7. *Commutativity of finite matrices*, by Dr. Olga Taussky-Todd, National Bureau of Standards, introduced by the Secretary.

Various generalizations of the concept of commutativity of a pair A, B of finite matrices are being discussed. One such generalization consists in assuming that all polynomials $p(A, B)$ have as eigenvalues $p(\alpha_i, \beta_i)$ where α_i, β_i are the eigenvalues of A and B taken in a special order. A more recent generalization assumes that only all linear combinations $\lambda A+\mu B$ have as eigenvalues $\lambda\alpha_i+\mu\beta_i$. A further generalization is obtained by studying the matrices in the pencil $\lambda A+\mu B$ which have multiple characteristic roots. The vanishing of the higher commutators of A and B also plays a big role. If in particular $B=A^*$ (the complex conjugate and transpose of A) then the vanishing of the higher commutators sheds light on the nature of A itself.

R. P. BAILEY, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-ninth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Utah, Salt Lake City, Utah, on May 4 and 5, 1956. Professor C. R. Wylie, Jr., Chairman of the Section, presided at all three sessions.

There were 77 persons registered for the meeting, including 48 members of the Association.

Officers elected at the meeting for 1956–1957 were: Chairman, Professor R. R. Gutzman, Colorado School of Mines; Vice-Chairman, Professor J. S. Leech, Colorado College; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *Connectedness in partially ordered sets*, by Professor L. E. Ward, Jr., University of Utah.

A partially ordered space is a triple $(X, T, <)$ where (X, T) is a topological space, $(X, <)$ is a partially ordered set, and there obtains some harmonious relation between T and $<$. Toward the end of characterizing various partially ordered spaces theoretically, results of the following type are of interest: order hypothesis→topological conclusion. Sample theorem: if the set of $a \leqq x$ is compact for each $x \in X$, if the partial order is continuous and dense, and if each pair of points of X have a common predecessor, then X is connected.

2. *Convergence of sequences of linear fractional transformations*, by Professor W. J. Thron, University of Colorado.

Let $\{T_n(z)\}$ be a sequence of nonsingular linear fractional transformations. Let Z be the set in the complex plane, such that $\{T_n(z)\}$ converges for all $z \in Z$. Finally, let $T(z)$ be the function to which the sequence converges on Z. Possible domains of convergence Z and corresponding limit function $T(z)$ are determined. Among other results it is proved that $T(z)$ must either be a constant for all $z \in Z$, or a constant for all but one value of Z, or a linear fractional transformation (in this case Z is the whole plane).

3. *Orthonormal functions used for the approximate solution of integro-differential equations*, by Professor F. M. Stein, Colorado Agricultural and Mechanical College.

Under certain conditions the integro-differential equation $U(u) \equiv L(u) - \int_a^b h(x, t)u(t)dt = f(x)$

with boundary conditions $U_i(u)=0(i=1, 2, \cdots, m)$ has a unique solution of the form $u(x) = \int_a^b H(x, t)f(t)dt$, where $H(x, t)$ is the Green's function of the problem. Let $S_n(x)$ be a sum of functions of the complete set $\phi_i(x)$ which are orthonormal with respect to the weight function $\rho(x)$ and which satisfy the boundary conditions. Under the criterion that $\int_a^b |f(x)-U[S_n(x)]|^r dx$ shall be least for $r>0$, this paper examines the sufficient condition that $S_n(x)$ as well as its first m derivatives be uniformly convergent to the corresponding derivative of the solution $u(x)$ as n becomes infinite.

4. *The Dirichlet series transformation*, by Professor J. R. Britton, University of Colorado.

Let $F(t)$ be a complex valued function defined for nonnegative integral values of t. If a is a positive constant, the Dirichlet series $\sum_{t=0}^{\infty} a^{-st}F(t)$ is the Dirichlet series transform, $D\{F(t)\}$. The series converges, for example, if $F(t)=\theta(k^t)$, $k>0$, $t>t_0$. Simple properties of the transform were developed, in particular, the convolution theorem

$$[D\{F(t)\}][D\{G(t)\}] = D\left\{ \sum_{t=0}^{\tau} F(t-\tau)G(\tau) \right\}.$$

Application was made to "summation" equations of the convulution type $\sum_{t=0}^{\tau} F(t-\tau)G(\tau) = H(t)$, where $G(t)$ and $H(t)$ are given functions, and $F(t)$ is to be found.

5. *Series solution of linear differential equations*, by Mr. C. A. Grimm, South Dakota School of Mines and Technology.

By the use of the exponential shift formula,

$$P(\Delta)e^{mt}f(t) = e^{mt}P(\Delta + m)f(t)$$

and the substitution $x-a=e^t$, one easily arrives at the formula $P(\Delta)(x-a)^m = P(m)(x-a)^m$. Then by writing (if possible) a linear differential equation in the form $f(\Delta)y+(x-a)^k g(\Delta)y=0$ with the assumed solution $y=\sum_{n=0}^{\infty} c_n(x-a)^{\lambda+Un}$, $c_0\neq 0$, the recurrence relation, standardly found by the method of Frobenius, is readily obtained.

6. *Homogeneous continua*, by Professor C. E. Burgess, University of Utah.

A brief history of results on homogeneous continua was given, and some related unsolved problems were mentioned.

7. *Finite geometries and difference sets*, by Professor B. W. Jones, University of Colorado.

Cyclic finite geometries were defined and it was shown that any such geometry leads to a perfect difference set. Conversely, any perfect difference set leads to a finite geometry. One or two generalizations were indicated.

8. *Decimal expansions*, by Dr. Bodo Volkmann, University of Mainz, Visiting Professor, University of Utah. (By invitation.)

Borel proved in 1909 that almost all real numbers are normal, *i.e.*, their decimal expansion involves each of the possible digits with the same asymptotic frequency. A number of generalizations of Borel's theorem to the digit distribution of non-normal numbers were obtained by Besicovitch, Knichal, Eggleston, and the speaker. These theorems describe certain sets of real numbers in terms of their Hausdorff (or fractional) dimension. Among the unsolved problems on decimal expansions are questions such as whether numbers like e, π, or $\sqrt{2}$ are normal in any scale, and what conclusions can be drawn from the normality of a real number in a given scale about its digits distribution in any other scale.

9. *Some constants associated with the Riemann zeta-function*, by Dr. W. E. Briggs, University of Colorado.

The Riemann zeta-function has the representation

$$\zeta(s) = \frac{1}{s-1} + \sum_{0}^{\infty} \frac{(-1)^n \gamma_n}{n!} (s-1)^n$$

where

$$\gamma_n = \lim_{n \to \infty} \left[\sum_{t=1}^{N} \frac{\log^n t}{n} - \int_1^N \frac{\log^n x}{x} dx \right]$$

and satisfies the functional equation $\zeta(s) = 2^s \pi^{s-1} \sin s\pi \Gamma(1-s)\zeta(1-s)/2$. By letting $s = 1-2m$, where m is a positive integer, it can be shown that infinitely many γ_n are positive, and infinitely many are negative. From a representation of γ_n as an infinite series, it can be shown that $\gamma_n = O\{(n/2e)^n\}$. Using the fact that $\zeta(s) - 1/(s-1)$ is an entire function of order one, it can be shown that if $\epsilon > 0$, then $n^{-\epsilon n} < |\gamma_n| < n^{\epsilon n}$ is true for infinitely many n.

10. *Production delay period*, by Professor E. A. Davis, University of Utah.

In the production of economic goods a "production delay period" elapses between the time factors are joined in a firm's productive mechanism and the time resulting output materializes. The delay period, "time", acts somewhat as a productive factor whose cost is expressed through the interest rate. The author considered adjustments of a firm attempting to maximize "profit" integrals, strict competition being assumed, under the assumption that the production delay period for the firm was: a) of zero length; b) of fixed length; c) of length subject to entrepreneur control. The latter two cases lead naturally to a theory involving producer expectations.

11. *Summing of series with missing terms*, by Dr. H. J. Fletcher, Brigham Young University.

A method is presented to sum many series of the form $\sum a_n b_n$ where the series $\sum a_n$ is known and b_n is a periodic function of n. The method consists of expressing b_n as a finite sum of sines and cosines, and summing the corresponding trigonometric series. In particular, if the series $\sum a_n \sin nx$ can be summed to a known function, then the corresponding sum which has terms missing periodically can be summed to a known function.

12. *Fitting empirical equations to data from small orifice fluid meters*, by Professor S. R. Smith, University of Wyoming.

Two empirical equations, $C = R/(a+bR)$ and $C = d + k/\sqrt{R}$ were fitted to the orifice meter data (C, R data was determined by Professor Eric Lindahl, University of Wyoming). Water was the fluid used in each case. C is the coefficient of discharge of the meter and R its corresponding Reynolds number, both dimensionless.

13. *Errors in linear systems*, by Professor C. A. Hutchinson, University of Colorado. (Presented by title.)

14. *Coordination of college and high school mathematics topics*, by Professor O. M. Rasmussen, University of Denver.

The author expressed the need for coordination of mathematics programs in the high schools and colleges, especially with respect to topics of modern mathematics. This need was shown by a brief review of some of the present activity concerning modern topics in mathematics. It was emphasized that coordination should take place on national, regional, and local levels with each individual aiding in any manner possible.

F. M. CARPENTER, *Secretary*