The May Meeting of the Rocky Mountain Section

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novelty to see computation applied to rules of numerical analysis. From tables of Gauss, Laguerre, Hermite, and Lobatto quadrature formulas which have recently been computed with great accuracy, and using the "method of inspection," P. Rabinowitz and the lecturer have formulated and proved a number of asymptotic results relating the abscissas to the weights. Other conjectures, including one by G. Szegö, seem plausible numerically, but proofs are still to be supplied.

7. *Nets and calculus*, by Professor B. J. Pettis, University of North Carolina. (By invitation)

D. B. LLOYD, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-third annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the United States Air Force Academy, Colorado Springs, Colorado, May 6 and 7, 1960. On Saturday the Section enjoyed a joint meeting and lecture with the Rocky Mountain Section of the Society for Industrial and Applied Mathematics. The meeting was divided into several sessions with Professors J. W. Ault, R. M. Elrick, J. W. Querry, J. S. Leech, and A. W. Recht presiding. There were 132 persons registered for the meeting, including 102 members of the Association.

Officers elected at the meeting for 1960–1961 were: Chairman, Professor L. W. Rutland, University of Colorado; Vice-Chairman, Professor L. C. Barrett, South Dakota School of Mines and Technology; Secretary-Treasurer, Professor Leota Hayward, Colorado State University; and Director of High School Mathematics Contest, Professor D. C. B. Marsh, Colorado School of Mines.

The following papers were presented:

1. *Cauchy's theory of characteristics*, by Professor R. W. McKelvey, University of Colorado.

This was a discussion of the recent studies of A. Plis and T. Wazewski on the domain of existence of solutions of the first order nonlinear partial differential equation $F(x, u, u_x)=0$, $x=(x_0, \cdots, x_n)$. Wazewski's derivation [Bull. Acad. Polon. Sci. Cl. III No. 4, 1956, pp. 131–135] of Plis' results [same source, pp. 125–129] is in the context of the classical Cauchy theory of characteristics. The method can be applied, by analogy, to the quasi-linear equation

$$\sum a_i(x, u)u_{x_i}, -b(x, u) = 0,$$

and is simple enough to find a place in elementary textbooks.

2. *Variation of parameters by vector methods*, by Professors L. C. Barrett and C. A. Grimm, South Dakota School of Mines and Technology, presented by Professor Grimm.

For the equation, $a_0y'''+a_1y''+a_2y'+a_3y=f(x)$, where the a's and $f(x)$ are functions of x continuous on a closed interval over which $a_0\neq 0$, whose reduced equation has the independent solutions y_1, y_2, y_3, we assume the particular solution $y=u_1y_1+u_2y_2+u_3y_3=\vec{U}\cdot\vec{Y}$, $\vec{U}$ to be determined by substitution. From this dot product representation it follows immediately that $y=\int_{x_0}^{x}[\vec{Y}(x)\cdot\vec{w}(t)f(t)][a_0(t)W(t)^{-1}]dt$, $\vec{w}=\vec{Y}\times\vec{Y}'$, $W=\vec{w}\cdot\vec{Y}''$, the Wronskian. It was shown how to generalize the results to other orders of equations.

3. *Existence and stability of periodic solutions of weakly nonlinear differential equations*, by Dr. H. R. Bailey, The Ohio Oil Company, Denver Research Center.

Existence and stability theorems for periodic solutions of weakly nonlinear differential systems have been given recently in a number of papers using a convergent method of successive approximations. This method was originally considered by Lamberto Cesari in 1940 for linear systems.

In the present paper the existence and stability theorems are specialized to the case of a weakly nonlinear differential equation and then applied to a nonlinear Mathieu equation. A summary of the corresponding results in more general situations is given.

4. *On solutions of second order differential equations by changing variables*, by Professor F. M. Stein and Mr. R. D. Finley, Colorado State University, presented by Mr. Finley.

In this paper the authors consider the various methods of changing the dependent and independent variables in the equation $y''+P(x)y'+Q(x)y=0$, where it is assumed that the solutions are known to exist, to reduce it to some classical form or to an equation which may be solved by standard means.

5. *Particular solutions for systems of:* (1) *nonhomogeneous, linear, ordinary differential equations*, (2) *nonhomogeneous, linear, ordinary difference equations*, by Professors L. C. Barrett and R. A. Jacobson, South Dakota School of Mines and Technology, presented by Professor Barrett.

In this paper Lagrange's identity and the bilinear concomitant, so familiar in ordinary differential equation theory, are generalized and then applied, as an alternative to variations of parameters, in solving systems of nonhomogeneous linear ordinary differential equations. A parallel treatment of difference equations is also given.

6. *An integral transform*, by Professor F. M. Hudson, Western State College.

If it is possible to write a differential equation in the form (1) $g(gF')'+gF'+F=0$, where g represents any function of x, then the solution of the equation can usually be found by use of the integral transform $T[F(x)]=\int_0^\infty K(m, x)F(x)dx$. The kernel $K(m, x)$ will depend on g and will be given by the equation $K(m, x)=g^{-1}\exp[\int -(m/g)dx]$. This transform will have as its basic differentiation property $T[gF']=mT[F]$. If an equation is written in the form (1), then the most convenient transform will be suggested by the function g. The Laplace and Mellin Transforms are special cases of the transform $T[F(x)]$.

7. *School Mathematics Study Group experimentation in Colorado*, by Professors W. E. Briggs and L. W. Rutland, Jr., University of Colorado, presented by Professor Rutland.

The objectives and accomplishments of the School Mathematics Study Group and of the Colorado Junior High Center and Colorado Geometry Center were reviewed. An outline was given of the SMSG texts for grades seven through twelve.

8. *The power series coefficients of certain L-functions*, by Professor W. E. Briggs, University of Colorado, and Professor R. G. Bushman, Oregon State College, presented by Professor Briggs.

An example is given to indicate a general method for determining the power series coefficients of functions defined by Dirichlet series. If χ is a principal character modulo k and $h=\phi(k)/k$, then $L(s)=\sum_{n=1}^\infty \chi(n)n^{-s}$ can be written as $h/(s-1)+\sum_{r=0}^\infty(-1)^r V_r(s-1)^r/r!$. Let $L_1(s)=L(s)-hs/(s-1)$. Using an integral representation of $L(s)$, the authors derive the Theorem: *If* $u<0$, *then* $\sum_{n\leq x} n^u\chi(n)\log^r n=h\int_1^x t^u \log^r t dt+(-1)^r L_1^{(r)}(-u)+0(1)$. By setting $u=-1$ and letting $x\to\infty$, it follows that $V_0=\lim_{x\to\infty}[\sum_{m\leq x} n^{-1}\chi(n)-h\log x]+h$ and

$$V_r=\lim_{x\to\infty}\left[\sum_{n\leq x} n^{-1}\chi(n)\log^r n-\{h/(r+1)\}\log^{r+1}x\right], \qquad r=1, 2, \cdots.$$

9. *New teacher education program in mathematics at Colorado State College*, by Professor D. O. Patterson, Colorado State College.

The program in mathematics at Colorado State College is to become, beginning in September 1960, more extensive in its offering and follow fairly closely the recommendations of the National Commission on Mathematics. New courses for secondary school teachers include such titles as "set theory," "modern algebra," "probability theory," "analysis," and "statistics." For the elementary school teachers new courses in mathematics are "arithmetic for elementary teachers" and "foundations of arithmetic."

10. *Astronautics program at USAF Academy*, by Colonel R. C. Gibson, USAF Academy.

One of the principal values of the astronautics program at the Air Force Academy is pedagogic, in that mathematics, physics and chemistry are brought together in a challenging and timely way during the two senior semesters.

The astronautics courses are using most, if not all of the mathematics taught in the regular cadet mathematics sequence. Both the mathematics department and the astronautics department are finding it extremely challenging to design the courses to maximize the cadet's ability to use and appreciate the magnificence of mathematics.

11. *Are mathematicians playing fair with Uncle Sam?* by Professor A. W. Recht, University of Denver.

Recent Russian advances in science forced a hysterical Uncle Sam into a crash program to support so-called "modern mathematics." NSF millions have been spent, and perhaps squandered, to support mathematical study groups and institutes to indoctrinate high school teachers. College professors have gone on a binge of riding their mathematical hobbies at government expense. Payola in mathematics is not on the grand scale of TV payola and rigged programs, but has had a profound effect on the morale of mathematics education. Have the mathematical touts had Uncle Sam gamble on the wrong horses? Have the mathematicians a real solution or a racket?

12. *Applications of mathematics in technology*, by Professor N. W. McLachlan, Visiting Professor of Applied Mathematics, University of Colorado (Invited address)

13. *Anti-associative systems*, by Professor C. H. Cunkle and Mr. D. R. Rogers, Utah State University, presented by Mr. Rogers.

A binary operation defined on a set S is anti-associative provided that $a(bc)\neq(ab)c$ for each a, b, c in S. Examples of anti-associative systems of n elements $(n>1)$ with one binary operation were constructed, and these were characterized for sets of 2 and 3 elements. A generalization of associativity was defined for binary operations mapping a finite set onto itself. These operations fell into classes which formed the elements of a permutation group whose identity element was the class of associative operations.

14. *A new nomographic treatment of quartic equations*, by Professor C. R. Wylie, Jr. and Mr. Elbert Johnson, University of Utah, presented by Professor Wylie.

The equation $\phi_1(t)+a\phi_2(t)+b\phi_3(t)+c=0$ can be written as the pair of equations $\phi_1(t)+a\phi_2(t)-p=0$, $b\phi_3(t)+c+p=0$, for each of which a nomogram can be constructed, provided that in the second equation $c+p$ be treated as one of the variables. The contributions of this paper are (1) a description of a mechanical device to facilitate the simultaneous use of these nomograms in the trial and error case when a, b, and c are given and t is required, and (2) the application of this procedure to the solution of the quartic equations $Ut^4+t^3+Vt^2\pm t+W=0$ which are obtained from the general quartic equation $x^4+a_1x^3+a_2x^2+a_3x+a_4=0$ via the substitutions $x=(\pm a_3/a_1)^{1/2}t$, the plus or minus sign being chosen according as $a_1a_3>0$ or $a_1a_3<0$.

15. *Probability in differential equations*, by Captain R. L. Eisenman, USAF Academy.

The solution $y=a-b\cos(Ct+d)$, where a, b, d are constants and C is a random variable, is eventually distributed as $F(k)=\cos(a-k)/b$ regardless of the distribution of C. The problem was motivated by perturbation of a satellite in circular orbit.

16. *An elementary method of determining initial conditions for missile trajectories*, by Professor C. H. Cunkle, Utah State University.

A missile trajectory can be predicted by solving a set of simultaneous differential equations. Such solutions can be very costly, and, in the process of designing a missile, methods are sought for reducing the number of solutions required. By assuming that the range is a quadratic function and using three-point interpolation, the proper initial conditions for a desired range are approximated in a minimum number of trials. The process involves only elementary analytic geometry.

17. *A matrix application of Newton's identities*, by Professor D. W. Robinson, Brigham Young University.

A novel proof, which is based upon Newton's identities, is given of the following theorem. *Let A be an n-by-n matrix over a field of characteristic zero or prime $p>n$. Then A is nilpotent if and only if trace $A^k=0$, $k=1,\cdots,n$.* Other applications are also suggested.

18. *An application of Markov processes in inventory theory*, by Professor P. W. Zehna, Colorado State College.

In recent investigations in the area of inventory depletion, it was possible to define a sequential model for issuing items from a stockpile. By considering the ages of the items to be random variables, a stochastic process was determined for each of two issuing schemes that were of interest. Further examination revealed imbedded Markov processes in each scheme. It was then possible to find unique stationary absolute probability distributions for each scheme and compare them on the basis of the statistical equilibrium afforded by the stationary distributions.

19. *Reflections of a mathematician*, by Professor L. J. Mordell, Visiting Professor of Mathematics, University of Colorado. (Invited address—SIAM)

The topics discussed were selected from the following: What is mathematics, and what are the difficulties in its study? How are mathematicians made, and how do they work? How do problems arise, and how are they solved? What help is given by the electronic computers? What part is played by memory and luck, and what kind of mistakes and errors do mathematicians make? Finally there are the aesthetic and international aspects of mathematics.

The following papers were presented by title:

20. *Limits of iterated discontinuous functions*, by Professor Emeritus A. J. Kempner, University of Colorado.

21. *On Whitworth's Exercise 667*, by Lt. D. R. Barr, USAF Academy.

At least four solutions of problems equivalent to Exercise 667 in Whitworth's *DCC Exercises in Choice and Chance* have appeared in the literature since the publication of the latter in 1897 (see references in J. O. Irwin, J. Roy Statist. Soc., vol. 118, pp. 393–396). A new solution, using only basic definitions and the formula for the simultaneous occurrence of exactly m among N events, is presented.

22. *Green functions for systems of differential equations*, by Professors L. C. Barrett and R. A. Jacobson, South Dakota School of Mines and Technology.

The present note is concerned with extending the familiar concept and usage of Green's function to nth order systems of ordinary linear differential equations with two-point boundary conditions.

23. *Methods for calculating principal idempotents*, by Mr. J. C. Higgins, Brigham Young University.

24. *On a generalization of Pythagorean theorem*, by Professor Aboulghassem Zirankzade, University of Colorado.

The generalized Pythagorean theorem, in Euclidean n-space, is well known and a simple proof, using methods of vector analysis, exists. A more elementary proof, using only plane and solid Euclidean geometry, is given for the case $n=3$ or $n=4$. It is also shown how this method could be modified to prove the generalized theorem for the case $n>4$. Because of the existence of this simple proof, the generalized theorem becomes a suitable topic to be offered in secondary school.

F. M. CARPENTER, *Secretary*

THE MAY MEETING OF THE WISCONSIN SECTION

The twenty-eighth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at Mount Mary College, Milwaukee, Wisconsin, on May 7, 1960. Professor C. B. Hanneken, Chairman of the Section, presided. This meeting was held jointly with the May meeting of the Wisconsin Mathematics Council and there were 129 present, including 52 members of the Association and 62 members of the Wisconsin Mathematics Council.