ORGANIZATION MEETING OF THE UPPER NEW YORK STATE SECTION

A meeting to organize the Upper New York State Section of the Mathematical Association of America was held at Colgate University, Hamilton, New York, on Saturday, May 11, 1940. Sessions were held both in the morning and in the afternoon. Luncheon and dinner were served to members and guests in the James C. Colgate Student Union. Professor H. T. R. Aude of Colgate University presided at the morning session, Professor F. F. Decker of Syracuse University at the afternoon session, and Professor W. M. Carruth of Hamilton College at the dinner. The president of the Association, Professor W. B. Carver of Cornell University, presided at the business meeting which concluded the the afternoon session.

The attendance was ninety-six, including the following forty-three members of the Association: R. P. Agnew, E. B. Allen, H. T. R. Aude, A. B. Brown, H. S. Brown, F. J. H. Burkett, I. S. Carroll, W. M. Carruth, W. B. Carver, T. F. Cope, F. F. Decker, H. A. DoBell, W. H. Durfee, A. S. Gale, H. M. Gehman, H. N. Hubbs, W. A. Hurwitz, B. W. Jones, Caroline A. Lester, L. L. Lowenstein, Harriet F. Montague, D. S. Morse, C. W. Munshower, W. R. Murray, E. E. Nash, Abba V. Newton, E. R. Ott, B. C. Patterson, V. E. Pound, J. F. Randolph, C. E. Rhodes, M. A. Scheier, Joseph Seidlin, Virgil Snyder, Ellen C. Stokes, R. E. Street, Mary C. Suffa, R. J. Walker, A. K. Waltz, J. F. Wardwell, C. W. Watkeys, A. E. Whitford, W. D. Wray. Representatives were present from twenty colleges and universities.

A petition signed by forty-three members of the Association was addressed to the Board of Governors requesting the authority to organize a Section. A proposed set of by-laws had been prepared and was adopted, subject to the approval of the Board of Governors of the Association. The following officers were elected for the year 1940–1941: Chairman, H. M. Gehman, University of Buffalo; Vice-Chairman, A. D. Campbell, Syracuse University; Secretary, C. W. Munshower, Colgate University.

The following papers were read:

1. "A plan for freshman mathematics" by Professor T. F. Cope, Queens College.

2. "An application of analytic geometry to cryptography" by Dr. R. E. Huston, Rensselaer Polytechnic Institute, introduced by Professor Allen.

3. "Errors in American college mathematics texts" by Professor A. B. Brown, Queens College.

4. "Jacobian circles of the biquadratic" by Professor B. C. Patterson, Hamilton College.

5. "Complex roots of a polynomial equation" by Professor H. M. Gehman, University of Buffalo.

6. "Representation of a class of functions by Stieltjes integrals" by Dr. B. A. Lengyel, Rensselaer Polytechnic Institute, introduced by Professor Allen.

7. "A locus associated with the Pascal line" by Professor E. R. Ott, University of Buffalo.

8. "The convergence and summability of series" by Professor R. P. Agnew, Cornell University.

Abstracts of the papers follow, the numbers corresponding to the numbers above:

1. The plan described by Professor Cope has been tried for a year at Queens College. The three features of the plan are: (1) sectioning on the basis of a placement test into three groups; (2) in the case of the middle group a differentiation of content in the second semester according as the students are arts or science majors; (3) the encouragement of the able students of the superior group by putting them into the regular sophomore mathematics class in the second semester. The advantages claimed are: (1) that it greatly simplifies the teaching of freshman mathematics because of the homogeneity of the classes; (2) that the pace of the work is adapted to the ability of the students; and (3) that the content of the courses is adapted to the needs of the students.

2. A simple transposition code yields only a single representation for each letter; it is usually easy to decipher even a short message in such a code. Using no idea more complicated than translation of axes, Dr. Huston examined four codes in which each letter had more than a single representation. In the first of these, every letter was represented by every other letter with the representation in a particular sequence depending on the preceding letter. In the second, every sequence of two letters had 26^2 representations ranging from AA to ZZ. In the third, every sequence of three letters had 26^3 representations ranging from AAA to ZZZ, so that every three-letter word represented every other three-letter word. In the fourth code, every six-letter word had 26^6 legitimate spellings which ranged from $AAAAAA$ to $ZZZZZZ$.

3. The paper appeared in the June-July issue of the MONTHLY.

4. In the inversive plane the biquadratic curves include all curves which determine four points with a circle. It is known that such a curve is self-inversive (anallagmatic) with respect to four circles, its Jacobian circles. On the basis of the theory of bipolars, Professor Patterson presented a method for determining the Jacobian circles of a biquadratic and, as a consequence, the three polarities (homographies of period 2) under which the biquadratic is invariant.

5. Professor Gehman gave a graphic interpretation of a pair of complex roots of a polynomial equation. The theorem given was a generalization of a well known theorem for the case of the cubic equation.

6. A direct proof was given by Dr. Lengyel for the following theorem of Koopman and Doob: Let $w=u+iv$ be an analytic function in the upper half-plane, $y>0$, which satisfies the conditions $0<vy<K$, where K is a constant. Then $w(z)$ can be represented by a Stieltjes integral $\int_{-\infty}^{+\infty} d\psi(\lambda)/(\lambda-z)+a$, where a is a real constant, $\psi(\lambda)$ is a monotone increasing function; in fact,

$$\psi(\lambda) = \lim_{y\to+0} \frac{1}{\pi} \int_{-\infty}^{\lambda} v(x+iy)dx.$$

The proof was based on the Poisson integral theorem. First the existence of the limit defining $\psi(\lambda)$ was established, then the representation theorem was proved by a reasoning adapted from a paper of R. Nevanlinna.

7. When four fixed points and two variable points on a conic are considered as the vertices of an ordered hexagon, under certain restrictions the Pascal line of the hexagon envelops an algebraic curve. Professor Ott obtained parametric equations for certain of these curves, located their singularities, and obtained their Plücker characteristics.

8. Professor Agnew gave a general discussion of convergence and other methods of summability of series. Emphasis was placed upon relations between different methods of summability and upon Tauberian theorems.

C. W. MUNSHOWER, *Secretary pro tempore*

THE TWENTY-FOURTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-fourth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College of Agriculture and Mechanic Arts, Fort Collins, Colorado, April 19 and 20, 1940. There were three sessions. Professor D. F. Gunder presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Educational Association.

The attendance was seventy-six, including the following twenty members of the Association: C. F. Barr, J. R. Britton, I. M. DeLong, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, W. V. Lovitt, S. L. Macdonald, A. E. Mallory, W. K. Nelson, Greta Neubauer, M. G. Pawley, G. B. Price, O. H. Rechard, A. W. Recht, C. H. Sisam.

At the business meeting the following officers were elected for next year: Chairman, W. V. Lovitt, Colorado College; Vice-Chairman, J. C. Fitterer, Colorado School of Mines.

The following papers were presented:

1. "The line integral of curvature as a measure of its associated central angle" by Professor C. F. Barr, University of Wyoming.

2. "Determination of the differential equation and the equation of the orbit of a central force when the law of the force is known" by Professor J. R. Everett, Colorado School of Mines.

3. "Expansions of determinants of order four and five" by Professor W. V. Lovitt, Colorado College.

4. "The Gaussian solution of the trinomial equation" by Professor A. J. Lewis, University of Denver, read by A. M. Kahan.

5. "Projective representation of an affinely connected space" by T. C. Doyle, University of Wyoming, introduced by Professor Rechard.

6. "Some famous problems of modern mathematics" by Professor G. B. Price, University of Kansas.

7. "Mathematical Reviews" by Professor G. B. Price, University of Kansas.

8. "Report of the college-secondary mathematics coördinating committee" by Professor D. F. Gunder, Colorado State College.

9. "Discussion of trends in the teaching of mathematics in the junior high school" by Dr. G. S. Willey, Director of Instruction, Denver Public Schools; Professor L. Edwards, Colorado State College of Education; Superintendent M. W. Jessup, Bennett, Colorado, by invitation of the program committee.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. The condition that a line integral of curvature along a polar curve be a constant multiple of its associated central angle is expressed by an ordinary differential equation of the second order, which is completely solvable. Embedded in this family of solutions are some of the best known angle-measurement theorems of elementary geometry. Professor Barr advanced the suggestion that the line integral under consideration may be used to unify and greatly extend these theorems. (A preliminary report.)

3. Professor Lovitt obtained the expansion of a determinant of order four by means of a schematic diagram comparable to that used for a determinant of order three.

4. Professor Lewis outlined Gauss's method of finding the real roots of trinomial equations by the use of addition and subtraction logarithms. He showed how the method could be simplified by modern computing machines.

5. Given an affinely connected space of N dimensions with connection components $\Gamma^i_{jk}(x)$ and curvature tensor $\Gamma^i_{jkl}(x)$, the partial differential equations

$$\frac{\partial^2 y}{\partial x^j \partial x^k} = \Gamma^r_{jk}\frac{\partial y}{\partial x^r} - \frac{1}{N-1}\Gamma^r_{jkr}y \tag{1}$$

will admit a system of $N+1$ fundamental solutions $y^\alpha(x)$ determined to within a projective transformation with constant coefficients providing the integrability conditions are satisfied. These solutions $y^\alpha(x)$ serve to define homogeneous coördinates of the point (x^i) and the geometry of the resulting projective representation will find its analytical expression in the invariant theory of (1) under the combined transformations $x^{-i}=x^{-i}(x)$ and $y=\phi(x)y$, where ϕ is an arbitrary non-vanishing factor.

6. Professor Price gave the history and present status of Waring's Problem, the Four-Color Problem, the Jordan Curve Theorem, and the Problem of Plateau. The talk was illustrated with various models and demonstrations, including paper and rubber models of surfaces, a map on a wooden torus which required seven colors for its coloring, and soap film models for the Problem of Plateau. These problems were used: (1) to emphasize the great progress that has taken place in mathematics in recent times; (2) to illustrate the nature and source of problems in mathematics; (3) to point out the difference between a proof in mathematics and a proof in physics. The oldest of the four problems was first studied in 1636; although great progress has been made in recent years,

all four of them are still the subject of research. Problems in mathematics arise from (a) conundrums dealing with the positive integers, (b) a study of the physical world, and (c) from generalizations of simpler problems. Proof in mathematics consists of logical deduction; proof in physics consists of an induction from a large number of experiments.

8. The coördinating committee found that there was little discrepancy between the material presented in high school courses in mathematics and that required or expected of entering college freshmen. As a consequence, the present lack of preparation of college freshmen was attributed to lack of retention. Six suggestions for improving the general situation were offered by the committee. These were:

(1) Teach with emphasis on understanding rather than mechanical manipulation.

(2) Teach the correct terminology to further promote understanding.

(3) Teach the material in larger units with frequent repetition to prevent loss of sight of the subject as a unified whole.

(4) Induce other departments to use mathematics understandingly.

(5) Supplement or replace formal college entrance requirements by placement examinations.

(6) Improve the quality of teachers by requiring of them more and broader education in both mathematics and other fields.

9. The main purpose of education in the junior high school is the furthering of wholesome growth and development of the whole child through broad, meaningful experiences. Mathematics teachers, concerned with the total development of boys and girls, must consider how their subject will contribute to the concerns and problems of youth. The trend for teaching mathematics in the junior high school was summarized by this discussion as follows:

(1) Teaching all children only that which we know all children will use. Children interested in vocational phases may pursue mathematics materials in those fields.

(2) Choosing units of subject-matter through pupil-teacher planning.

(3) Permitting the "natural" method of learning, which is the only good teaching technique.

(4) Not carrying drill beyond the limits of its use by average adults in the community.

(5) Giving ample opportunity for experiences employing the four fundamental processes, simple fractions, percentage, and interest, stressing accuracy in each case.

(6) Permitting many informational problem-solving experiences that are meaningful to the pupils.

(7) Providing a program of diagnostic checking and remedial teaching.

(8) The placing of general, or social, mathematics through the ninth grade, with opportunity for electing algebra in the ninth grade for the present at least.

A. J. LEWIS, *Secretary*

THE TWENTY-SIXTH ANNUAL MEETING OF THE KANSAS SECTION

The twenty-sixth annual meeting of the Kansas Section of the Mathematical Association of America was held at the University of Wichita on Saturday, March 30, 1940. In the morning there was a joint session with the Kansas Association of Teachers of Mathematics. A social hour and luncheon at noon was followed by the showing of a sound film, "Rectangular Coördinates." After this the two organizations met for separate programs. Professor C. B. Read, chairman of the Section, presided at the morning session as well as at the Section meeting.

The attendance was one hundred and twenty-three, including the following thirty-five members of the Association: Sister Mary N. Arnoldy, R. W. Babcock, Wealthy Babcock, E. A. Beito, Lois E. Bell, Talmon Bell, C. V. Bertsch, Florence L. Black, E. E. Colyer, R. D. Daugherty, Lucy T. Dougherty, W. H. Garrett, W. A. Harshbarger, A. J. Hoare, Emma Hyde, W. C. Janes, H. E. Jordan, C. F. Lewis, Anna Marm, U. G. Mitchell, O. J. Peterson, G. B. Price, C. B. Read, C. A. Reagan, B. L. Remick, D. H. Richert, J. A. G. Shirk, D. T. Sigley, G. W. Smith, R. G. Smith, W. T. Stratton, C. B. Tucker, Gilbert Ulmer, J. J. Wheeler, A. E. White.

The officers elected for the coming year are: Chairman, G. B. Price, University of Kansas; Vice-Chairman, C. V. Bertsch, Southwestern College; Secretary, Lucy T. Dougherty, Junior College, Kansas City. The time and place of the next meeting were left to the executive committee.

The major portion of the program was given to the reports of the committee appointed at the 1939 meeting to arrange for a uniform test in mathematics. The test was prepared by the committee, and was given in September 1939 to entering freshmen in most of the colleges and universities in the state. Several of the institutions repeated the test at the end of the first semester. The committee through its different members presented the results of the test by summary, analysis, and interpretation. These reports are included in the abstracts below. In the free and informal discussion that followed, the problems developed were so many that the committee was continued with instructions to go on with the study another year.

The following eight papers and reports were presented:

1. "The Peano-Baker method and mean coefficients" by Professor W. C. Janes, Kansas State College.

2. "Mean deviation of ungrouped variates" by Professor J. A. G. Shirk, Kansas State Teachers College, Pittsburg.

3. "Mathematics journals" by Professor D. T. Sigley, Kansas State College.

4. "Results of Kansas mathematical tests as they apply to college courses"—reports and round table discussion.

   a. Professor U. G. Mitchell, University of Kansas.

   b. Professor W. H. Garrett, Baker University.

c. Professor J. A. G. Shirk, Kansas State Teachers College, Pittsburg.
d. Professor W. T. Stratton, Kansas State College.
e. Professor C. V. Bertsch, Southwestern College.

Abstracts of the papers and reports follow, the numbers corresponding to the numbers in the list of titles:

1. Professor Janes gave a brief résumé of the Peano-Baker method of solving differential equations, together with a discussion of the procedure for obtaining approximate solutions by replacing the matrix of coefficients by a matrix of constants.

2. Professor Shirk showed that quite frequently the median is a more significant average than the arithmetic mean as a measure of the central tendency when the number of variates is rather small. Mean deviation is defined as the arithmetic mean of the absolute values of the deviations of the variates from any central tendency. It may therefore be taken from the median as well as from the arithmetic mean. He then developed a simple formula for finding mean deviation by the use of an addition-subtraction machine. The process is much simpler than the calculation of standard deviation. He also gave an easy proof that the mean deviation from the median is less than from any other point.

3. Professor Sigley, after a short philosophical discussion on mathematics journals, spoke particularly on *Mathematical Reviews*. The accomplishments and policies, as interpreted from the first three numbers of the publication, were discussed. Statistics covering number of subscriptions, papers reviewed, and so on, were presented.

4. a. Professor Mitchell presented the results of Kansas Mathematics Test, Number One, given to 4351 students entering Kansas colleges and universities in the fall of 1939. The test was prepared by a committee of the Kansas Section of the Mathematical Association of America and consisted of twenty questions in arithmetic and thirty-five questions in elementary algebra. Tabulations and analyses are not yet complete, but distributions of scores for 2903 students having one year or less, and for 1448 students having more than one year of high school algebra were presented. Also the total scores of a combined group of 4045 students, and a diagnostic analysis of answers to each of the fifty-five questions asked, for 2542 students. It is expected that the complete results will be published in the October 1940 issue of the *Bulletin* of the Kansas Association of Teachers of Mathematics.

b. Professor Garrett presented the results of a study made by the committee, showing the correlation of the grades made in the freshman classes in mathematics in twenty colleges and universities of Kansas the first semester of 1939–1940, and the scores made in the Kansas Test given at the beginning of the semester. Several charts were presented showing the relation of the class grades to the test scores, the distribution of grades in the various classes, the relation of the maxima and minima scores, and so forth. The data included the class records of fifty-one sections in five-hour college algebra, totaling 1150 students; seventeen sections of trigonometry, 358 students; thirty-eight sections of three-hour college algebra, 784 students; and three sections of general mathematics, 80 students.

c. Professor Shirk expressed the opinion that the principal value of this test has been to reveal the very meager mathematical skills in the possession of the average college freshman. From a study of the first semester grades received in college mathematics in Kansas State Teachers College of Pittsburg, it was found that the correlation of the grades with the scores of the mathematics test was too low to be of the significance desired, and therefore this test would hardly serve as a means of dividing freshman students into sections. The correlation with the grades in college algebra was .411; the results of this test might be used with some value as additional information to the high school record, which was found to give a correlation with college mathematics of about .65. The arithmetical portion of the test showed great deficiencies in ordinary computations, and it was suggested that in forming a test for 1940, business men be asked to aid in devising a test which would be in line with what they felt their employees should know. This would also enlist the support of patrons of the public system in insisting upon better accomplishment in preparatory mathematics.

d. Professor Stratton said that at Kansas State College the test was given at the close of the semester, as well as at the beginning, and Professor A. E. White had worked out a number of correlations between the term grades and the test, for sections of three-hour and five-hour college algebra. The correlations were high. Professor Stratton also gave the average scores made by students from a number of the larger high schools and suggested the possibility of ranking the high schools as to the quality of their work done in mathematics.

e. Professor Bertsch said that the low scores in the arithmetic part of the test, and the general correlation between the arithmetic and the algebraic parts, led him to believe that the cause of the low results may be partially inherent in the work preceding the high school. He mentioned the trend in grade school children of excellent abilities to profess a dislike of arithmetic, while seemingly quite enthusiastic over other subjects. The early development of such an attitude cannot but have a bad influence on a student's success in mathematics. He then suggested some changes in the tests for the coming year, that may make them more valuable for diagnostic purposes.

LUCY T. DOUGHERTY, *Secretary*

THE SEVENTEENTH ANNUAL MEETING OF THE NEBRASKA SECTION

The seventeenth annual meeting of the Nebraska Section of the Mathematical Association of America was held at Creighton University, Omaha, on Saturday, May 11, 1940. Professor A. K. Bettinger of Creighton University was chairman.

The attendance was thirty-six, including the following twelve members of the Association: M. A. Basoco, A. K. Bettinger, W. C. Brenke, C. C. Camp, A. L. Candy, H. M. Cox, J. M. Earl, J. D. Fitzpatrick, M. G. Gaba, A. L. Hill, F. E. Marrin, Lulu L. Runge.

Officers elected for the coming year were: Chairman, A. L. Candy, University of Nebraska; Secretary-Treasurer, Lulu L. Runge, University of Nebraska; Member of Executive Committee, A. K. Bettinger, Creighton University. The next meeting will be held at the University of Nebraska in Lincoln, May 1941.

After a one o'clock luncheon at the Omaha Athletic Club, Professor Brenke presided at a round table discussion on (1) unified mathematics examinations; and (2) classification tests in mathematics. There was also a discussion of Professor Weil's paper.

The following program was presented:

1. "On an identity by Bailey" by J. A. Daum, University of Nebraska.

2. "How can experimental psychological facts be utilized for teaching mathematics in secondary schools?" by Professor Hermann Weil, Nebraska Central College, introduced by Professor Bettinger.

3. "Algebraic proofs of Dwyer's identities" by T. E. Oberbeck, University of Nebraska, introduced by the Secretary.

4. "A certain one-to-one correspondence" by E. P. Coleman, University of Omaha, introduced by Professor Bettinger.

5. "What does a mathematics examination examine?" by H. M. Cox, University of Nebraska.

6. "Rectification of the ellipse" by Professor M. G. Gaba, University of Nebraska.

7. "Grades in freshman algebra as indicative of later success in engineering courses" by Professor C. C. Camp, University of Nebraska.

8. "Certain pseudo-periodic functions" by Professor W. A. Dwyer, Creighton University, introduced by Professor Bettinger.

9. "Note on perfect numbers" by Professor T. A. Pierce, University of Nebraska, by title.

Abstracts of the papers follow, numbered in accordance with their numbers listed above:

1. The identity

$$\theta_3(\alpha)\theta_3(\beta+\gamma)\theta_3(\beta+\delta)\theta_3(\alpha+\gamma+\delta) - \theta_3(\beta)\theta_3(\alpha+\gamma)\theta_3(\alpha+\delta)\theta_3(\beta+\gamma+\delta)$$
$$= \theta_1(\gamma)\theta_1(\delta)\theta_1(\alpha-\beta)\theta_1(\alpha+\beta+\gamma+\delta),$$

(W. N. Bailey, *Quarterly Journal of Mathematics*, 1936) was discussed by Mr. Daum. It was shown that this identity is equivalent to Jacobi's fundamental formula involving products of four theta functions. Relations involving certain of the functions

$$\phi_{\alpha\beta\gamma}(x, y) \equiv \frac{\theta_1'\theta_\alpha(x+y)}{\theta_\beta(x)\theta_\gamma(y)}$$

were also obtained.

2. Psychologists of the University of Marburg, Germany, have discovered that there are some children who have the ability to see very vividly objects which are not present to their sight at the moment. Such a child, after having been asked to look attentively at an object, is able, with eyes open or closed to *see* this object again. This is possible either immediately or after a certain lapse of time, even after the passage of several years. Although the stimulus object may be reproduced with almost photographic fidelity, eidetic images, as these images are called, differ from the original stimulus object as to color, form, and a number of details. They differ from hallucinations as well as from after-images. They differ from memory-images in that the phenomena are really seen. Dr. Weil's paper dealt with eidetic images as observed in plane and solid geometry.

3. Mr. Oberbeck used the arithmetic method of Uspensky to prove four of the arithmetic identities of W. A. Dwyer which appear in vol. 45 of the *Bulletin of the American Mathematical Society*. He also showed by simple arithmetic considerations that these four identities form a fundamental set of a certain type of identity.

4. Mr. Coleman considered the relation $t^2+yt+x=0$, where x and y refer to the rectangular coördinates of a point in the plane and t refers to a point on a line. By this relation an eminent contact between certain notions of the one-dimensional geometry and the corresponding notions of the two-dimensional geometry is shown. For an assigned point in the plane the relation becomes a quadratic in t which yields two points on the line. When a point is chosen in the plane so that the discriminant, y^2-4x, is equal to zero, the quadratic equation leads to one real distinct point on the line. A one-to-one correspondence is shown to exist between the points of the parabola, $y^2-4x=0$, and the points on the t-line.

5. Mr. Cox illustrated the concepts of examination reliability, examination validity, question difficulty, and question validity by data obtained from the use of the University of Nebraska mathematics classification examination in the Nebraska high schools. He considered the meaning of the "single score" as obtained from an achievement examination.

6. By elementary geometric methods Professor Gaba showed that the length of an ellipse of semi-major axis a and eccentricity e is

$$\lim_{k\to 0}(1+2k)\sum_{n=0}^{n=k-1}\frac{\sqrt{[(2k+1)^2+(2n+1)^2]^2-4e^2[(2n+1)^2-(2k+1)^2]^2}}{(k^2+k+n)(k^2+k+(n+1)^2)}.$$

7. This is a statistical study of prerequisite courses. Should the prerequisite course be given special consideration? Should credit be given on the same basis as for other courses? With the passing mark 60, in view of a rule that requires 4/5 of the grades for graduation to be 70 or higher, certain courses may need to be repeated. Would the mortality in future work be appreciably lessened if the credit in prerequisite courses was withheld for any mark under 70? Professor Camp made frequency tables and calculated correlation coefficients for grades

in algebra with subsequent courses. The records of 1010 students over a period of 6 years were tabulated by a student assistant. Among other things the ratios of students receiving grades less than 70 in one or more subsequent courses for the algebra grade groups 60–69 and 70–79 were calculated. These ratios did not differ sufficiently to justify withholding credit in prerequisite courses for students in the 60–69 group.

8. Professor Dwyer showed that a certain pair of identities, given by Uspensky (*Bulletin of the American Mathematical Society*, vol. 36, identities I, II), involving incomplete numerical functions in three variables result from the paraphrase of a certain theta function identity. The method of establishing the theta identity is that used by Basoco (*American Journal of Mathematics*, vol. 54).

9. By use of Gauss's theorem that the sum of the totients of the divisors of a number is equal to the number, Professor Pierce proved that a number is perfect if, and only if, the sum of the totients of the proper divisors of the number is equal to the totient of the number.

LULU L. RUNGE, *Secretary*

THE MARCH MEETING OF THE SOUTHEASTERN SECTION

The eighteenth annual meeting of the Southeastern Section of the Mathematical Association of America was held at the University of Georgia, Athens, Ga., on Friday and Saturday, March 29–30, 1940.

There were in attendance about three hundred persons from fifty-three institutions, including the following eighty-one members of the Association: Martha E. Allen, D. H. Ballou, T. A. Bancroft, G. F. Barnes, D. F. Barrow, W. S. Beckwith, R. C. Blackwell, R. V. Blair, J. W. Blincoe, E. T. Browne, Iris Callaway, E. A. Cameron, C. L. Carroll, Jr., T. C. Carson, J. B. Coleman, R. W. Cowan, Orpha A. Culmer, Forrest Cumming, R. T. Donnell, B. F. Dostal, L. A. Dye, E. D. Eaves, W. W. Elliott, Floyd Field, Tomlinson Fort, W. A. Gager, L. L. Garner, Leslie J. Gaylord, J. J. Gergen, M. E. Gillis, L. J. Green, J. T. Hains, C. L. Hair, R. A. Hefner, Archibald Henderson, Ruby U. Hightower, P. R. Hill, C. W. Hook, J. V. Howell, H. M. Hughes, R. O. Hutchinson, J. A. Hyden, J. B. Jackson, E. S. Kennedy, F. W. Kokomoor, G. B. Lang, J. W. Lasley, Jr., F. A. Lewis, J. B. Linker, T. G. Loudermilk, E. L. Mackie, S. W. McInnis, W. J. Mays, J. F. Messick, E. R. C. Miles, Verdie F. Miller, W. B. Moye, W. P. Ott, C. R. Phelps, C. G. Phipps, W. W. Rankin, M. C. Rhodes, H. A. Robinson, C. C. Sams, C. L. Seebeck, Jr., C. Eucebia Shuler, T. M. Simpson, Augustus Sisk, A. R. Sloan, H. E. Spencer, F. H. Steen, Ruth W. Stokes, J. M. Thomas, Louise Thompson, T. L. Wade, Jr., J. A. Ward, D. L. Webb, C. W. Williams, W. L. Williams, F. L. Wren, R. C. Yates.

Sessions were held Friday afternoon and evening, and Saturday morning. Professor C. L. Hair, chairman of the Section, presided, except Friday evening, and part of Saturday morning when the Section was divided into sub-groups according to the nature of the papers presented. Sub-groups were presided over by Professors F. L. Wren, J. B. Jackson, Forrest Cumming, F. A. Lewis, Archibald Henderson, and D. H. Ballou. On Friday evening a dinner was given in honor of the visiting speaker, Dean Tomlinson Fort of Lehigh University. At this time Professor Forrest Cumming presided.

At the business session on Saturday the following officers were chosen for 1940–41: Chairman, Forrest Cumming, University of Georgia; Vice-Chairman, J. W. Lasley, Jr., University of North Carolina; Secretary-Treasurer, H. A. Robinson, Agnes Scott College; Members of Executive Committee: D. H. Ballou, Georgia School of Technology, T. M. Simpson, University of Florida, F. L. Wren, George Peabody College for Teachers; Governor, Region No. 5, J. M. Thomas, Duke University. The next meeting was scheduled for March 1941, at the University of North Carolina.

The following forty-three papers were presented:

1. "The marked ruler" by Dr. R. C. Yates, Louisiana State University.

2. "Four models on conic sections for use in projective geometry" by Professor Ruth W. Stokes, Winthrop College.

3. "Freshman preparation" by Professor C. L. Hair, The Citadel.

4. "A problem in baseball" by Professor W. S. Beckwith, University of Georgia.

5. "Analysis of an ancient Hindu game" by Dr. L. D. Rodabaugh, University of Alabama, introduced by the Secretary.

6. "Southeastern mathematics text-book authors" by Professor H. A. Robinson, Agnes Scott College.

7. "*Mathematical Reviews* for the mathematician in a small college" by Professor F. W. Kokomoor, University of Florida.

8. "The story of the parallelogram" by Dr. R. C. Yates, Louisiana State University.

9. "The study of geometry in high school" by Professor W. W. Rankin, Duke University.

10. "The Lehmus-Steiner problem" by Professor Archibald Henderson, University of North Carolina.

11. "Mathematics and the sciences" by Dean Tomlinson Fort, Lehigh University.

12. "A note on the average deviation" by Dr. H. H. Germond, University of Florida, introduced by the Secretary.

13. "The Plate of Zones of Ghiyathu'd Din al-Kashi" by Dr. E. S. Kennedy, University of Alabama.

14. "The resolvents of a polynomial" by Professor J. M. Thomas, Duke University.

15. "A program in freshman mathematics designed to care for a wide variation in student ability" by Professor E. A. Cameron, University of North Carolina.

16. "A new course for commerce students" by Professor J. W. Lasley, Jr., University of North Carolina.

17. "*Mathematical Reviews* for the research mathematician" by Professor J. J. Gergen, Duke University.

18. "Abel's lemma and applications in infinite series" by Dean Tomlinson Fort, Lehigh University.

19. "A mathematical program for junior college students who do not plan to continue work in mathematics" by Professor W. A. Gager, St. Petersburg Junior College.

20. "Problems of adapting instruction in mathematics to the students entering the junior college" by Professor C. Eucebia Shuler, Georgia Southwestern College.

21. "Aids to motivation in junior college mathematics" by Professor Ruth W. Stokes, Winthrop College.

22. "Mathematical inadequacies of students entering the senior college" by Professor W. A. Beckwith, University of Georgia.

23. "The status of mathematics in a liberal curriculum" by Miss Curtis Ledford, Griffin, Georgia, High School, introduced by the Secretary.

24. "Models for illustrating special products and factoring" by T. G. Loudermilk, Decatur, Georgia, Boys High School.

25. "Correlating mathematics in the junior high school" by Martha E. Allen, Atlanta Junior High School.

26. "A diagonal property of determinants" by Professor E. R. C. Miles, Duke University.

27. "On the characteristic roots of a circulant" by Professor H. S. Thurston, University of Alabama, introduced by the Secretary.

28. "Some generalizations of the fundamental theorem of algebra" by Dr. Witold Hurewicz, University of North Carolina, introduced by the Secretary.

29. "Solution of a certain linear difference equation of the second order with polynomial coefficients" by Dr. R. W. Cowan, University of Alabama.

30. "Automorphisms of the Lie algebra of order 28" by C. L. Carroll, Jr., Georgia School of Technology.

31. "The number and computation of idempotents for any modulus" by H. M. Hughes, University of Tennessee.

32. "Line configurations associated with a group of order 48 and degree 6" by Dr. W. G. Warnock, University of Alabama, by title.

33. "An involutorial Cremona transformation associated with a pencil of quartic surfaces" by Dr. L. A. Dye, The Citadel.

34. "On geometries and invariants" by Dr. T. L. Wade, Jr., University of Alabama.

35. "Loci associated with certain circles in the plane" by Professor L. L. Garner, University of North Carolina.

36. "Twisted cubics associated with a space curve" by Dr. L. J. Green, Georgia School of Technology.

37. "Line coördinates" by R. T. Donnell, Castles Heights Military Academy, Tennessee.

38. "On the reduction of a matrix to rational canonical form" by Professor E. T. Browne, University of North Carolina.

39. "A note on the quartic and its Hessian" by Dr. J. A. Ward, Tennessee Polytechnic Institute.

40. "Formulas for coefficients in equations whose roots form certain sequences" by Dr. G. B. Lang, Emory University.

41. "An invariant of the set of affine connections defining vectors normal to every arbitrary curve of a Riemann space" by Dr. C. L. Seebeck, Jr., University of Alabama.

42. "A note on cube roots of complex numbers" by Dr. J. N. Mallory, Union University, introduced by the Secretary.

43. "A theory of runs of luck" by W. J. Mays, Masseyville School, Tennessee.

Abstracts of some of the papers follow, numbered in accordance with their listing above:

1. Dr. Yates gave a discussion of the constructional possibilities of the double-edged marked ruler as a geometrical tool. By inserting a marked portion between two given lines, similar to the trammel of Archimedes and the conchoid trisection trammel mentioned by Pappus, Dr. Yates showed that all problems leading to equations of degree not higher than four whose coefficients represent possessed lengths are solvable by this tool alone.

2. Professor Stokes displayed four projective geometry models and demonstrated their construction. She also presented a sound film on the parabola.

3. In his retiring address, Chairman Hair discussed the problem of freshman mathematics and the preparation of the pupil, asserting that the right kind of pre-college direction has been lacking.

4. Professor Beckwith discussed a problem of "minimum time" for throwing a ball from an outfielder to a catcher. He showed that when a ball is caught on the fall before a bounce, the time is longer than if caught on the rebound.

5. In this paper, the solution of a generalized game of "Kim" was presented from a matric view-point. Dr. Rodabaugh's technique was applied to a broad class of games.

6. Some seventy mathematics books published by authors living in the Southeastern Section were listed by Professor Robinson. Thirty books published recently were displayed in an exhibit.

7. Professor Kokomoor gave the history of "*Mathematical Reviews*," and showed the need for the publication by mathematicians located in institutions isolated from the research centers.

8. Dr. Yates gave an account of the dominant rôle the parallelogram has played in the theory of dissection, in restricted euclidean construction, and in the theory of linkages and mechanical motions.

9. Professor Rankin reviewed the status of the study of geometry in the high schools and pointed out some of the more recent efforts for improving the work in geometry. He invited college professors to give serious thought as to

ways and means of giving better preparation for the teachers of geometry.

10. Professor Henderson presented his subject in the form of a centennial anniversary paper, as it was in 1840 that Lehmus gave to Steiner the problem of equal internal base-angle bisectors of a triangle. Dr. Henderson sketched the history of the problem, and presented a number of solutions believed to be new, based on a uniform technique.

11. This paper will appear in this MONTHLY.

12. Dr. Germond illustrated certain common errors found in statistics textbooks. He showed that unless a modification is made in the manner of computing the average deviation, it is not necessarily the least when computed about the median.

13. Dr. Kennedy gave a preliminary report on a study being made of an anonymous 15th century Persian manuscript which describes the construction and operation of an astronomical instrument invented by al-Kashi.

14. This paper will appear in this MONTHLY in November.

15. This paper appeared in the August-September, 1940, issue of this MONTHLY.

16. Professor Lasley gave an account of a new course in mathematics for commerce students which was developed at his university by the combined efforts of both mathematics and commerce departments.

18. In this address, Dean Fort was concerned with the formula for summation by parts and applications in the theory of infinite series. Applications were first pointed out in the study of the convergence of Dirichlet series, factorial series, Lambert series, and other related types. The second portion of the address was concerned with generalizations of the formula for summation by parts which are applicable in the theory of summability. Particular reference was made to recent research of the lecturer.

19. Professor Gager showed how he believed a mathematics program may be integrated in order to give the junior college terminal student the minimum essential tool mathematics for effective citizenship.

20. Professor Schuler's test showed that preparation of incoming students indicates a decrease in quality of present day training. She urged that pupils be taught to think analytically, to read reflectively, and to reason effectively in order that they might arrive at true conclusions with controlled emotions.

21. Professor Stokes discussed visual aids to motivation in junior college mathematics. She showed certain lantern slides, and a sound film on "Rectilinear Coördinates." Finally, she called attention to an electrically lighted stereographic instrument which may be used to illustrate propositions in solid geometry.

22. Professor Beckwith listed certain mathematical inadequacies found in students entering the senior college. He suggested that more emphasis should be placed on the fundamental operations of algebra and arithmetic.

23. Miss Ledford showed that the demands of our technical civilization make a study of mathematics a vital necessity to everyone.

25. Miss Allen gave a brief analysis of the type of mathematics a progressive

junior high school should carry on in order to have an integrated curriculum.

26. In a determinant of order n, the number of terms in the expansion which are free of elements of a particular diagonal is $\phi_1(n)=n!\Sigma(-1)^k/k!$. Professor Miles showed that if $\phi_k(n)$ denotes the number of terms free of elements of k parallel diagonals, then $\phi_k(n)$ is determined as the solution of a linear difference equation of order $k+1$ with polynomial coefficients.

29. Dr. Cowan assumed his solution to be in the form of a series in ascending powers of a parameter with undetermined coefficients. When the series was inserted in the difference equation, a set of non-homogeneous difference equations with constant coefficients were obtained which were solved by using the operator E. The resulting solution was simplified by means of Zeta-functions.

31. Mr. Hughes gave a simple formula for the number of idempotents for a given modulus and demonstrated some devices for simplifying their computations.

32. The operators of the imprimitive group of order 48 and degree 6 were applied to the fundamental identity of line geometry in S_3. The resulting identities were interpreted as lines.

34. In this paper, Dr. Wade pointed out certain advantages, particularly from the invariant view-point, of considering sub-geometries of projective geometry from the theories of tensors, as opposed to the view-point of regarding such geometries as the theories of sub-groups of the general linear projective transformation group.

35. Dr. Garner discussed a sequence of mutually equal circles which were tangent to each other in consecutive pairs, each circle being also tangent to a fixed circle. The point of contact of a typical pair of circles generated a continuous curve. Certain interesting properties of this locus were mentioned.

36. A canonical tetrahedron of reference can be found which yields a simple pair of power series expansions for a space curve in the neighborhood of an ordinary point. Dr. Green showed that geometical characterizations of them can be obtained from a study of the two-parameter family of five-point osculating twisted cubics.

37. Mr. Donnell gave the results of a study of the class of certain cubic curves by means of line coördinates.

38. Let A be an n-square matrix with elements in a field F. By invoking a knowledge merely of the theory of linear dependence and of the Hamilton-Cayley theorem, and without presupposing any knowledge of the notion of invariant factors, Professor Browne showed how to obtain a non-singular matrix T, with elements in F, such that $T^{-1}AT$ is in rational canonical form.

39. Dr. Ward derived a formula that expressed the roots of a quartic as functions of the roots of its Hessian in the special case where the Hessian is a cubic. Since all quartics may be so reduced, it is a general solution of the quartic. Similarities were pointed out between this method and a similar formula for the solution of a cubic.

41. Dr. Seebeck gave a necessary and sufficient condition for a symmetric affine connection of the vector associated with every arbitrary curve of a Rie-

mann space by means of covariant differentiation with respect to this affine connection to be normal to its associated curve.

42. Professor Mallory's work led to a method of finding certain solutions of a cubic equation. By making certain simplifications of the "irreducible case," a formula for the solution of the cubic by Tartaglia's method was derived.

43. The theory of runs of luck was developed from the solution of a certain difference equation. Mr. Mays developed an expression which gives approximately the number of repeated trials of an event of doubtful outcome necessary to obtain a given probability for a given run of luck.

H. A. ROBINSON, *Secretary*

BISECTING CIRCLES

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1. Introduction. If a circle x (center X) cuts the circle a (center A) in the ends of a diameter, the circle x is said to bisect the circle a. In the geometry of inversion, orthogonality and bisection play analogous rôles. If A is a given point and x a given circle, and if the power p of A with respect to x is positive, then A is the center of a circle a orthogonal to x, and its radius is $t=\sqrt{p}$, where t is the length of the tangent from A to the circle x. An inversion with respect to the circle a leaves the circle x invariant. If, however, the power p of A with respect to x is negative, then A is the center of a circle a bisected by x, and its radius is $c=\sqrt{-p}$, where c is the minimal half-chord through A. An inversion with respect to the imaginary circle, center A, radius$=\sqrt{-c^2}$, leaves the circle x invariant. We call this imaginary circle the conjugate of a, and represent it by the symbol $\bar{a}$. The inversion with respect to $\bar{a}$ may be interpreted as a real inversion with respect to the circle a, followed by a reflection on the point A. For if any chord through A cuts x in P_1 and P_2, we have

$$AP_1\cdot AP_2 = -c^2.$$

If the inverse of P_1 with respect to a is P_1', and the reflection of P_1' on A is P_1'', then the equations

$$AP_1\cdot AP_1' = +c^2, \qquad AP_1\cdot AP_1'' = -c^2,$$

show that P_1'' and P_2 are identical, so that the circle x remains invariant.

It is the purpose of this paper to study circles with respect to this bisection property, and as illustrations to discuss the ruler and compass construction of circles determined by three conditions, when some of these conditions are this bisection property. The other conditions will be orthogonality and tangency,* represented by the symbols O and T, respectively.

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* For circles involving these properties only, see a paper by the author, this MONTHLY, vol. 34, 1927, pp. 357–359.

All of the results of this article are not new; for example, a number of them are given or suggested at various places in Altshiller-Court, College Geometry (referred to hereafter as A.-C.), and in Coolidge, Treatise on the Geometry of the Circle and the Sphere (referred to hereafter as C.). Some specific references will be given later.