5. *A representation symbol applied to Waring's theorem, modulo p,* by Professor J. D. Elder, St. Louis University.

The author gave an expository account of the representation symbol, $[a, b, c]$, introduced by Sr. M. F. Torline, C.S.J. in her doctoral dissertation. Implicative properties were given, and their uses in problems connected with Waring's theorem were discussed.

6. *Electronic computers, information and education,* by Professor P. C. Hammer, University of Wisconsin. (By invitation.)

If it is agreed that a proof is in a chain of symbols, machines can and do prove theorems. More generally, they answer questions. The principal role of computing machines is to prove propositions and answer questions. It is an obstacle to the use of machines that mathematicians consider they are dealing with infinite processes, which they say cannot be done by machine. While it may be debated whether there are infinite processes, there is no doubt that no one deals with any process with infinite means. Hence, there are no stated proofs which could not be duplicated on a computer.

MARY L. CUMMINGS, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-first annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College, Greeley, Colorado, on Friday afternoon and evening and Saturday forenoon, May 9 and 10, 1958. Professor D. O. Patterson, Chairman of the Section, presided at all three sessions. On Saturday morning the Section held a joint meeting and luncheon with the Colorado Council of Teachers of Mathematics.

There were 107 persons registered for the meeting, including 67 members of the Association. Officers elected at the meeting for 1958–1959 were: Chairman, Professor N. C. Hunsaker, Utah State Agricultural College; Vice-Chairman, Professor J. W. Ault, United States Air Force Academy; and Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *Solving boundary value problems by use of Green's function in conjunction with the Laplace transform and separation of variables,* by Professor L. C. Barrett, South Dakota School of Mines.

The primary purpose of this paper is to point out how an influence function, *i.e.* Green's function, may be utilized together with the Laplace transform and separation of variables to facilitate a solution of boundary value problems of engineering and physics. Among the notable features of the method are: (a) Its capacity to yield the inverse of certain Laplace transforms without requiring recourse to complex variable theory. (b) The method enables one to escape the tedium of the step-by-step procedure, and subsequent use of superposition, usually followed in solving such problems by separation of variables. (c) Time-dependent boundary conditions present no special difficulty to the method.

2. *The radiation of waves from a point source,* by Professor R. W. McKelvey, University of Colorado, introduced by the Secretary.

The object of the paper is to obtain by a new method, a known expression for a *radiation solution* of the generalized wave equation,

$$\frac{\partial^2 u}{\partial t^2} = \sum_{i,j=1}^{3} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{3} a_i \frac{\partial u}{\partial x_i} + au.$$

The coefficients a_{ij}, a_i, a are variable functions of space, but are constant in time. The matrix (a_{ij})

is positive-definite. [For an exact definition of a radiation solution, see Courant-Hilbert, *Mathematischen Physik II*, Berlin, 1937, p. 453]. The formula in question has been obtained by the methods of Hadamard. [*loc. cit.*, p. 154]. The procedure given here avoids many of the complications of those methods, while preserving the spirit. It consists of a construction process, resembling Hadamard's construction of the fundamental solution. [J. Hadamard, *Lectures on Cauchy's Problem*, New York.]

3. *Coset spaces in topological groups and their relation to the group*, by Mr. D. A. Ford, Graduate Assistant, University of Utah.

A topological group is an abstract group defined on the elements of a topological space where x^{-1} is continuous in x, and the product xy is continuous in x and y simultaneously. If H is a subgroup of a topological group G, then the space of left cosets G/H is a topological space and if H is invariant, G/H is a topological group. The paper deals with the relation of topological properties among the group, subgroup and coset space. For example, if H and G/H are compact, then G is compact.

4. *Geometries based on the undefined terms "sets" and "inclusion,"* by Professor Aboulghassem Zirakzadeh, University of Colorado.

E. V. Huntington, in 1913, introduced a set of axioms for Euclidean Geometry which was based on the undefined terms "set" and "inclusion." Using these undefined terms and changing the given axioms, it is possible to find other geometries, finite and infinite. A minimum set of axioms is given to insure that the resulting geometries are sufficiently regular. The consistency and independence of these axioms is proven.

5. *Variation of parameters in solving systems of difference equations*, by Professor L. C. Barrett and Professor F. E. Dristy, South Dakota School of Mines, presented by Professor Dristy.

The primary purpose of this paper is to illustrate how the method of variation of parameters, so familiar in finding particular integrals of nonhomogeneous linear differential equations, may be extended to determine a particular solution of a nonhomogeneous system of linear difference equations. At the same time a technique is developed for solving such a system which, in contrast to the usual procedure, may be applied directly to the system. Thus, the usual reduction of the system to a single difference equation before solving becomes unnecessary.

6. *Heat conduction with variable thermal conductivity in a sphere*, by Professor Nathan Schwid, University of Wyoming.

When the dependence of the diffusivity coefficient $k/c\rho$ of the heat conduction equation upon the temperature is sufficiently significant to warrant its consideration, the equation becomes nonlinear. The quantity k, the thermal conductivity, is then a function of temperature with $c\rho$, the product of specific heat and density, constant. If we take $k/c\rho$ as $\alpha+\beta u$, where β/α is small, and u is the temperature, an approximation to the temperature in a sphere under simple boundary conditions is obtained which approximates the solution for substantial values of t.

7. *The geometry of* $f(n, \alpha)=\sum e^{ik\alpha}$, $k=0, \cdots, n$, by Professor Emeritus A. J. Kempner, University of Colorado.

An obvious vector construction is combined with the geometrical multiplication in the plane of complex numbers of all points on one curve by all points of a second curve to obtain results of which the following is representative: The "Wertevorrat" (Set of values assumed) of $\sum\sum e^{i(k_1\alpha_1+k_2\alpha_2)}=f(n_1, n_2)$, α_1/π, α_2/π irrational, k_1, k_2 independently over $0, 1, \cdots, n_1$, and $0, 1, \cdots, n_2$, respectively, $n_1, n_2<\infty$ is given by the cardioid $a\rho \sin(\alpha_1/2)\sin(\alpha_2/2)=1-\cos(\dot{\theta}+\alpha_1/2+\alpha_2/2)$. In this cardioid the functional values are distributed everywhere densely.

8. *Mathematics program for outstanding cadets at USAFA*, by Professor W. Milliken, United States Air Force Academy.

Three levels of mathematics have been established at the Air Force Academy. The regular course is the usual two-year engineering mathematics course with spherical trigonometry, some statistics and differential equations added. A cadet may advance from this course to the accelerated course at the beginning of the second semester. The accelerated course covers the same material plus a course in elementary statistics in a year and a half. The super-accelerated course covers this material in one year. Thus, there is extra time available for cadets in the faster programs to take additional advanced mathematics courses or other electives.

9. *Mathematical education in Europe, Britain, and the United States,* by Professor W. W. Rogosinski, King's College, Durham University, England; Visiting Professor, University of Colorado.

An attempt is made to point out and to explain the striking differences in mathematical education, both at high school and university level, as seen in Continental Europe, Britain, and the United States of America. The explanation is sought in a different philosophy of education in general which, in turn, is conditioned by different history and tradition: the scholastic idealism of Europe, its realistic variant in Britain, and the social (and materialistic) trend in American education.

10. *A sequel to Euclid*, by Professor H. S. M. Coxeter, University of Toronto. (Invited Address.)

11. *Coaxal circles and inversion*, by Professor H. S. M. Coxeter, University of Toronto.

Any two given circles belong to a pencil of coaxal circles consisting of all the circles orthogonal to any two circles, α and β, orthogonal to the two given circles. The arbitrariness of α and β is established by inverting the two given circles into straight lines or concentric circles. When inverted with respect to a sphere whose center is outside the plane, two orthogonal pencils of coaxal circles yield sections of a sphere (the inverse of the plane) by pencils of planes through two polar lines. Such a pencil of circles is hyperbolic (*i.e.*, intersecting), parabolic (touching), or elliptic (disjoint) according as the common line of the planes is a secant, a tangent, or an exterior line.

12. *Oscillation and non-oscillation of second order complex differential equations*, by Mr. R. W. Hunt, Graduate Assistant, University of Utah.

The primary object of this paper was to investigate the zeros on $a \leqq x < \infty$ of solutions of the differential equation $(py')' + fy = 0$, with p and f complex-valued continuous functions of the real variable x. By the use of an associated system of two real, second-order equations obtained by writing p, f, and y in polar form, two sufficient conditions for disconjugacy (at most one zero on $a \leqq x < \infty$) of all nontrivial solutions were obtained. Then a special form of this equation, $(y'/q)' + \bar{q}y = 0$, q complex-valued, was changed to the first order system $y' = q\bar{z}$, $z' = -q\bar{y}$, with solutions $s(x) = s[a, x; q]$ and $c(x) = c[a, x; q]$ corresponding to the boundary conditions $y(a) = 0$, $z(a) = 1$. Finally $s[a, x; q]$ and $c[a, x,; q]$ were shown to have two properties analogous to well-known properties of the real sine and cosine functions; namely, $|s|^2 + |c|^2 \equiv 1$ and, for $k = 1$, $s[a,x; kq] = ks[a, x; q]$, $c[a, x; kq] = c[a, x; q]$.

13. *A multiple integral approach to Taylor's theorem*, by Professor L. C. Barrett and Mr. D. W. Willett, Student, South Dakota School of Mines, presented by Mr. Willett.

This note presents several elementary geometrical considerations, involving lengths, areas, and volumes, which lead quite naturally to a multiple integral approach to Taylor's theorem.

14. *Evaluation of a limit from the theory of heat flow*, by Dr. H. R. Bailey, Mathematician, Ohio Oil Company Research Center, Littleton, Colorado, introduced by the Secretary.

The problem of heat conduction in an infinite homogeneous medium from the surface of a cylinder whose radius is increasing with time is solved by the Green's function method. The solution is obtained as an integral of the form $I=\int_0^t f(t, \tau)d\tau$. A method is given to obtain an explicit evaluation of this integral for $t\to\infty$ for the case of the cylinder radius increasing at a constant velocity. It is shown that the integral can be divided into two parts, $I=\int_0^{t/N} f(t,\tau)d\tau+\int_{t/N}^t f(t,\tau)d\tau$, where the last integral goes to zero as $t\to\infty$ and the integrand in the range $[0, t/N]$ can be replaced by an asymptotic expression which can be integrated explicitly as a function of N. Finally the desired limit is obtained by passing to the limit as $N\to\infty$.

15. *An application of the decomposition of a matrix into principal idempotents*, by Professor D. W. Robinson, Brigham Young University.

As a simple application of the decomposition of a (diagonable) matrix into principal idempotent elements, this note provides a proof of the following well-known result: if the nth derivative of a function f exists at α, then it can be computed as the limit of $h^{-n}\sum_{m=0}^{n}\binom{n}{m}(-1)^m f[\alpha+(n-m)h]$ as h approaches zero.

16. *Families of Sturm-Liouville systems*, by Mr. E. L. Dunn, Colorado State University, introduced by Professor F. M. Stein, Colorado State University.

A family of Sturm-Liouville systems is defined as the collection of all Sturm-Liouville systems whose equations can be obtained by repeatedly differentiating and integrating a Sturm-Liouville equation. For each system, similar boundary conditions apply to the same interval. In this paper the conditions are developed such that a Sturm-Liouville system may generate a family. It is shown that (1) if a Sturm-Liouville system generates a family, the kth derivatives and antiderivatives of its eigenfunctions form orthogonal sets, and (2) if the eigenfunctions of the generating system are not polynomials the sets of eigenvalues for all members of a family are identical. The case when the eigenfunctions are polynomials must be considered separately.

17. *Let's not go off the deep end*! by Professor A. W. Recht, University of Denver.

The general theme is in opposition to the idea of introducing Boolean algebra, sets, and similar types of theoretical mathematics into high school and elementary college courses. We are already teaching too much "gifted" mathematics to the general student, and not teaching successfully the kind of mathematics the 85 per cent or perhaps the 100 per cent, ought to have before they go into the so-called superior pure mathematics. Maybe we have an inferiority complex, and are running away from our real job, which is to bring up all people in our democracy to their full potentialities with a more democratic kind of mathematics.

18. *The work of the Commission on Mathematics*, by Professor Henry Van Engen, University of Wisconsin, Madison.

See this MONTHLY, Report of the May Meeting of the Wisconsin Section.

F. M. CARPENTER, *Secretary*

MAY MEETING OF THE WISCONSIN SECTION

The twenty-sixth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at Carroll College, Waukesha, Wisconsin, on May 3, 1958, Professor R. D. Wagner, Chairman, presiding. Sixty-nine attended the meeting, including forty members of the Association.

At the business meeting of the Section the following officers were elected for the coming year: Chairman, Prof. J. V. Finch, Beloit College, Beloit, Wisconsin; Vice-Chairman, Prof. C. B. Hanneken, Marquette University, Milwaukee, Wisconsin; Secretary-Treasurer, Sister Mary Felice, Mount Mary College, Milwaukee, Wisconsin.

Mr. J. W. Kennedy gave the following report of the 1958 high school mathematics contest: A preliminary contest was held on Feb. 27, in 237 schools in the state, with