THE TWENTY-SECOND ANNUAL MEETING OF THE ILLINOIS SECTION

The twenty-second meeting of the Illinois Section of the Mathematical Association of America was held at Bradley Polytechnic Institute on Friday and Saturday, May 9–10, 1941. Professor Mildred Hunt, chairman of the Section, presided at all sessions.

The meeting was attended by fifty persons, including the following thirty-eight members of the Association: Edith I. Atkin, H. G. Ayre, S. F. Bibb, G. A. Bliss, A. O. Boatman, C. E. Comstock, D. R. Curtiss, J. E. Davis, W. M. Davis, Edna M. Feltges, Elinor B. Flagg, L. R. Ford, A. E. Gault, G. D. Gore, M. R. Hestenes, W. N. Huff, Mildred Hunt, R. N. Johanson, E. C. Kiefer, J. M. Kinney, W. C. Krathwohl, Luise Lange, J. R. Mayor, C. N. Mills, G. E. Moore, Mary W. Newson, I. E. Perlin, J. W. Peters, E. W. Ploenges, Ruth B. Rasmusen, W. T. Reid, J. M. Sachs, R. G. Sanger, H. A. Simmons, F. C. Smith, N. W. Wells, F. E. Wood, Alice K. Wright.

The Section accepted the invitation of James Millikin University, Decatur, Illinois, for the meeting on May 8–9, 1942. The following officers were elected: Chairman, R. N. Johanson, Bradley Polytechnic Institute; Vice-Chairman, E. W. Ploenges, James Millikin University; Secretary, C. N. Mills, Illinois State Normal University.

The Section passed a resolution to coöperate with the Regional Governors in any plan worked out for financial aid to the Sections.

The following fourteen papers were read:

1. "Solution of a class of Diophantine problems including an unsolved problem of Tanzo Takenouchi" by Professor H. A. Simmons, Northwestern University.
2. "The generalized hypergeometric equation" by Professor F. C. Smith, College of St. Francis.
3. "Predicting class quality by means of orientation test" by Professor W. C. Krathwohl, Illinois Institute of Technology.
4. "Evaluation of mathematical instruction" by Dr. W. H. Erskine, Wright Junior College, introduced by Professor Hunt.
5. "Some aspects of junior college mathematics" by N. W. Wells, Springfield Junior College.
6. "Sturm's theorem for functions of two variables" by Professor D. R. Curtiss, Northwestern University.
7. "Complex numbers and wing profiles of airplanes" by Dr. J. M. Dobbie, Northwestern University, introduced by Professor Curtiss.
8. "Mathematical aspects of meteorology" by Professor C. G. Rossby, Assistant Chief of U. S. Weather Bureau, introduced by Professor Hunt.
9. "Mathematics south of the border" by Professor Rufus Oldenburger, Illinois Institute of Technology.
10. "When to teach theory of equations" by Professor J. E. Davis, Central Y.M.C.A. College.

11. "Mathematics in exterior ballistics" by Professor G. A. Bliss, University of Chicago.

12. "The theorem of Morley" by Dr. J. W. Peters, University of Illinois.

13. "Boundary value problems" by Professor W. T. Reid, University of Chicago.

14. "A classification of classifications" by Professor F. E. Wood, Northwestern University.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. By modifying a set of inequalities of Curtiss, a sieve was developed for catching the individual maximum number that exists in any E-solution (defined in *Trans. of Am. Math. Soc.*, Nov., 1932) of many equations of the form $\sum(1/x_1)=b/a$, $(a, b)=1$, $a\geqq b$. Professor Simmons showed that the sieve functions in the case of the equation $\sum(1/x_1)=5/11$, suggested by Tanzo Takenouchi. The process is found to work equally well for any equation in the form $\sum(1/x_1)=(5+9t)/(11+20t)$, $(t=0, 1, 2, \cdots)$.

2. Professor Smith reviewed the well known solutions of Gauss's hypergeometric equation, and then outlined the methods used to obtain the corresponding solutions of a $q+1$ by $q+1$ hypergeometric equation in both non-logarithmic and logarithmic cases.

3. Professor Krathwohl showed by means of correlation coefficients and equations of lines of regression that it is sometimes possible to predict from a 45-minute mathematics aptitude test what the average final grade of a class in mathematics will be as late as two years after the test has been taken. The degree of prediction depends on the consistency of grading and the standards of the instructor. The average correlation coefficients lie between 0.45 and 0.65. For individual instructors, correlation coefficients have been found as high as 0.90. The exceptional cases are those of instructors who grade on a normal frequency curve and hence have a correlation coefficient of zero. The correlation coefficients seem to increase with the teaching experience of the instructor.

4. Dr. Erskine reviewed a study in the evaluation of the instruction in mathematics at Wright Junior College, based upon the index numbers on individual questions in a series of tests given in the first-year mathematics course over a period of several semesters. The index numbers were calculated by the formula $(C-I)/(C+I)$, where C and I are respectively the number of correct and incorrect answers. The study showed that although the index numbers of different classes vary considerably (one-half the possible range), the index numbers of the classes of each instructor are remarkably stable, with an average variation of from 1/10 to 2/10 of the possible range, and indeed comparable to the stability of the index numbers for the whole department (400 students). It was pointed out that, because of the degree of stability, index numbers may be used as a guide in improving tests and also in improving instruction.

5. Mr. Wells stated that the effectiveness of junior college mathematics is dependent upon thoroughness and effectiveness in the teaching of high school

mathematics. This can be achieved partially by a large number of problems to develop technic, and also by a good analytical reading ability on the part of the student. Another desideratum is an effective vigorous facility in arithmetic. The junior college can achieve similar effectiveness by numerous problems and also by showing the possibilities of the application of mathematics to some of the sciences. The aim should be a teaching of fundamental principles with the development in the student of confidence in his ability to use mathematics as a tool.

6. If F and Φ are power series in x and y which are convergent in a neighborhood of the origin and vanish for $x=0$, $y=0$, then Φ may not be a divisor of F in the sense used by Weierstrass and others. In this case there may be various definitions of quotient and remainder. Professor Curtiss used one such definition to set up a greatest common divisor process which, applied to F and its derivative with respect to y, produces a set of functions having properties analogous to those of the Sturm's functions of the elementary theory of equations. They can be used to locate branches of the curve $F=0$ that pass through the origin, and to determine whether F has a maximum or minimum at the origin.

7. Dr. Dobbie gave a brief introduction to the theory of conformal mapping as applied to the shaping of wing profiles, including a discussion of a class of profiles developed recently. A review of the potential theory for the flow past a circular cylinder of infinite span was included.

9. In Mexico the emphasis of university learning in the past has been on the arts. It was only three years ago that mathematics was taken from the department of philosophy in the National University and made a separate department. Since then there has been definite encouragement for the faculty to engage in research. Professor Oldenburger stated that a journal of science to inspire research in both physics and mathematics will soon be started. The National University gives master's degrees but not doctor's degrees in mathematics. The library facilities are poor, and there is a great need for journals and advanced books. This year the first Mexican scientist joined the American Mathematical Society, and it is likely that others will follow.

10. Professor Davis suggested that a first course in theory of equations may profitably be given to fourth-semester college students, concurrently with the second semester of calculus. The advantages claimed are that greater coherence, systematization, and economy of effort in subsequent undergraduate courses in mathematics are thereby made possible. The remarks were based upon nine years' experience with such practice at the Central Y.M.C.A. College.

11. Professor Bliss described some of the mathematics used in the control of artillery fire, beginning with a brief description of military maps, the construction of which is a highly technical problem of the theory of surfaces. The determination of the position of a battery on the map, of the map range from the battery to a target, and of the azimuth of the line of fire, require only relatively elementary applications of the mathematics of surveying. The use of a range table to find the corrections to the map range due to various types of abnormal

conditions, and to find the elevation corresponding to the corrected range, is also mathematically simple. The mastery and rapid coördination of all these operations in the field is the fine art of the fire control officer. Perhaps the most serious mathematical problem connected with artillery fire is the construction of range tables. Various methods of computation of trajectories, by the approximation of Siacci, by short arc methods of computation, and mechanically by means of the differential analyzer of Bush, together with methods of computing differential corrections, were discussed.

12. "If the trisectors of the interior angles of a triangle A, B, C be drawn, and if those trisectors adjacent to BC meet at P, those adjacent to AC meet at Q, and those adjacent to AB meet at R, then P, Q, R is an equilateral triangle." This is known as the theorem of Morley. Dr. Peters discussed the history of this theorem from its discovery by Frank Morley until the present.

13. Professor Reid was concerned with fundamental relationships and analogs that exist between certain problems for pencils of quadratic forms and the principal results for definitely self-adjoint and H-definitely self-adjoint differential systems as developed by Bliss and the speaker.

14. Professor Wood discussed the desirable properties which a classification should have, and gave illustrations of classifications with those properties and of classifications which did not possess certain stated properties.

C. N. MILLS, *Secretary*

THE TWENTY-FIFTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-fifth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado College, Colorado Springs, Colorado, April 18–19, 1941. There were three sessions. Professor W. V. Lovitt, chairman of the Section, presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Education Association.

There were thirty-four present, including the following twenty-five members of the Association: C. F. Barr, M. T. Bird, Jack Britton, I. M. DeLong, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. E. Mallory, W. K. Nelson, Greta Neubauer, M. G. Pawley, G. B. Price, O. H. Rechard, A. W. Recht, C. H. Sisam, V. J. Varineau, G. A. Whetstone.

At the business meeting the following officers were elected for next year: Chairman, J. C. Fitterer, Colorado School of Mines; Vice-Chairman, A. E. Mallory, Colorado State College of Education; Regional Governor for Region 12, 1942–43, O. H. Rechard, University of Wyoming.

The joint session held on Saturday morning consisted of a discussion of the two following reports: (1) "The place of mathematics in secondary education" by the Joint Commission of the M. A. A. and the N. C. T. M.; (2) "Mathematics in general education" by a commission of the Progressive Education Association. The discussion was led by Dr. H. R. Douglass, Director of the College of Education, University of Colorado.

The following papers were presented:

1. "Various types of singular points of differential equations of the first order" by Professor J. R. Everett, Colorado School of Mines.

2. "A note on Klein's determinant approach to the line integral of area" by Professor C. F. Barr, University of Wyoming.

3. "Generalized euclidean rings" by Dr. V. J. Varineau, University of Wyoming.

4. "Specification of elastic strain" by Dr. G. A. Whetstone, Amarillo College.

5. "Interpolation with the calculating machine" by Professor A. W. Recht, University of Denver.

6. "Excursions from the beaten path of undergraduate mathematics" by Professor M. T. Bird, Utah State Agricultural College.

7. "The inherent error in extension of Newton's method for approximating real roots" by Professor M. G. Pawley, Colorado School of Mines.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. Professor Everett discussed (a) nodal points, (b) vortex points, (c) spiral points, and (d) saddle points, arising in differential equations of the first order.

2. Klein develops a special statement of Green's theorem in a plane. His approach is by a determinant of triangular area. The suggestion of generality is obvious. Professor Barr extended this approach to a surface integral of volume.

3. Dr. Varineau defined a class of generalized euclidean rings. He showed that the class of euclidean rings, as defined in the literature, is included in this class of generalized euclidean rings. He also demonstrated that the ring of matrices with elements in a proper euclidean ring is a generalized euclidean ring.

4. In an elastic body in space account must be taken of six components of strain, not all of which can be independent since they are defined as linear combinations of the first order partial derivatives of the three displacements. Dr. Whetstone proved that with the exception of the three sets (e_x, e_y, e_{xy}), (e_y, e_z, e_{yz}), and (e_z, e_x, e_{zx}) we may select any three strains arbitrarily and may then determine which of the coefficients in the Taylor expansions of the other three are arbitrary. These results were obtained by the methods of Riquier.

5. Professor Recht demonstrated the use of the calculator in ordinary interpolation using first and second differences; also, a special method of subdivision of tables to fifths of intervals using up to fourth differences.

6. The results of these excursions as given by Professor Bird are probably not new, but they may be found in novel settings. The values of $\log_{10}2$, $\log_{10}3$, and $\log_{10}7$ were found to four digits directly from the definition of logarithm. The relations between the law of sines, law of cosines, *etc.*, were made explicit. A construction for the axes of the ellipse $Ax^2-2Bxy+Cy^2=D$ was related to the

lines $Ax=By$ and $Bx=Cy$. A construction for the hyperbola $b^2x^2-a^2y^2=a^2b^2$ was related to the parametric form $x=a(1+t^2)/(1-t^2)$, $y=2bt/(1-t^2)$. The series for $\log_e N$ was exhibited as the result of certain rearrangements of the series $1-1/2+1/3-1/4+\cdots$. Unusual weighted sums were considered as approximations for a definite integral and contrasted with the usual "rules." Finally, π was computed by the use of the inverse sine.

7. Professor Pawley described extensions of Newton's method for approximating real roots in which the desired root is approximated by x intercepts of curves of higher order of contact than the tangent. He derived an upper bound to the error involved in these approximations. In particular, he simplified the well known parabolic approximation by expanding an x intercept of the parabola into a convergent alternating series. An upper limit to the error involved in this approximation was derived and illustrated by an example.

A. J. LEWIS, *Secretary*

EQUATIONS IN QUATERNIONS

IVAN NIVEN, University of Illinois

1. Introduction. We prove the existence of a quaternion root of the equation

$$a(x) = x^m + a_1x^{m-1} + a_2x^{m-2} + \cdots + a_m = 0, \qquad a_m \neq 0, \tag{1}$$

with coefficients from the algebra of real quaternions. The writer had proved this result when m is odd, but the proof was rendered obsolete when Nathan Jacobson pointed out that the result (without restriction on m) can be obtained as a simple consequence of some work of Ore [1]. This is given in detail in §2.

In §3 we give a method for obtaining the roots of (1), which is not very practical in the sense that it involves the simultaneous solving of two real equations of degree $2m-1$. The method used is a generalization of Sylvester's treatment [2] of the quadratic equation corresponding to (1). Sylvester's conclusion that a quadratic equation has six roots is incorrect because he neglects to show that they exist, and also overlooks the possibility of an infinite number of roots; a complete analysis is given in §4 (Theorem 2). The number of roots of (1) is discussed in §5 (Theorem 3), necessary and sufficient conditions being given for an infinite number of roots.

The proof given here of the existence of a root of (1) is stated for the general case where the coefficients of the equation are quaternions over any real-closed field R (*i.e.*, no sum of squares in R is equal to -1, and no algebraic extension of R has this property).

Reinhold Baer, on hearing of this existence proof, proved the converse, so that we have the following strong result:

THEOREM 1. *Let D be a non-commutative division algebra with centrum C. Then every equation* (1) *with coefficients from D has a solution in D if and only if C is a real-closed field, and D is the algebra of real quaternions over C.*