

The April Meeting of the Rocky Mountain Section Source: The American Mathematical Monthly, Vol. 62, No. 7 (Aug. - Sep., 1955), pp. 547-550 Published by: <u>Mathematical Association of America</u> Stable URL: <u>http://www.jstor.org/stable/2307390</u> Accessed: 18/01/2015 20:48

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at http://www.jstor.org/page/info/about/policies/terms.jsp

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



*Mathematical Association of America* is collaborating with JSTOR to digitize, preserve and extend access to *The American Mathematical Monthly*.

http://www.jstor.org

THE MATHEMATICAL ASSOCIATION OF AMERICA

9. A note on Fourier coefficients, by Professor R. P. Gosselin, Youngstown College.

Let c(g) be the *n*th Fourier (exponential) coefficient of g(x). Let f(x) belong to  $L^q$ , q an integer  $\geq 2$ . Let  $c_n(f)$  be positive and decrease with 1/|n|. By use of Parseval's formula, it is shown that  $(c_n(f))^q \leq A_q c_n(f^q)/(|n|+1)^{q-1}$ , where  $A_q$  is a constant depending only on q. As an application of this inequality, a proof of the following result, due to Hardy and Littlewood (*Journal of the London Math. Soc.*, vol. 6, 1931, pp. 3-9), is obtained: If f(x) belongs to  $L^r$ ,  $r \geq 2$ , and  $c_n(f)$  decreases, then

$$\sum_{n=-\infty}^{+\infty} |n|^{r-2} (c_n(f))^r \leq B_r \int^{2\pi} |f(x)|^r dx.$$

10. The tensor form of the equations of hydrodynamics, by Mr. W. H. Lane, Wright Air Development Center, introduced by the Secretary.

The tensor forms of the general energy equation and of the Navier-Stokes equations of motion for a viscous incompressible fluid are considered. The special case for spherical coordinates in which the velocity and temperature fields are assumed to be inversely proportional to the power of a radial vector is developed, and the resulting class of exact solutions is discussed.

11. On ideals in the ring of linear multidifferential polynomials, by Mr. Frank Levin, University of Cincinnati, introduced by Professor H. D. Lipsich.

The ring of linear multidifferential polynomials is a noncommutative ring of polynomials in several indeterminates. This ring is not a principal ideal ring, and, therefore, the results are stated ideal-theoretically. A basis of an ideal of multidifferential polynomials which corresponds to the basis of an ideal in the principal ideal ring of ordinary differential polynomials is the *canonical basis* of the ideal. With this basis one is able to provide an upper bound for the length of a basis of the ideal and to give a necessary and sufficient condition for solvability of multidifferential equations.

FOSTER BROOKS, Secretary

## THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-eighth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, Wyoming, on Friday afternoon and evening and Saturday forenoon, April 22 and 23, 1955. Professor Nathan Schwid, Chairman of the Section, presided at all three sessions.

There were 62 persons registered for the meeting, including the following 46 members of the Association:

J. W. Ault, G. E. Bardwell, C. F. Barr, B. C. Bellamy, W. E. Briggs, J. R. Britton, R. K. Butz, F. M. Carpenter, Sarvadaman Chowla, E. L. Crow, W. E. Dorgan, F. N. Fisch, H. T. Guard, Leota C. Hayward, Anna S. Henriques, Archie Higdon, J. E. Householder, Sr., P. F. Hultquist, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, R. B. Kriegh, L. J. Lange, E. B. McLeod, Jr., M. L. Madison, W. D. Marsland, Jr., W. E. Mientka, W. K. Nelson, Greta Neubauer, D. O. Patterson, J. W. Querry, O. M. Rasmussen, O. H. Rechard, A. W. Recht, Calvin A. Rogers, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, P. O. Steen, W. J. Thron, E. P. Tovani, V. J. Varineau, W. W. Varner, C. R. Wylie, Jr.

Officers elected at the meeting for 1955-1956 were: Chairman, Professor

1955]

C. R. Wylie, Jr., University of Utah; Vice-Chairman, Professor R. R. Gutzman, Colorado School of Mines; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. Remarks on the functional equation  $f[f(z)] = e^z - 1$ , by Professor W. J. Thron, University of Colorado.

If the functional equation has a solution f(z) which is holomorphic in a sufficiently large neighborhood of the point z=0 then it can be shown that f(0)=0. Using this one can determine a unique formal power series solution  $f(z) = \sum c_n z^n$ . Let r be the radius of convergence of this series. By means of Picard's theorem, Hadamard's factorization theorem, and a result of Pólya, it is established that  $r < \infty$ . The statement that r=0 but that there exists a solution of the functional equation which is holomorphic for all z, not on the negative real axis, concludes the paper.

2. Note on hemispheric numerical integration of the barotropic model, by Major J. F. Blackburn, USAF Academy, and W. L. Gates, presented by Major Blackburn, who was introduced by the Secretary.

Under the assumption of frictionless, adiabatic flow in hydrostatic and quasi-geostrophic equilibrium, the barotropic equation was solved numerically for hemispheric flow. The method of solution was similar to the scheme outlined by Charney, Fjørtoft and von Neumann (*Tellus*, November, 1950) applied to a smaller area. The procedure consists of the cyclical calculation of the absolute vorticity advection at time t for each point of a finite difference grid, the solution of the resulting finite difference equations for  $\partial z/\partial t$  at each point by a method of relaxation, and the calculation of the heights z at time  $t + \Delta t$  using centered differences over a short time interval.

3. Some simple geometrical properties of the space  $L^2$  by Mr. A. E. Labarre, Ir., University of Wyoming, introduced by the Secretary.

Geometrical interpretations of the Parseval and Riesz-Fischer theorems are given. The space  $L^2$  is the direct product of the even functions of  $L^2$  and the odd functions of  $L^2$ . How the operation of differentiation in  $L^2$  can be interpreted as an orthogonal transformation is explained.

4. The number of lattice points in an n-dimensional tetrahedron, by Professors Sarvadaman Chowla and W. E. Mientka, University of Colorado, presented by Professor Mientka.

Let the  $a_i(1 \le i \le n)$  be positive integers relatively prime in pairs, and  $A = \prod_{i=1}^n a_i$ . In this paper we find exact expressions (which are polynomials in  $\eta/A$  and the  $a_i$ ) for (i) the number of solutions in non-negative integers  $x_i$  of  $\sum_{i=1}^n a_i x_i = \eta$  whenever  $\eta \equiv 0 \pmod{A}$ , (ii) the number of lattice points in the tetrahedron bounded by the planes  $\sum_{i=1}^n a_i x_i = \eta \pmod{A}$ .

5. A possible measure of asymmetry in a line, by Professor Calvin A. Rogers, Colorado Agricultural and Mechanical College.

Eight requirements were set up, formally expressing intuitive convictions about the asymmetry in the x-axis of two points  $P_1$  and  $P_2$  with same abscissa and ordinates,  $y_1$  and  $y_2$ . From these, it was deduced that one of the simplest functions satisfying all requirements was the fraction  $(y_1+y_2)^2/(y_1^2+y_2^2)$ .

6. Remarks on the distribution of primes, by Professors A. J. Kempner and Sarvadaman Chowla, University of Colorado, presented by Professor Kempner.

From the extended Prime Number Theorem two formulae are derived:

$$\frac{\pi(\delta x) - \pi(\gamma x)}{\pi(\beta x) - \pi(\alpha x)} = \frac{\delta - \gamma}{\beta - \alpha} \cdot \left\{ 1 + \mathcal{C}(\alpha, \beta, \gamma, \delta) \cdot \frac{1}{\log x} \right\} + o\left(\frac{1}{\log x}\right)$$

and

$$[\pi(\delta x) - \pi(\gamma x)] - [\pi(\beta x) - \pi(\alpha x)] = [(\delta - \gamma) - (\beta - \alpha)] \cdot \frac{x}{\log x} + \mathcal{C}'(\alpha, \beta, \gamma, \delta) \cdot \frac{x}{\log^2 x} + o\left(\frac{x}{\log^2 x}\right).$$

Specialization of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$  leads to results concerning the distribution of primes.

7. Knots and quadratic forms, by Professor K. A. Hirsch, University of London and University of Colorado. (By invitation).

The speaker discussed certain topological properties of knots by considering the invariants of related quadratic forms.

8. The power series coefficients of L-series, by Dr. W. E. Briggs, University of Colorado.

Consider  $L_k(s) = \sum_{1}^{\infty} \chi(n)n^{-s}$  where  $\chi$  is a real non-principal character mod k. This series can be presented by the power series  $\sum_{1}^{\infty} L^{(r)}(1)(s-1)^r/r!$ . These coefficients can be determined by evaluating the *r*-th derivative of the defining series to obtain

$$L^{(r)}(1) = (-1)^{r} \sum_{1}^{k} \chi(t) \gamma_{r,k,i}, \text{ where } \gamma_{r,k,i} = \lim_{x \to \infty} \left[ \sum_{n \leq x, n \equiv i(k)} \frac{\log^{r} n}{n} - \frac{\log^{r+1} x}{k(r+1)} \right]$$

Similarly the power series coefficients of the zeta function can be determined by considering

$$h(s) = \zeta(s) - \frac{s}{s-1} = s \int_1^\infty \frac{|x| - x}{x^{s+1}} dx = \sum_0^\infty \frac{h^{(r)}(1)}{r!} (s-1)^r$$

and evaluating the expression obtained by differentiating the integral r times. This gives  $h^{(r)}(1) = (-1)^r \gamma_r$  for r > 0 and  $h(1) = \gamma_0 - 1$  where  $\gamma_r = \gamma_{r,1,0}$ .

## 9. A problem in interpolation, by M. L. J. Lange, University of Colorado.

Given a polynomial of degree n with coefficients in a field K,  $f(x) = (x - \alpha_1)^{m_1}(x - \alpha_2)^{m_2} \cdots (x - \alpha_k)^{m_k}$ , and with the  $\alpha_i$  in the root field K' of f(x), and given a polynomial g(z) of arbitrary degree with coefficients in K', the problem is to find a polynomial h(x) of degree  $\leq n-1$  with coefficients in an extension field K'' such that F(x) = g(h(x)) - x is divisible by f(x). The author showed that an h(x) with the required properties exists if and only if for all  $\alpha_i$  with  $m_i > 1$  the equation  $g(z) - \alpha_i = 0$  has at least one simple root. He also gave a method for actually constructing such a polynomial h(x).

10. An outline of the mathematics curriculum and schedule at the USAF Academy, by Colonel Archie Higdon, USAF Academy.

The United States Air Force Academy mathematics curriculum consists of courses in college algebra, plane and spherical trigonometry, analytical geometry, differential and integral calculus, applied calculus, and elementary differential equations for a total of 21 quarter hours.

Students will be sectioned according to demonstrated ability in mathematics with an average of 12.5 students per section. They will be graded almost every day and daily preparation is mandatory. The top sections will cover some advanced topics in each course not required for those with less aptitude for mathematics. These top sections will contain many students who would be admitted with advanced standing in civilian schools. All students are required to complete all four years at the U. S. Air Force Academy regardless of previous college training.

11. A new Poisson equation analog computer, by Mr. W. W. Varner, University of Colorado.

A new computer has been completed at the University for the very rapid solution of the general second order partial differential equation

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + ku + F(x, y, u) = 0$$

with appropriate boundaries. It also solves the Poisson equation in three dimensions with a grid of 960 nodes or mesh points which can be arranged very easily and quickly into Cartesian, cylindrical, spherical, and other coordinate systems. It can handle very complicated boundaries, source and sink conditions, and transients.

12. A review of the 1954 Oregon Summer Conference, by Professor F. M. Carpenter, Colorado School of Mines.

F. M. CARPENTER, Secretary

## CALENDAR OF FUTURE MEETINGS

Thirty-ninth Annual Meeting, Rice Institute, Houston, Texas, December 30, 1955.

Thirty-seventh Summer Meeting, University of Washington, Seattle, Washington, August 20–21, 1956.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

- Allegheny Mountain
- ILLINOIS, Eastern Illinois State College, Charleston, May 11–12, 1956.
- INDIANA, Wabash College, Crawfordsville, May 5, 1956.
- Iowa, Grinnell College, Grinnell, April 20–21, 1956.

Kansas

- Kentucky
- LOUISIANA-MISSISSIPPI, McNeese State College, Lake Charles, Louisiana, February 17–18, 1956.
- MARVLAND-DISTRICT OF COLUMBIA-VIRGINIA, Catholic University, Washington, D. C., December 3, 1955.

Metropolitan New York

- MICHIGAN, University of Michigan, Ann Arbor, March, 1956.
- MINNESOTA, South Dakota State College, Brookings, October 15, 1955.
- MISSOURI, Fontbonne College, St. Louis, Spring, 1956.

Nebraska

NEW ENGLAND, Organizational Meeting, Uni-

- versity of New Hampshire, Durham, November 26, 1955.
- NORTHERN CALIFORNIA

Оню, April, 1956.

- OKLAHOMA, Oklahoma City University, October 28, 1955.
- PACIFIC NORTHWEST, Oregon State College, Corvallis, June, 1957.
- PHILADELPHIA, University of Pennsylvania, Philadelphia, November 26, 1955.

Rocky Mountain

- SOUTHEASTERN, UNIVERSITY OF GEORGIA, Athens, March 16–17, 1956.
- SOUTHERN CALIFORNIA, Pomona College, Claremont, March 17, 1956.
- SOUTHWESTERN, New Mexico College of Agriculture and Mechanical Arts, Las Cruces, April, 1956.
- TEXAS, Southwest Texas State Teachers College, San Marcos, April, 1956.
- UPPER NEW YORK STATE, Alfred University, Alfred, April 28, 1956.
- WISCONSIN, Marquette University, Milwaukee, May, 1956.