puter program, introduced by Ramon E. Moore in "range" arithmetic, is a very hopeful step in the right direction. The need for seeking processes that are numerically stable must be emphasized. Finally, some implications of function theoretical concepts were touched upon.

6. *Minimum variance of estimates under stratified sampling*, by Professor W. R. Van Voorhis, Fenn College.

When a population is stratified for purposes of sampling, the variance of the estimated sample mean, x, depends not only upon the allocation of the sample of size n to the several k strata, but also upon the location of x_i, the points of stratification. Necessary and sufficient conditions to yield minimum variance have been established by Dalenius but these conditions do not yield the explicit values of x_i that must be known before an optimum stratification can be made. It is shown that no general proof of the existence of uniqueness is possible. For the case of "proportional" allocation, it is shown that there exists at least one optimal solution for k strata. A method of successive approximations beginning with a first feasible solution is discussed, and examples are given for several well-known distributions.

7. *Mutation view of conics* (*shadow transformations and primal states*), by Dr. Beckham Martin, Owens-Illinois Glass Company, Toledo, Ohio.

In the presentation the following salient remarks were made: (A) There had to be a clean break with conventional geometry, which has reached a state of stagnation, before one could ever hope to reach new pinnacles of achievement. (B) Mutation Geometry is the science of intangible change (shadow-transformations). The discussion began with the general conic equation: (1) $Ax^2+Bxy+Cy^2=Dx+Ey+F$. A primal state number P was calculated: (2) $P=2/(\sqrt{B^2+(A-C)^2}+A+C)$ by which (1) was transformed shadow-wise to its primal state: (3) $ax^2+bxy+cy^2=dx+ey+f$ from which the properties of the representative conic may be read off at sight. Example: The eccentricity is given by (4) $e^2=2-(a+c)$

8. *The 1959 mathematical program*, by Professor R. L. Wilson, Ohio Wesleyan University.

A contrast is made between the type of mathematics currently being used in the physical and nonphysical sciences and the type of mathematics so applied a decade or more ago. Implications are drawn for the mathematical education of students in the various fields of specialization. Alternative suggestions for meeting this situation are made.

9. *Composite pattern of primitive Pythagorean triangles formulated by arithmetical progression*, by Mr. R. J. Irwin, Eddie Painton Associates, Inc., Cleveland, Ohio.

The method presented is believed to be easier and quicker to compile than the methods more commonly used. The non-Pythagorean Triangles are eliminated very readily as they follow a rhythmic appearance in these tables. Periodic checks throughout the tables automatically correct preceding calculations. These tables discovered two (probably typographical) errors in existing published tables. The interesting relation of Pythagorean Triangles to prime numbers is also shown.

FOSTER BROOKS, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-second annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Utah State University, Logan, Utah, on Friday afternoon and evening and Saturday forenoon, May 8 and 9, 1959. The meeting was divided into several sessions with Professors N. C. Hunsaker, Joe Elich, J. H. Barrett, and Harvey Fletcher presiding. There were 94 persons registered for the meeting, including 60 members of the Association.

Officers elected at the meeting for 1959–1960 were: Chairman, Colonel J. W. Ault, United States Air Force Academy; Vice-Chairman, Professor L. W. Rutland, University of Colorado; and Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *A method of approximating the roots of an equation by quadratic formulae*, by Professor Stephen Kulik, Utah State University.

Two zeros of an analytic function $f(z)$ are approximated with a prescribed accuracy by one application of a quadratic formula. The coefficients of the quadratic equation depend on $D_n(z)$ which is calculated recursively,

$$D_n = f'D_{n-1} - f''fD_{n-2}/2! + \cdots + f^{(n-1)}(-f)^{n-2}D_1/(n-1)! + f^{(n)}(-f)^{n-1}D_0/(n-1)!,$$
$$D_0 = 1, D_1 = f', D_2 = f' - f''f; \quad \text{where} \quad D_n = D_n(z), f^{(n)} = f^{(n)}(z), \ n = 0, 1, \cdots.$$

The roots of the quadratic equation converge to the two zeros of $f(z)$ which are nearer to the number z than the remaining zeros.

2. *A note on a simple matrix isomorphism*, by Professor D. W. Robinson, Brigham Young University.

Let C be the field of complex numbers. Let $\phi : \alpha\phi = A$ be the well-known ring isomorphism of C onto a real subsystem of 2-by-2 matrices over C. Let $f(x)$ be a polynomial over C. It is shown that $\phi(f(\alpha)) = f(\phi(\alpha))$ if and only if $f(\bar{\alpha}) = \overline{f(\alpha)}$, where $\bar{\alpha}$ is the complex conjugate of α. This result is then generalized by considering (1) a ring isomorphism of the n-by-n matrices over C onto a real subsystem of the $2n$-by-$2n$ matrices over C, and (2) functions of matrices.

3. *Definition of "plus" and "times" for the natural numbers*, by Mrs. Jean J. Pederson, Olympus Senior High School of Salt Lake City and University of Utah.

A function may be defined as a set of ordered pairs such that no two distinct pairs of the set have the same first element. Introducing the natural numbers according to the technique of Peano, explicit definitions, as sets of ordered pairs, may then be exhibited for the addition and multiplication functions as applied to the natural numbers.

4. *An extreme value problem for honor students*, by Captain R. C. Rounding and Captain R. L. Eisenman, United States Air Force Academy.

This paper explores the relationship between a problem in which the volume of a cylindrical solid is given and the relative dimensions are to be found to minimize surface area, and the corresponding problem of a rectangular region wherein the area is constant and the perimeter is to be minimized. It is presented as a problem for Honor Students as an example of a technique in mathematical research.

5. *The multiplicity and positiveness of the characteristic numbers of second order Sturm-Liouville systems involving generalized boundary conditions*, by Professor L. C. Barrett, South Dakota School of Mines and Technology.

This paper is concerned with Sturm-Liouville systems that possess boundary conditions involving left and right hand limits of the dependent variable and its first derivative at one or more interior points of a given fundamental interval. Two theorems are cited which provide introductory information concerning the nature of the characteristic functions of such systems. A third theorem desoribes the orthogonality of the characteristic functions. It is then shown that the characteristic numbers are simple roots of the characteristic equation. Sufficient conditions that these characteristic values be non-negative are also given.

6. *Distribution of zeros of solutions of complex differential equations*, by Mr. N. H. Mines, University of Utah.

A zero-free region of a nontrivial solution of the complex differential equation $[K(z)W']'$

$+G(z)W=0$ for $K(z)\equiv 1$ was obtained by E. Hille (*Transactions American Mathematical Society*, vol. 23, 1922). The same method can be used to obtain a zero free region of a nontrivial solution of the general equation requiring only analyticity of the coefficients and $K(z)\neq 0$.

7. *A vector solution of simultaneous linear equations*, by Professor C. A. Grimm, South Dakota School of Mines and Technology.

Three points in the plane, $Ax+By+Cz=D$, $D\neq 0$, are sufficient to determine, to a scalar multiple, A, B, C. The vector $[A, B, C]$ is orthogonal to the plane, and is a scalar multiple of the cross product of two vectors in the plane determined by the three points in the plane. By generalizing this idea a method is developed by which any set of n nonhomogeneous linear equations in n variables may be solved by evaluating one n by n determinant.

8. *Homogeneous production function*, by Professor E. A. Davis, University of Utah.

9. *Lattice points*, by Professor T. M. Apostol, California Institute of Technology (Lecture sponsored by Mathematical Association of America).

10. *The Caltech experiment in calculus*, by Professor T. M. Apostol, California Institute of Technology. (Invited Address).

11. *An undergraduate course on the topology of a line*, by Professor C. E. Burgess, University of Utah. (Invited Address).

The author described an introductory undergraduate topology course which is based upon axioms that describe a linearly ordered, separable, connected space with no first point and no last point. Such a course has been offered at the University of Utah each year for the last several years. A similar address was given before the Wisconsin Section at Whitewater, Wisconsin, May 11, 1957 (this MONTHLY, vol. 64, 1957, p. 627).

12. *Particular solutions for nonhomogeneous, linear, ordinary difference equations*, by Professors Forrest Dristy and L. C. Barrett, South Dakota School of Mines and Technology, presented by Professor Dristy.

In this paper an identity is derived which relates adjoint difference expressions in much the same way that Lagrange's identity of differential equation theory relates adjoint differential expressions. It is then shown how this identity may be used to determine a particular solution of a nonhomogeneous linear ordinary difference equation once the complementary function is known. Thus, the method provides an alternative to the familiar method of variation of parameters.

13. *Application of Fourier series to difference equations*, by Lieutenant J. N. Christiansen, United States Air Force Academy.

A method for obtaining general solutions to linear difference—differential equations is presented. The method is applied to a simple example and the solution is plotted for a special case. The method presented is valuable in that it requires no knowledge of mathematics beyond that usually gained from a course in advanced calculus. It is easy to apply and reduce the solution to quadratures in a very few steps. Also, the solution contains the initial conditions explicitly. The method is analogous to the use of the Fourier transform for finding solutions to partial differential equations.

14. *Geometrical motivations for determinant type proofs of mean value theorems*, by Mr. R. A. Jacobson and Professor L. C. Barrett, South Dakota School of Mines and Technology, presented by Mr. Jacobson.

The usual proofs of the mean value theorems involve the process of applying Rolle's Theorem to functions or determinants happily designed to yield the desired conclusions. The determinants thus employed are usually introduced without comment. In this paper it is shown that these determinants may be motivated by an analysis originating in a geometrical setting.

15. *An approach to the foundations of intuitionism*, by Mr. David Drake, University of Colorado.

The concept of a formal proof was adapted to intuitionist philosophy by replacing axioms and primitive rules of inference with other assertability criteria, such as metamathematical observation. The primitive symbols of logic were defined by means of words having reference to finite and perceptually concrete situations. An axiom of formalized intuitionist logic was then derived—for the given interpretation of symbols—by the modified proof method.

16. *The program of advanced placement in mathematics at Colorado State University*, by Professor F. M. Stein, Colorado State University.

Many students enter Colorado State University with more than the minimum background to enter the regular sequence of mathematics courses. This is first an outline of the method used to select those in this group who could start the sequence at an advanced level, and second a report on the success of the program thus far.

17. *A method of solving Diophantine quadratic equations*, by Professor B. W. Jones, University of Colorado.

Here a method, originating in some ideas of Edgar Emerson, is given for finding all the integral solutions of certain equations of the form $ax^2-by^2=c$ where a, b, and c are positive integers, given a finite number of such solutions. Geometrically this is equivalent to use of a zigzag pattern of lines. This applies also to some quadratic indefinite forms with cross products. In some respects this method is an improvement over the traditional use of the Pell equation.

F. M. CARPENTER, *Secretary*

THE MAY MEETING OF THE WISCONSIN SECTION

The twenty-seventh annual meeting of the Wisconsin Section of the Mathematical Association of America was held on May 2, 1959, at Wisconsin State College, Platteville, Wisconsin, Professor J. V. Finch, Chairman, presiding. There were 69 present, including 33 members of the Association and 30 members of the Wisconsin Mathematics Council, 15 of whom are also members of the Association.

At the business meeting, the following officers were elected for the coming year: Chairman, Professor C. B. Hanneken, Marquette University; Vice-Chairman, Professor Henry Van Engen, University of Wisconsin; Secretary-Treasurer, Sister Mary Felice, S.S.N.D., Mount Mary College.

The following report of the Section's fourth annual mathematics contest for high school students was given by the Contest Committee Chairman, Professor Earl Swokowski, Marquette University:

For the second consecutive year a preliminary contest examination consisting of multiple-choice questions was given for the purpose of enabling the teachers to better select the participants in the final contest. This examination was given to 12,600 students in 283 high schools on February 26, 1959. The top contestant in each school was awarded a certificate.

The final contest was held on April 11, in 27 centers distributed throughout the State, with 900 participating from 187 schools. Cash prizes of \$50, \$25, \$10, and \$1 were awarded to each student in the top 15 percent of the contestants, divided into four groups respectively. In addition initialed M.A.A. award pins were given to the twenty in the top two groups and a certificate to the rest of these groups.

After some discussion during which various suggestions were offered for subsequent contests, the Section Chairman was instructed to continue to carry on the contest as in the past two years.

After an address of welcome by Professor Bjarne Ullsvik, President of Wisconsin