2. *A geometric definition of an analytic function*, by Professor A. W. Goodman, University of Kentucky.

3. *The* (F_n, a_{nk}) *topological space*, by Mr. T. R. Westbrook, University of Louisville.

4. *Eigenvalue problems of ordinary differential equations*, by Mr. James Rolf, University of Kentucky.

V. F. COWLING, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-fifth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at The South Dakota School of Mines and Technology, Rapid City, South Dakota, May 4 and 5, 1962. The meeting was divided into several sessions with Professors F. M. Carpenter, P. O. Steen, and Lawrence Fearnley, presiding. There were 68 persons registered for the meeting.

Officers elected at the meeting for 1962–1963 were: Chairman, Professor H. J. Fletcher, Brigham Young University; Vice-Chairman, Col. J. W. Ault, United States Air Force Academy; Secretary-Treasurer, Professor Leota C. Hayward, Colorado State University; and Director of High School Mathematics Contest, Professor D. C. B. Marsh, Colorado School of Mines.

The following papers were presented:

1. *Quasi-resolutions of the identity*, by Professor E. R. Deal, Colorado State University.

For certain nonspectral operators, if the conditions that $\{E(\delta)\}$ be a resolution of the identity be weakened to the condition that $\{E(\delta)\}$ be an operator measure, it is still true that T may be represented in the form $T=\int_{\sigma(T)} \lambda E(d\lambda)+N$ where N is a generalized nilpotent operator. Such an operator T is called a quasi-spectral operator. Examples of quasi-spectral operators are given, and sufficient conditions for an operator to be quasi-spectral are given.

2. *Convergence and stability in the numerical integration of ordinary differential equations*, by Professor R. A. Hansen, Brigham Young University.

The use of the class of difference equations $y_{n+k}+a_{k-1}y_{n+k-1}+\cdots+a_0y_n=h(b_ky'_{n+k}+\cdots+b_0y_n')$ for the numerical solution of the initial value problem for an ordinary differential equation $y'=f(x, y)$, $y(a)=y_0$, is considered. Sufficient conditions are indicated which guarantee the convergence of the solution of the difference equation to the solution of the differential equation as the tabular interval h approaches zero. Stability of solution is defined and sufficient conditions are given which insure stability.

3. *Some n-dimensional coverage problems*, by Professor W. C. Guenther, University of Wyoming.

The center of a sphere is aimed at a point target in an n-dimensional coordinate system, with aiming errors being governed by a p.d.f. $f(X)$. Before the sphere arrives the point selects a new position according to a probability law whose p.d.f. is $g(X')$. The probability that the sphere covers the point target when the sphere comes to rest is computed for several choices of $f(X)$ and $g(X')$.

4. *A linear congruence with side conditions*, by Professor David Rearick, University of Colorado.

For positive integral r and n, and integral m, denote by $\phi_r(n, m)$ the number of distinct solutions of the congruence $x_1+x_2+\cdots+x_r=m \pmod n$ with x_i relatively prime to n for all i. It is shown that $\phi_r(n, m)$ and the rth power of Ramanujan's exponential sum $C_n(m)$ form a Fourier transform pair. From this is deduced a formula for $\phi_r(n, m)$ in terms of the Euler ϕ-function.

5. *An integral transform*, by Professor R. H. Niemann, Colorado State University.

The Riemann Stieltjes integral from zero to infinity of $g(z+t)/g(z)$ with respect to $c(t)$ has several interesting special cases. Here $g(z)$ is assumed to be analytic and $e^{pz^q}g(z)$ can be represented in an asymptotic series in a sector of the complex plane that includes the positive real axis. The exponents p and q are polynomials in z and $c(t)$ is a function of bounded variation. If $g=\exp(-z^2/2)$ the integral reduces to the Laplace transform. If g is the reciprocal of the gamma function the integral reduces to the factorial transform and the factorial series if $c(t)$ is chosen properly.

6. *Concave functions and points of inflection*, by Professor L. C. Barrett, South Dakota School of Mines and Technology.

In this note we give an analytic definition for concavity of a function at a point and then extend the definition to concavity over an interval. Generalizations of the concept are noted and point of inflection is also defined. These ideas are enlarged upon by means of theorems and illustrative examples. It is pointed out that the definition of concavity may be formulated in terms of a determinant, or in terms of second order central differences.

7. *Matrices of basis vectors*, by Captains R. L. Eisenman and D. R. Barr, United States Air Force Academy, presented by Captain Barr.

A linear combination of basis vectors can be written as a formal product RC_R, where R is a row matrix of basis vectors and C_R is a column matrix of coefficients. This notation has been used at the Air Force Academy, and has advantages in unifications of concepts and notations of vector analysis and linear algebra, and in formulation and testing of conjectures. These advantages are illustrated by the finding of the relations between the matrices of a linear transformation of a vector space into itself in two different bases and by a generalization of the Coriolis theorems, respectively.

8. *The Fibonacci matrix modulo m*, by Professor D. W. Robinson, Brigham Young University.

The periodic properties of the Fibonacci sequence modulo m (see D. D. Wall, this MONTHLY 67 (1960) 525–532) are studied by considering integral powers of the 2-by-2 matrix with first row (0, 1) and second row (1, 1).

9. *Some convergence results for continued fractions*, by Professor K. L. Hilliam, University of Colorado.

10. *On the convergence criterion of Du Bois Reymond and the theory which has evolved from it*, by Professor Alexander Peyerimhoff, University of Utah.

The theory of convergence—and summability factors is discussed as a generalization of the convergence criterion of Du Bois Reymond. This generalization is obtained by weakening one of the assumptions of the criterion through the idea of summability. Complete results are obtained if the method of summability is connected with a certain mean value theorem—as was observed first by L. S. Bosanquet in the case of the Cesáro method.

11. *Some observations regarding binomial coefficients*, by Dr. T. C. Fry, Consultant to the Director, National Center for Atmospheric Research.

12. *A multiphase diffusion problem*, by Lt. C. F. Lutz, U. S. Air Force Academy.

The diffusion equation is $D_i\partial^2 C_i/\partial X^2 = \partial C_i/\partial t$ where the diffusivity, D, is considered a constant. The solution of the diffusion equation is derived for the experimental situation of an infinite bar of constant cross section, where the bar may be regarded as extending along the x-axis. After time $t=0$ it is found that the diffusion causes a separation of the bar into n-segments corresponding to n pure phases. Thus, there are phase boundaries at points, X_i, where discontinuous changes in C_i are observed. The solution is generalized for the case where D is variable.

13. *A mathematical treatment of the eutectoid in the W-C system*, by Professors G. W. Orton and Rudolph Speiser, United States Air Force Academy and The Ohio State University, presented by Lt. Col. George Orton.

An analysis is made of the three univariant curves about the W-C eutectoid to relate temperature, enthalpy and the activity of carbon in the reactions. The activity of carbon determined experimentally is related to the free energy change in the reaction and the free energy is expressed in terms of enthalpy and entropy. Values are presented for ΔF^0, ΔH^0 and ΔS^0 of each reaction.

14. *The college training of high school teachers*, by Professor W. R. Orton, University of Arkansas.

LEOTA C. HAYWARD, *Secretary*

DISTRIBUTION OF MAA FILMS

Effective September 1, 1962, the Association will discontinue the free distribution of the films by Henkin, McShane, and Hewitt, which were produced by the MAA Committee on Production of Films. These films may be rented from Modern Learning Aids, 3 East 54th Street, New York 22, N. Y.

Schools and individuals wishing to purchase the films should also write to Modern Learning Aids.

CALENDAR OF FUTURE MEETINGS

Forty-sixth Annual Meeting, University of California, Berkeley, January 26–28, 1963.

Forty-fourth Summer Meeting, University of Colorado, Boulder, August 26–28, 1963.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

ALLEGHENY MOUNTAIN, Pennsylvania State University, University Park, May 4, 1963.

ILLINOIS, Northern Illinois University, De Kalb, May 10–11, 1963.

INDIANA, Evansville College, October 5, 1962.

IOWA, Iowa State University, Ames, April 19–20, 1963.

KANSAS, Kansas State University, Manhattan, April 20, 1963.

KENTUCKY

LOUISIANA-MISSISSIPPI, Buena Vista Hotel, Biloxi, Mississippi, February 15–16, 1963.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Howard University, Washington, D.C., December 1, 1962.

METROPOLITAN NEW YORK

MICHIGAN, Michigan State University, East Lansing, March 23, 1963.

MINNESOTA, Bemidji State College, November 3, 1962.

MISSOURI

NEBRASKA, University of Nebraska, Lincoln, May 3–4, 1963.

NEW JERSEY, Rutgers, The State University, New Brunswick, November 3, 1962.

NORTHEASTERN, Connecticut General Life Insurance Company, Bloomfield, Connecticut, November 24, 1962.

NORTHERN CALIFORNIA, University of California, Berkeley, January 1963.

OHIO, Ohio State University, Columbus, May 4 1963.

OKLAHOMA, Oklahoma City University, November 10, 1962.

PACIFIC NORTHWEST, Western Washington College, Bellingham, June 14, 1963.

PHILADELPHIA, Franklin and Marshall College, Lancaster, Pennsylvania, November 24, 1962.

ROCKY MOUNTAIN, Brigham Young University, Provo, Utah, Spring, 1963.

SOUTHEASTERN, University of Chattanooga, Chattanooga, Tennessee, March 29–30, 1963.

SOUTHERN CALIFORNIA, University of California, Riverside, March 9, 1963.

SOUTHWESTERN, Arizona State College, Flagstaff, April, 1963.

TEXAS, North Texas State University, Denton, April 19–20, 1963.

UPPER NEW YORK STATE, University of Buffalo April 27, 1963.

WISCONSIN, Carroll College, Waukesha, May 4, 1963.