sion of the other two, with "relevance" running a very bad second, and "reasonableness" a much worse third. It is my contention that this order should be reversed; in any case "reasonableness" and "relevance" deserve at least as much importance as "techniques." For students whose major interest is in the sciences, all three aspects should be virtually tied for first place. By this, I do not mean at all to belittle the importance of "techniques" for technical students, but I am convinced that when their reasonable justification is properly stressed, the techniques themselves require far less attention than is ordinarily given them. Understanding really does improve technical facility.

For students with little or no interest in the sciences, who really have little need for such facility, "techniques" should trail considerably behind. It is definitely my experience that these students can become very interested in the logical nature of mathematics, whereas they have no personal interest whatever in solving problems of a practical (to a technician) nature, no matter how interesting such problems may seem to a mathematician. It may seem surprising to many teachers, but it is nevertheless a fact that to many college freshmen the idea that algebra is a reasonable subject comes as a complete revelation. If there is danger to the status of mathematics, it does not arise from overemphasis of its "reasonableness." It comes from the deadly overemphasis on routine "techniques," and the unwholesome neglect of its "reasonableness" and of its "relevance" to the real world.

THE FIRST ANNUAL MEETING OF THE METROPOLITAN NEW YORK SECTION

The first annual meeting of the Metropolitan New York Section of the Mathematical Association of America was held at Hunter College, New York City, on Saturday, April 18, 1942. Professor F. H. Miller presided at the morning session and Professor T. F. Cope, chairman of the Section, presided at the afternoon meeting.

The attendance was about one hundred and thirty-six, including the following seventy members of the Association: R. Lucile Anderson, R. G. Archibald, L. A. Aroian, Aaron Bakst, Brother Bernard Alfred (Welch), Frank Boehm, A. B. Brown, Jewell Hughes Bushey, J. H. Bushey, S. S. Cairns, H. R. Cooley, Elizabeth M. Cooper, T. F. Cope, W. H. H. Cowles, Marguerite D. Darkow, D. R. Davis, Carolyn Eisele, W. H. Fagerstrom, J. M. Feld, Edward Fleisher, R. M. Foster, B. P. Gill, Marion C. Gray, Etta Greenberg, Harriet M. Griffin, J. I. Griffin, C. C. Grove, N. A. Hall, L. S. Hill, J. H. Hlavaty, R. A. Johnson, L. S. Kennison, E. H. Koch, Jr., Edna E. Kramer-Lasser, Helen L. Kutman, A. W. Landers, Mary K. Landers, Nathan Lazar, C. H. Lehmann, C. C. MacDuffee, H. F. Mac Neish, J. J. McCarthy, P. H. McGrath, Mary McKenna, D. May Hickey Maria, A. E. Meder, F. H. Miller, E. C. Molina, Philip Newman, M. A. Nordgaard, Walter Penney, Mina S. Rees, Selby Robinson, S. G. Roth, H. D. Ruderman, Arthur Sard, Edna C. Schnefel, L. P. Siceloff, Lao G. Simons, James Singer, C. S. Stuckey, J. A. Swenson, J. J. Tanzola, H. E. Wahlert, Louis Weisner, Mary E. Wells, A. Marie Whelan, D. E. Whitford, John Williamson, Jack Wolfe.

At the beginning of the morning session President G. N. Shuster of Hunter College welcomed the Section to Hunter College. At the beginning of the afternoon meeting the following officers were elected for the coming year: Chairman, H. F. Mac Neish, Brooklyn College; Vice-Chairman, Edna E. Kramer-Lasser, Thomas Jefferson High School; Secretary, H. E. Wahlert, New York University; Treasurer, F. H. Miller, Cooper Union.

The following four papers were presented at the morning session:

1. "An application of matrix theory to cryptography" by Professor L. S. Hill, Hunter College.

2. "The teaching of mathematics at the defense training institute" by C. H. Lehmann, Cooper Union.

3. "On the principles of statistical inference" by Dr. Abraham Wald, Columbia University and Queens College, introduced by Professor Cope.

4. "Mathematical training for aeronautical engineers" by Dr. N. A. Hall, Vought-Sikorsky Aircraft Company.

At the afternoon session a symposium on "Integrated mathematics in high school" was held. Dr. J. A. Swenson of Andrew Jackson High School presided at this symposium and the following four papers were presented:

5. "The significance of Δx in secondary mathematics" by Agnes Morley, Andrew Jackson High School, introduced by Dr. Swenson.

6. "Integrated mathematics with special application to the tenth year (geometry)" by Harry Sitomer, New Utrecht High School, Brooklyn, introduced by Dr. Swenson.

7. "Spatial and probable relationships in secondary mathematics" by Dr. Edna E. Kramer-Lasser, Thomas Jefferson High School, Brooklyn.

8. "Integrated mathematics in Catholic high schools" by Brother Anselm, St. Joseph's Normal Institute, introduced by Brother Bernard Alfred.

Abstracts of the papers follow.

1. Professor Hill's presentation was based upon two articles published in this MONTHLY: *Cryptography in an algebraic alphabet*, July, 1929, and *Certain linear transformation apparatus of cryptography*, March, 1931. The articles attracted wholly unanticipated attention in the United States and abroad. Their interest was strengthened by the circumstance that the author's learned, ingenious, and mechanically-minded colleague of some fifteen years, Professor Louis Weisner, was able to plan a machine for the speedy and effortless operation of those simpler types of transformation which were considered in the 1929 article. These transformations include, as special cases several of the more prominent military cipher systems, and afford extensions which seem to be quite beyond cryptanalytic approach.

2. Mr. Lehmann described the origin, organization, and scope of the Defense Training Institute of the Engineering Colleges of Greater New York. This new

school, in actual operation since February 10, 1941, is the only one in the country having a full daytime coordinated curriculum set up primarily to train engineering personnel for war industries. The address featured the place of mathematics in the Institute's curriculum and described some of the teaching problems in mathematics encountered in what is probably one of the most highly concentrated and intensive technical programs ever undertaken.

3. Dr. Wald discussed some modern developments of the theory of statistical estimation. Suppose x is a random variable and its probability distribution function $f(x, \theta)$ involves an unknown parameter θ. In the theory of estimation the value of the unknown parameter θ has to be estimated on the basis of N observations $x_1, \cdots, x_N$ on the variable x, *i.e.*, we have to construct a function $t(x_1, \cdots, x_N)$ of the observations which can be considered as a "good statistical estimate" of the parameter θ. In the modern theory definitions for a "good" or "best" statistical estimate are formulated and solutions are given for certain classes of cases. If the number of observations N is large, the so-called maximum likelihood estimate provides a satisfactory solution of the problem of estimation under fairly general conditions.

4. Dr. Hall discussed the mathematical training required by aeronautical engineers from the viewpoint of the engineer, whose primary consideration is the production of the most efficient airplane. He presented a brief picture of the program of development of airplane design from the original conception in terms of function and specifications to the final detail design for production. The various places where mathematics entered the program were pointed out to indicate the type of mathematical training required. It was observed that two distinct types of ability were demanded. First, the average engineer was called upon to use mathematics of a difficulty up to the grade of the calculus with absolute accuracy and dependability. The need for training in thoroughly reliable work as a basic requirement was stressed. The second demand was for a preparation in advanced fields of mathematical analysis with emphasis on numerical methods and the possession of the viewpoint of mathematical service to the engineering program. "When we are training aeronautical engineers, we must train men to develop airplanes and not mathematics."

6. Mr. Sitomer pointed out that, whether in the tenth year or not, integrated mathematics must not be confused with simultaneous, correlated, comprehensive, or general mathematics, which refer to the scope of instructional materials but not to the methods of presentation and solution. Integrated mathematics may be defined as a program, not a syllabus, which aims to enlist any branch of mathematics which is best fitted to solve a given problem, whether it be computation or proof. The more branches in use, the better the integration. The simpler the computation or proof, the better the integration. In the tenth year we may select methods from algebra, plane and solid geometry, trigonometry, calculus, function theory, number theory, vector analysis and topology. Courageous experimentation in the classroom by teachers, well-read in mathematics, will determine possible syllabi. In the tenth year special emphasis on the

nature of proof and on a postulational system demands a simple synthetic introduction with a gradual transition to analytic geometry and other methods of proofs. Integrated mathematics results in a rich set of concepts arranged spirally in a syllabus.

7. Dr. Kramer-Lasser showed how integrated mathematics is used to develop general space concepts. Even as early as the seventh year, elementary procedures form a foundation for more advanced work. For example, in working with paper patterns, proper numbering of vertices and edges eventually leads pupils to think of the surfaces they construct as sets of triangles with suitable identification of vertices and edges. In the ninth and tenth years, space geometry is studied along with plane. By rotating and translating $3, 4, \cdots, n-1$ dimensional forms, pupils obtain an abstract realization of $4, 5, \cdots, n$ dimensional figures. Combinatorial methods are also used to obtain the concept of higher dimensions. In the eleventh and twelfth years, elementary analytic geometry of three dimensions is studied and analytic generalization to higher dimensions is made.

8. Brother Anselm stated that, in general, the Catholic High Schools in and around New York City adhere to the traditional courses in mathematics. They do this because they think that their present curriculum is the best for their particular needs. They are reluctant to change to the integrated course because most of the leading educators in this field are not certain that integrated mathematics courses are better than the traditional courses. Moreover, they fear that lack of drill and understanding of the reasons for manipulative processes may be brought about by the integrated program. They are convinced also that the much desired purpose of an integrated program can be achieved with the traditional subjects. Hence the Catholic schools look, not to a change of courses to improve secondary mathematics, but rather to better teaching of the present courses.

H. E. WAHLERT, *Secretary*

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-sixth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado School of Mines, Golden, Colorado, April 17 and 18, 1942. There were three sessions. Professor J. C. Fitterer presided at the first two and at the business session of the third. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Education Association. Mr. H. W. Charlesworth of East Denver High School presided at this meeting.

The attendance was forty-four, including the following fifteen members of the Association: C. F. Barr, J. R. Everett, J. C. Fitterer, I. L. Hebel, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. E. Mallory, W. K.

Nelson, Greta Neubauer, M. G. Pawley, A. W. Recht, C. H. Sisam, and W. E. Wilson.

At the business meeting the following officers were elected for the coming year: Chairman, Professor A. E. Mallory, Colorado State College of Education; Vice-Chairman, Professor A. W. Recht, University of Denver.

The following papers were presented:

1. "An improved cosine-law slide rule" by Professor I. L. Hebel, Colorado School of Mines.

2. "On determining the 'best' critical region for testing the null hypothesis when the parent populations follow the Poisson law" by H. T. Guard, Colorado State College, introduced by Professor Macdonald.

3. "The use of Cauchy's integral formula in evaluating certain improper integrals" by Professor A. J. Lewis, University of Denver.

4. "A study of roulettes with the aid of the cathode-ray oscillograph" by Professor M. G. Pawley, Colorado School of Mines.

5. "Geometrical demonstration of a theorem on envelopes and its application to solve a maximum and minimum problem" by G. E. Uhrich, University of Colorado, introduced by Professor Kempner.

6. "The mathematical approach to the fundamentals of hydraulics" by C. P. Vetter, Senior Engineer, Bureau of Reclamation, introduced by the Secretary.

7. "Remarks on repeating decimal fractions" by Professor Emeritus I. M. De Long, University of Colorado. Read by Professor A. J. Kempner.

8. "Periodic decimal fractions, primitive roots and quadratic residues" by Professor A. J. Kempner, University of Colorado.

9. "A report of instruction and learning activities as observed in geometry class rooms" by Professor A. E. Mallory, Colorado State College of Education.

10. "The handmaiden spurned" by Professor O. H. Rechard and Professor C. F. Barr, the University of Wyoming.

11. "Some simple proofs of the addition law in trigonometry" by G. E. Uhrich, University of Colorado, introduced by Professor Kempner.

12. "The use of mathematics in industry" by L. A. McElroy, Denver public Schools, introduced by the Secretary.

13. "Constructions by means of a marked ruler and other instruments" by Professor Claribel Kendall, University of Colorado.

Abstracts of the papers follow.

1. Professor Hebel gave an extension of an earlier paper concerning a slide rule for the solution of the cosine law of spherical trigonometry, particularly as applied to distances on the earth. By properly manipulating the equation, an improved slide rule has been devised which gives a direct solution by purely mechanical means, as contrasted to the original model where it was necessary to "take out" intermediate answers to reach the final results. The operation was demonstrated on a specially constructed two meter long classroom model slide rule.

2. This problem was first considered by Przyborowski and Wilenski in *Biometrica*, 1939.

Given the random variables x_i where $\rho(x_i | \lambda_i) = \lambda_i^{x_i} e^{-x_i}/x_i!$, $(i=1, 2)$. To determine the best critical region for testing the composite hypothesis, H_0, that $\lambda_1 = \lambda_2$ with respect to the alternative hypothesis $\lambda_1 < \lambda_2$. The test consists in finding a region, w_0, in the sample space, W, such that, if E is the observed sample point,

1. $P\{E\epsilon w_0 | H_0\} \leqq \epsilon$, the desired level of significance.
2. $P\{E\epsilon w_0 | H_1\} > P\{E\epsilon w | H_1\}$

where H_1 is any alternative hypothesis and w is any other region that satifies 1.

The test reduces to rejection of H_0 when $x_1 \leqq x_\epsilon$ where x_ϵ is chosen so that

$$\sum_{x_1=0}^{x_\epsilon} \binom{s}{x_1} \left(\frac{1}{1+A}\right)^{x_1} \left(\frac{A}{1+A}\right)^{s-x_1} \leqq \epsilon$$

is satisfied. A is the ratio of the sample sizes and $s = x_1 + x_2$.

The power function is

$$B(\theta | s) = \sum_{x_1=0}^{x_\epsilon} \binom{s}{x_1} \left(\frac{1}{1+A\theta}\right)^{x_1} \left(\frac{A\theta}{1+A\theta}\right)^{s-x} \quad \text{where} \quad \theta > 1.$$

The calculations of tables given the critical region and values of the power function for specified values of s and θ and also with s unspecified were demonstrated by Mr. Guard.

3. Professor Lewis outlined methods of using Cauchy's integral formula and Cauchy's residue formula in evaluation of certain improper integrals with real integrands. He illustrated the method by applying it to two examples.

4. Professor Pawley made a study of the parametric equation

$$x = a_1 \cos n_1 t + a_2 \cos n_2 t + a_3 \cos n_3 t,$$
$$y = a_1 \sin n_1 t \pm a_2 \sin n_2 t + a_3 \sin n_3 t.$$

The equation was shown to represent a group of roulettes, including epicycloids, hypocycloids, epitrochoids, and hypotrochoids, depending upon the relations between the a's and n's. The various curves were pictured on the flourescent screen of a cathode-ray oscillograph. Simple rules were given for anticipating the form of roulette obtained when the a's and n's in the equation are varied. An equation was given for a curve approximating the N-sided regular polygon and these curves were displayed on the screen of the oscillograph.

5. Mr. Uhrich dealt mainly with a geometrical proof of the theorem: The envelope of the family of chords which cut segments of equal area from a given curve is tangent to each chord at its midpoint. The converse of this theorem is proved in Salmon's *Analytische Geometrie der Kegelschnitte*, Kap. 13, with the purpose of application to conic sections; however the method seems to be general.

Besides some special examples, Mr. Uhrich used this theorem to prove: Of all the chords which can be drawn through a fixed point within a closed convex curve, that chord which is bisected by the point cuts off a segment of minimum area from the curve. This theorem was given as a problem in the *Analyst*, vol. 5.

6. Mr. Vetter outlined the various methods of mathematical approach to the solution of problems connected with fluid flow. These methods permit of practical solutions and of solutions that may be verified experimentally only in comparatively few and comparatively simple cases. He demonstrated further, that as an alternative it is possible to attack the problems from purely dimensional considerations without the necessity of simplifying assumptions. The latter method, without leading to final answers, nevertheless gives precise information as to the mathematical form of the functional relationship between the physical quantities involved and, in many instances, permits experimental verification or experimental evaluation of specific functional constants. He analyzed the relationship of the method to the theory of models.

7. Professor Emeritus Ira M. De Long of the University of Colorado is eighty-seven years old, and has been for four months in a Boulder hospital. He has maintained his life-long interest in the properties of repeating decimals, and never tires of telling me interesting and amusing relations which he has discovered for himself. Without claims of priority I present two examples.

(1) If we expand a fraction a/p, p a prime, say 3/7, we have $10\cdot 3=4\cdot 7+2$, $10\cdot 2=2\cdot 7+6$, $10\cdot 6=8\cdot 7+4$, $10\cdot 4=5\cdot 7+5$, $10\cdot 5=7\cdot 7+1$, $10\cdot 1=1\cdot 7+3$, with the digit period 428571′ and the remainder period 264513. The $7(4+2+8+5+7+1)=9(2+6+4+5+1+3)$. For a/p, $p\cdot\sum(\text{digits})=9\cdot\sum(\text{remainder})$. For a base g instead of 10, $g\neq 1$, $\not\equiv 0 \bmod p$, 9 is replaced by $g-1$.

(2) To obtain the period of 1/49 from the period 142857 or 1/7, proceed as follows: Write

.020408	020408	020408	020408	020408	020408	020408
	142857	285714	428571	571428	714285	857142
020408	163265	306122	448979	591836	734693	877550

which is (except for the last 0 which has to be replaced by 1) the correct period.

8. The examination of the period length and digit properties, etc., of an ordinary repeating decimal have a particularly special character, on account of the orientation with respect to the base 10. By considering simultaneously the representation of all fractions a/p, $(a/p)=1$, for a given p, with respect to *all* bases $g>1$, and by fixing attention on the periodic *remainder* sequences rather than upon the periodic *digit* sequences, the general pattern or relations appears clearly. For a/p, $a=1, 2, \cdots, p-1$ we have for each base $g>1$, $(p/g)=1$, the following theorems:

I. Let $d_1=1, \cdots, d_\nu, \cdots, d_\mu=p-1$ be the divisors of $p-1$. Then, denoting by $L(a/p)_g$ the period length in the remainder sequence, there are exactly $\phi(d_\nu)$ remainder sequences for which $L(a/p)_g = d_\nu$. (For $p=7$, $a=4$, $g=2, 3, \cdots, 6, 8$ we have remainder sequences 124, 513264, 214, 623154, 34, 4, respectively.) The same holds of course for the digit periods.

II. For a given g the remainders $1, 2, \cdots, p-1$, of a given a/p will either form exactly a remainder period, or the $p-1$ remainders break up into α sets of β each, $\alpha\beta = p-1$. In each of these $L(a/p)_g$ one of the three cases occur: Either each number of the period is quadratic residue of p, or each number is a non-residue; or the numbers are alternately residues or non-residues.

III. If $(g/p)=1$, $L(a/p)_g$, is a divisor of $(p-1)/2$, 1 and $(p-1)/2$ inclusive. But if $(g/p)=-1$, $L(a/p)_g$, which must divide $p-1$, does not divide $(p-1)/2$.

9. Professor Mallory gave a report of the type of work being done in certain geometry classes. The classes visited showed a high degree of understanding of the subject matter as revealed by the procedures.

10. The paper by Professor Rechard and Professor Barr grew out of the rejection by a geologist of a simple formula involving a few trigonometric functions on the basis that geologists would not use it. Instead, in his published paper, he described and illustrated a graphic method for solving a chain of right triangles. Where the fault lies for such a situation is the question raised by this paper, in the hope that out of it might come some clue to the old problem of how to make usable mathematics used.

11. In this paper Mr. Uhrich gave a presentation of three of the four proofs with slight variations for the addition theorem for the sine of the sum or difference of two angles which are given in Professor Gerhard Hessenberg's *Ebene und sphärische Trigonometrie*, Kap. IV.

12. The successful person in industry is the one who can separate essential and non-essential factors, get the job done and show a profit. Mathematical training is important to the extent that it applies to the job in hand. Pupils should be offered basic knowledge of mathematics with the problems that are *real to them*. This should help them to develop the habit of applying mathematical knowledge to practical situations. Industry specializes; mathematics essential to one industry has no value to another. But, since no one can predict with 100% accuracy the potentialities of an individual, it would seem wise to recommend training in mathematics for each pupil, at least until a choice of occupation has been made.

13. We are all familiar with constructions by means of an unmarked ruler and compasses, the euclidean instruments. Some or all of these constructions can be carried out by means of other instruments such as an angle ruler, a marked ruler, an unmarked ruler without compasses or by compasses alone. Professor Kendall paid particular attention to constructions by means of a marked ruler, including the trisection of the angle and the finding of a length which is the cube root of a given line segment.

A. J. LEWIS, *Secretary*