with which students solve textbook problems, perform mathematical experiments, and process scientific data.

9. *Polypack: a set of polynomial algorithms*, by R. A. Kahn, student, George Washington High School, Denver, Colorado.

Polypack is a set of algorithms which enables a computer to add, subtract, multiply, divide, or evaluate two polynomials. For example, the addition algorithm tells the computer how to add the coefficients of two given polynomials—$A(x)$ and $B(x)$—to obtain $(A+B)(x)$. These algorithms were derived by first taking specific polynomials and performing different operations on them. Usually a general pattern could be observed for obtaining the result in the specific cases. This pattern was then enlarged and a general algorithm for the operation was formed.

10. *BOOTRAN: A logical interpreter for a Control Data* 160-A *computer*, by Randy Levine, student, George Washington High School, Denver, Colorado.

BOOTRAN is an interpretive system for a Control Data 160 or 160-A computer. In an algebraic manner, it does logic or simple Boolean algebra. Hence the name, BOOTRAN. The talk concerns the most recent version, BOOTRAN III, which is the third in a series of programs to develop a system to do advanced Boolean algebra by computer in a method which can be easily understood by the programmer, even though he may not be acquainted with computers in detail. The main distinguishing feature of BOOTRAN III is that it does its logic using Polish Notation, instead of parentheses. Future BOOTRANs will eliminate this and make writing source programs considerably easier.

11. *Number theory*, by Larry Davis, student, Golden High School, Golden, Colorado.

12. *Generalized polynomials*, by Captain D. E. Helton, USAFA.

In 1959 Jan Mikusinski developed generalized functions through convolution quotients. By restricting the class of continuous functions to the Heaviside function $h(t)=1$ for $t\geqq 0$; $h(t)=0$ for $t<0$, one may obtain good partial results. Considering convolution powers of $h(t)$, as we do the integers in developing the rational numbers, then applying pointwise addition, scalar multiplication and last, forming ordered pairs, we arrive at the field of convolution quotients which we call "Generalized Polynomials." This field contains a class of impulse functions, including the Dirac δ function.

The simplicity of proving no zero divisions under convolution is a prime advantage of "Generalized Polynomials."

13. *An Undergraduate Research Program in Mathematics*, by F. M. Stein, Colorado State University.

An Undergraduate Research Program in Mathematics to be conducted at Colorado State University during the summer of 1964 similar to three previous programs is described. These eight week programs, supported by the National Science Foundation, attempt to show the nine well qualified undergraduate mathematics majors who participate how a mathematician works and what he does by directing their work on various non-coursework topics.

14. *General repeated exponentiation*, by R. A. Bruce, student, Colorado State University.

The convergence or divergence of the sequence $(x_n)_1^\infty$, where $x_1=c$, $x_2=x^{x_1}, \cdots, x_n=x^{x_{n-1}}, \cdots$, can be completely determined for $c\geqq 0$ and $x>0$. In particular, when $c=x$, this sequence converges for x, such that $1/e^e\leqq x\leqq e^{1/e}$, and diverges for all other positive x.

15. *Nondecreasing solutions of* $y''=f(x, y)$, by J. W. Bebernes, University of Colorado.

Consider the problem of finding a unique solution of class C^2 on $[a, \infty)$ of the infinite interval boundary value problem (*): $y''=f(x, y)$, $y(a)=-\alpha$ $(\alpha>0)$, $y(x)\leqq 0$, $y'(x)\geqq 0$, for $x\geqq a$. Assume as needed: (1) $f(x, y)$ is continuous on $\{(x, y)\mid x\geqq a, |y|<\infty\}$, (2) $f(x, y)$ is nondecreasing in y for each fixed x, (3) $f(x, 0)\equiv 0$, $x\geqq a$, (4) there exists a $\delta>0$ such that $y\leqq f(x, y)$ for all $x\geqq a$, $-\delta\leqq y\leqq 0$. By the use of the technique of subfunctions, the following theorems can be proved. THEOREM A: *If f satisfies* (1), (2), *and* (3), *then there exists a unique solution of* (*). THEOREM B: *If f satisfies* (1), (2), (3), *and* (4) *then there exists a unique negative solution of* (*).

LEOTA C. HAYWARD, *Secretary*