where $b$ and $d$ are the orthonormalized bisectors of the given angle and its supplement. The equation was transmuted to a primordial prototype

$$A \cdot r = 0,$$

whose solution was

$$r = 0/0.$$

A second view started from a single prototype which led to a factorable 4th degree equation with a free choice parameter with which to control its irreducibility. A mutation curve was drawn for this which was shown to trisect the angle.

Foster Brooks, Secretary

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-seventh annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado Agricultural and Mechanical College, Fort Collins, Colorado, on Friday afternoon and evening and Saturday forenoon, April 30 and May 1, 1954. Professor M. L. Madison, Chairman of the Section, presided at all three sessions.

Seventy-five registered for the meeting, including the following forty-nine members of the Association:


Officers elected at the meeting for 1954–1955 were: Chairman, Professor Nathan Schwid, University of Wyoming; Vice-Chairman, Professor C. R. Wylie, Jr., University of Utah; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. On expressing the matrix $A^t$ as a polynomial in $t$, by Professor R. K. Butz, Colorado Agricultural and Mechanical College.

This paper discusses the notion of $E_t F(X)$ matrices as introduced by G. B. Huff (Matrices such that $A^t$ is a polynomial in $t$ and principal idempotent elements, Bull. Amer. Math. Soc., vol. 59, 1953, p. 54). Emphasis is placed on the fact that the proofs of the main theorems require only the more elementary concepts of matrix theory and on methods of finding $F(X)$ given a matrix $A$ with elements in the field of complex numbers. The clarity with which some classical results follow by the use of this notion is pointed out.

2. An optimum solution of $N$ equations in $M$ unknowns with $N$ greater than $M$, by Mr. Leon Rutland, University of Colorado.

A problem in engineering design led to a consideration of the system of equations
\[
\sum_{j=1}^{m} a_{ij}x_j = t_i \quad (i = 1, 2, \ldots, n), \quad (n > m),
\]

where the desired optimum solution of the system is that set of \( x' \)'s for which the largest absolute value of any of the deltas in the set of equations

\[
\Delta t + \sum_{j=1}^{m} a_{ij}x_j = t_i \quad (i = 1, 2, \ldots, n), \quad (n > m),
\]
is as small as possible. Several theorems giving solutions to the problem under various conditions were either proved or stated with the proofs being omitted. A numerical example, carried through on a digital computer, was cited to indicate that even though there are as many as forty equations the method is feasible and the answer can be readily attained.

3. The method of Frobenius, by Professor R. H. Cook, South Dakota School of Mines and Technology, introduced by the secretary.

The usual textbook presentation of the method of Frobenius effectively camouflages two important points: (1) that the method involves a Taylor's expansion, and quite often leads to just a Taylor's expansion; (2) the conditions under which the method is applicable. This paper suggests a modified approach which has neither of the above disadvantages, is easily taught, and is sufficiently flexible to be applicable to many non-linear problems.


The following is investigated: Given a non-negative \( g(x) \), defined for all \( x > 0 \), and which is strictly monotonic increasing and everywhere differentiable, to find a closed-form \( f(x) \), reasonably computable, such that \( f[f(x)] \) is at least approximately equal to \( g(x) \). It was indicated how this approximation could be accomplished for certain \( g \)-functions, with examples.

5. Some infinite series, by Dr. W. E. Briggs, Professor S. Chowla, Professor (Emeritus) A. J. Kempner, and Research Assistant W. E. Mientka, University of Colorado, presented by Mr. Mientka.

It is proved that

\[
\sum_{i=1}^{n} \frac{\sigma_n}{n^2} = 2\gamma(3),
\]

where

\[
\sigma_n = \sum_{i=1}^{n} \frac{1}{i}.
\]

6. Effect of rotation on the normal mode frequencies of transverse vibration of a cantilever beam, by Professor R. H. Cook and Mr. L. J. Eatherton, South Dakota School of Mines and Technology, presented by Mr. Eatherton.

The differential equation,

\[
EI \frac{\partial^4 y}{\partial x^4} - \frac{\partial^2 y}{\partial x^2} \int_x^L \rho A \phi \partial \phi \partial y + \rho A \xi_0^2 \frac{\partial y}{\partial x} + \rho A \frac{\partial^2 y}{\partial t^2} = 0
\]
describes the vibrations, in a vertical plane, of a cantilever beam which rotates about a vertical axis through its clamped end. This equation and appropriate boundary conditions are considered.
by the use of Taylor's expansion. The normal mode frequencies are calculated in terms of $\Omega$, the rotational speed. Results show the dependence upon both $\Omega^2$ and $\Omega^4$ for the first and second modes and upon $\Omega^2$ for the third mode. They are in excellent agreement with existing experimental data.

7. **Block designs**, by Professor Burton W. Jones, University of Colorado.

The definition and significance of balanced incomplete block designs are briefly described and methods of exclusion sketched.

8. **The mapping of the circles of $S_2$ into the points of $S_3$**, by Professor C. R. Wylie, Jr., University of Utah. (By invitation).

If the coefficients $a$, $b$, $c$, $d$ in the general equation of a circle

$$d(x^2 + y^2) - 2ax - 2by + c = 0$$

are interpreted as homogeneous point coordinates in $S_3$, point circles are mapped into points on the paraboloid

$$V = a^2 + b^2 - cd = 0,$$

proper real circles are mapped into finite points outside $V$, improper real circles (lines) are mapped into real points at infinity, and imaginary circles are mapped into finite points within $V$. Pencils and bundles of circles are represented in $S_3$ by lines and planes, respectively, and may be classified according to the intersection of their images with $V$. Two circles which are orthogonal are represented by points each of which lies in the polar of the other with respect to $V$. Conjugate pencils of circles are represented by lines conjugate with respect to $V$. Various theorems from college geometry were interpreted in $S_3$, and the classical constructions for circles satisfying three conditions were considered as problems in descriptive geometry in $S_3$.

9. **The University of Colorado Engineering Experiment Station analog computer—The UCEESAC**, by Mr. Walter W. Varner, University of Colorado.

A description of the Boeing analog computer recently installed at the University of Colorado was given. Types of problems that can be solved as well as restrictions on their solution were given. Finally a simple pair of simultaneous differential equations were considered and the simplicity of forming the wiring diagram shown.

10. **Fitting empirical equations to fluid meter data**, by Professor S. R. Smith, University of Wyoming.

Empirical equations of the form

$$C = \frac{R}{a + bR} + dR^*$$

were fitted to both flow nozzle and orifice meter data and residuals determined. $R$, $C$ curves were fitted to data for the fluids stream, oil and water for $0 < R \leq 3,040,000$. $C$ is the coefficient of discharge of the meter and $R$ its corresponding Reynolds number, both dimensionless.

11. **On $f(f(x)) = F(x)$, $F(x)$ given (real), $f(x)$ unknown**, by Professor (Emeritus) A. J. Kempner, Dr. W. E. Briggs, Professor S. Chowla and Mr. W. E. Mientka, University of Colorado, presented by Professor Kempner.

For the real case the geometrical interpretation employed in studying $f(x) = x = 0$ can be extended so as to lead immediately to results such as: there exist totally discontinuous functions $f(x)$

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* Equation first used by I. D. Ruggles, former graduate student in mathematics, University of Wyoming.
for which $f(f(x))$ is single-valued and continuous; or, the function $y = g(x)$ given by

$$x - y + \pi/2 = \mu \cos (x + y), \ |\mu| \leq 1/2,$$

is a single-valued inverse iterate of $f(x) = x + \pi$. It also makes plain the plausibility of introducing iterations of $f^{(n)}(x)$ of any rational (or even any real) index $n$.

12. Figure it out for yourself, by Professor A. W. Recht, University of Denver.

Trained mathematicians are at a premium; everyone realizes the great part mathematics plays in a technical civilization. Yet election of mathematics in high schools is waning, despite efforts of government and mathematical groups stressing urgent need of training of the talented. The problem is to reach the 85% who will use mathematics in normal pursuits and also train the talented 15%. The paper suggested revisions of policy, including training of really professional mathematics teachers, emphasis on students doing own work, insistence on 100% accuracy, promotion of "first the problem, then the mathematics," and strengthening of fundamental concepts to avoid mathematical accidents in home and industry.

13. Textbooks on elementary mathematics, by Professor J. S. Leech, Colorado College.

Attention is called to the fact that in a large majority of textbooks many terms are defined erroneously or ambiguously. Many theorems are stated and "proved" without any or with incomplete hypotheses. In all of these cases, the author believed that the correct definitions and statements of theorems not only do not increase the difficulty of the subjects, but serve to clarify concepts and contribute greatly to understanding.

14. Freshman mathematics separation, by Professor I. L. Hebel, Colorado School of Mines.

This paper dealt with the mathematics department's experience over the last seven years in assigning and classifying freshman mathematics students.

F. M. Carpenter, Secretary

THE APRIL MEETING OF THE SOUTHWESTERN SECTION

The fourteenth annual meeting of the Southwestern Section of the Mathematical Association of America was held at Arizona State College, Tempe, Arizona, on April 16, 1954. Professor M. S. Hendrickson, Chairman of the Section, presided.

Forty persons attended the meeting including the following twenty-seven members of the Association:


The following were elected officers for the year 1954: Chairman, Professor D. L. Webb, University of Arizona; Vice-Chairman, Professor R. L. Westhafer, New Mexico College of Agriculture and Mechanic Arts; Secretary-Treasurer, Professor W. W. Mitchell, Jr., Phoenix College (four year term); Lecturers, Professor Max Kramer, New Mexico College of Agriculture and Mechanic Arts (one year), Professor D. G. Duncan, University of Arizona, (two years).