THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-third annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, on Friday and Saturday, April 28–29, 1939. There were three sessions. Professor C. F. Barr, chairman of the Section, presided at each.

The attendance was forty-one, including the following eighteen members of the Association: C. F. Barr, J. R. Britton, I. M. DeLong, G. W. Gorrell, D. F. Gunder, C. A. Hutchinson, Dunham Jackson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, W. K. Nelson, Greta Neubauer, O. H. Rechard, A. W. Recht, W. M. Stewart.

The following officers were elected for the coming year Chairman, D. F. Gunder, Colorado State College; Vice-Chairman, W. V. Lovitt, Colorado College; Secretary-Treasurer, A. J. Lewis, University of Denver. The 1940 meeting is to be held at Colorado State College, Fort Collins, Colorado.

The Section was honored in having Professor Dunham Jackson of the University of Minnesota as its guest speaker. The Saturday morning session was a joint meeting with the Mathematics Section of the Eastern Division of the Colorado Educational Association.

The following papers were presented:

1. "Teaching college mathematics to large classes" by Professor O. H. Rechard, University of Wyoming.

2. "On the Diophantine equation $x(x+1)\cdots(x+n-1)=y^k$" by Dr. L. Louise Johnson, University of Colorado.

3. "Leading differences for backward interpolation" by Professor W. V. Lovitt, Colorado College.

4. "Operational calculus and the theory of numbers" by Professor C. A. Hutchinson, University of Colorado.

5. "Evaluating shear stresses without finding flexure function" by Professor D. F. Gunder, Colorado State College.

6. "On foci and branch points" by Professor A. J. Kempner, University of Colorado.

7. "Orthogonality" by Professor Dunham Jackson, University of Minnesota.

8. "Morley triangles" by Professor Claribel Kendall, University of Colorado.

9. "A new class of orthogonal polynomials" by Professor Dunham Jackson, University of Minnesota.

10. Report: "College-secondary mathematics coordinating committee" by Professor D. F. Gunder, Colorado State College.

11. Discussion: "The report of the Joint Commission" by Professor C. A. Hutchinson, University of Colorado; Professor G. W. Gorrell, University of Denver; and Miss Glennie Bacon, University High School, Laramie.

12. "Are we ready?" by Wendall Wolf, Morey Junior High School, Denver, introduced by the Secretary.

Abstracts of some of the papers follow, numbered in accordance with their place on the program:

1. Professor Rechard continued the report given the Section at its last meeting on the teaching of large classes in mathematics at the University of Wyoming. In addition to the two reported on last year, three other classes were divided into large (over 50) and small (under 30) sections. On the basis of percentile rank in the Ohio College Ability Test two pairs of sections, those in analytic geometry and college algebra, were found to be not comparable. In the third sections which were comparable, no significant difference was found in their mathematical attainments as measured by three one-hour tests and the final two-hour examination. Thus of five classes divided into large and small groups, the three in which comparability on the P.R. basis was established have shown no significant difference in mathematical attainment between the large and small groups.

2. Let the product of n consecutive positive integers be $P_n = x(x+1) \cdots (x+n-1)$. The equation $P_n = y^k$, y and k integers greater than 1, is known to be impossible if $n=2$, 3, or k; if $n \leqq 203$, $k=2$; if $n \leqq 13$, $k=3$, or 5; if $n=4$, k one of a certain infinite set of primes; if one of the integers is ap, p prime, $a \leqq 8$; or if $x < n^2$. Dr. Johnson showed that the equation is impossible also if $n=2k$; if $x \leqq (n+1)^k - n$; if one of the integers is $p_1^{\alpha_1}, p_2^{\alpha_2}, \cdots, p_\mu^{\alpha_\mu}$, the p's prime, $\sum_{i=1}^{\mu} \alpha_i < k$; or if one of the integers is a^k, $a < p$, p the greatest prime which divides P_n; and that the equation has at most one solution for $n=4$, 5, or 6, k a prime $\geqq 7$. There are a few other closely related results.

3. Leading difference formulas for subtabulation are derived from Newton's formula for negative interpolation. These leading differences at the end of a table being computed, the subtabulations can be made by a series of subtractions. Professor Lovitt has hence exposed the leading differences for the subtabulation in terms of the leading differences at the end of a table of differences instead of the leading differences at the beginning of the table.

4. Professor Hutchinson gave a critique of an article by B. van den Pol, in the December 1938 *Philosophical Magazine.*

5. The complete solution of the well known flexure problem requires the determination of a function x satisfying $\nabla^2 x = 0$ within the section and the condition that the derivative shall be a specified function on the boundary. However, for the actual determination of the stresses within the beam only the partial derivatives of x are needed. Professor Gunder found an accurate and relatively simple method of finding these derivatives by transforming the given boundary onto the unit circle and setting up the solution for x in terms of the usual integral involving the transformed value of the normal derivative. This expression is then differentiated under the integral sign and the resulting integral evaluated mechanically to give the required stresses.

7. Professor Jackson emphasized the fact that mathematical study is largely concerned with the acquisition of fundamental general ideas which recur in varied and progressively more complicated situations. Familiarity with such

fundamental principles is often vitally reinforced, and their significance clarified, by acquaintance with less elementary applications in which they are viewed under diverse aspects. This is illustrated by an outline of the development of the notion of orthogonality from the perpendicularity of elementary geometry through the algebraic and analytical formulations which appear in statistics, mathematical physics, and current research in pure mathematics.

8. Professor Kendall reported on an article, *Morley's Triangle*, given in the Mathematical Gazette of February, 1938. There W. J. Dobbs defines the Morley Triangle as one obtained by the intersections of certain trisectors of the angles of a given triangle. He shows that this triangle is isosceles, and extends the discussion to show that there are 27 such triangles, 18 of which are equilateral.

9. Professor Jackson's paper appeared in full in the October issue of the MONTHLY.

A. J. LEWIS, *Secretary*

THE ANNUAL MEETING OF THE MINNESOTA SECTION

The annual meeting of the Minnesota Section of the Mathematical Association of America was held at Carleton College, Northfield, Minnesota, on Saturday, May 13, 1939. A morning session was held at 10:30 o'clock and was followed by luncheon and an afternoon session at 2:15 o'clock. Professor W. H. Bussey of the University of Minnesota presided at each session.

Seventy-three persons attended the meeting, including the following twenty-eight members of the Association: C. J. Blackall, L. E. Bush, W. H. Bussey, E. J. Camp, C. S. Carlson, S. Elizabeth Carlson, Sister M. Claudette, H. H. Dalaker, J. H. Daoust, Brother Louis De La Salle, Margaret C. Eide, C. H. Gingrich, H. E. Hartig, J. S. Hickman, Dunham Jackson, Margaret P. Martin, W. R. McEwen, Sigurd Mundhjeld, F. J. Polansky, Inez Rundstrom, M. G. Scherberg, C. Grace Shover, F. J. Taylor, H. P. Thielman, Ella Thorp, A. L. Underhill, K. W. Wegner, Marion A. Wilder; and Sister Thomas à Kempis, institutional member representative.

At the business session officers were elected for the coming year as follows: Chairman, C. S. Carlson, St. Olaf College; Secretary, A. L. Underhill, University of Minnesota; Executive Committee, H. P. Thielman, College of St. Thomas, and Sister Thomas à Kempis, College of St. Teresa.

The following nine papers were presented:

1. "The stability of an unsymmetric top" by Professor E. J. Camp, Macalester College.

2. "A number problem" by Professor L. E. Bush, College of St. Thomas.

3. "An appreciation of Sophie Germain" by Sister M. Thomas à Kempis, College of St. Teresa.

4. "On the use of the complex exponential in the solution of differential equations" by Professor H. E. Hartig, University of Minnesota.

5. "The osculating conics to a point on a plane curve" by Dr. M. G. Scherberg, University of Minnesota.

6. "A system of orthogonal polynomials satisfying an auxiliary condition" by Professor Dunham Jackson, University of Minnesota.

7. "An alternate derivation of the equation of the director sphere" by Michael Norris, College of St. Thomas, introduced by Professor Bush.

8. "On roots of unity" by Dr. C. Grace Shover, Carleton College.

9. "Note on the integro-differential equations satisfied by certain generalized trigonometric functions" by Professor H. P. Thielman, College of St. Thomas.

Abstracts of these papers follow, the numbers corresponding to the numbers in the list of titles:

1. Professor Camp showed that a study of the motion of a rigid body with one point fixed can be reduced to a study of the Euler differential equations for a rigid body. If gravity is the only force acting on the body, there is a special solution in which one of the principal axes of inertia through the center of gravity is in the vertical position. A study of the solutions in the neighborhood of this special one is accomplished by making a transformation of variables involving a parameter ϵ in such a way that for $\epsilon=0$ the original variables reduce to the special solution mentioned above. The Euler equations are then solvable as a power series in the parameter ϵ, and the coefficients of the first degree terms in ϵ furnish the basis for the discussion of stability. The conditions for stability depend on the moments of inertia and the angular velocity of the body.

2. Let $S(r, N)$ be the sum of the digits of all numbers less than N when these numbers are expressed in the scale of notation of radix r. Then $S(r, N)$ is asymptotic to $(r-1)N(\log N)/(2\log r)$, in the sense that as $N\to\infty$ the limit of the quotient of $S(r, N)$ by the latter function is unity. Professor Bush showed that for N sufficiently large, the average sum of the digits of all numbers less that N is least when these numbers are expressed in the binary scale. In fact, if r_1 and r_2 are integers such that $2\leqq r_1<r_2$, and if N is sufficiently large, the average sum of the digits of all numbers less that N is less when the numbers are written in the scale of radix r_1 than when they are written in that of radix r_2.

3. In this paper Sister Thomas à Kempis gave the result of her investigation of letters in the Sophie Germain collection in the Bibliothèque Nationale. These letters reveal the unusual influence which Mme. Germain exerted on mathematical physics, especially on the theory of the vibrations of elastic surfaces. In particular they show her close association with contemporary savants, such as Biot, Legendre, Poisson, Lagrange, Laplace, Fourier, Delambre, Cauchy, and others, and contribute facts to substantiate Mozan's conclusion that "all things considered, she was the most profoundly intellectual woman France has produced."

4. The so-called complex number method of obtaining the particular solution of certain ordinary linear differential equations was discussed by Professor Hartig. By the introduction of an operator, the effect of which was to convert a

vector into its projection on a fixed reference line, the simple scalar character of the differential equation was retained, without sacrificing the simplicity and the interpretability inhering in the complex number method of solution.

5. On the hypothesis that the plane curve $y=f(x)$ is differentiable in an open interval containing $x=x_0$ and that the second derivative $f''(x_0)$ is not zero, Dr. Scherberg proved that each point in the plane not on the tangent line to the curve at (x_0, y_0) is the focus of a unique osculating conic. Further he proved that if C is a circle tangent to the curve $y=f(x)$ on the concave side at (x_0, y_0) and of radius one-fourth of the radius of curvature at (x_0, y_0), then each point of C is the focus of an osculating parabola; each point inside C is the focus of an osculating hyperbola; each point not in C but on the same side of the tangent line as C is the focus of an osculating ellipse; and finally each point not in C but on the opposite side of the tangent line from C is the focus of an osculating hyperbola.

6. At a recent meeting of another Section, Professor Jackson reported on a system of orthogonal polynomials $p_n(x)$, each satisfying the condition that $p_n(1)=p_n(-1)$. The present paper gave a corresponding discussion, similar in general outline but with some differences in detail, for a set of orthogonal polynomials with the boundary condition $p_n(-1)=-p_n(1)$.

7. Given a quadric surface, which is non-singular,

$$F \equiv \sum_{i,j=1}^{4} a_{ij}X_iX_j = 0, \qquad a_{ij} = a_{ji},$$

it is desired to find the locus of points from which three mutually perpendicular planes can be drawn tangent to the surface. Three such planes are assumed to meet in a point (x_1, x_2, x_3, x_4) of the locus. By using the equation of the quadric in plane coördinates, Mr. Norris obtained equations on the coördinates of the three planes. On elimination of these coördinates we get the necessary condition on (x_1, x_2, x_3, x_4). This is the usual $A_{44}(X_1^2+X_2^2+X_3^2)-2(A_{41}X_1+A_{42}X_2+A_{43}X_3)X_4+(A_{11}+A_{22}+A_{33})X_4^2=0$, where A_{ij} is the cofactor of a_{ij} in the matrix of the quadric.

8. The product of the n nth roots of unity is 1 or -1 according as n is odd or even, the product of the imprimitive nth roots of unity is 1 or -1 according as n is odd or even, and the product of the primitive nth roots of unity is 1. These results were proved by Dr. Shover by using simple lemmas from the theory of rational integers.

9. This paper is an extension of the article *A Generalization of Trigonometry*, H. P. Thielman, National Mathematics Magazine, vol. 11, 1937, pp. 349–351. Professor Thielman showed that in a general theory certain functions can be defined analogous to the trigonometric functions. These functions satisfy equations which in the theory of Volterra's functions of composition reduce to integro-differential equations, in the analytic function theory to differential equations. The results of L. E. Ward, this MONTHLY, vol. 34, 1927, and those

of V. B. Temple, National Mathematics Magazine, vol. 13, 1939, p. 263, are shown to be special cases of this general theory.

A. L. UNDERHILL, *Secretary*

THE SPRING MEETING OF THE ALLEGHENY MOUNTAIN SECTION

The twelfth regular meeting of the Allegheny Mountain Section of the Mathematical Association of America was held at Thiel College, Greenville, Pennsylvania, on Saturday, May 13, 1939. Professor H. L. Black, chairman of the Section, presided at both the morning and afternoon sessions.

The attendance was seventy-two, including the following twenty-two members of the Association: C. S. Atchison, B. R. Beisel, O. F. H. Bert, H. L. Black, A. M. Bryson, P. N. Carpenter, Elizabeth B. Cowley, L. L. Dines, H. L. Dorwart, V. V. Johnston, G. R. Kraus, David Moskovitz, Frederick Mosteller, L. T. Moston, F. W. Owens, Helen B. Owens, Elizabeth Renwick, R. G. Sturm, Sister M. Clotilda Sullivan, J. S. Taylor, E. D. Wells, R. T. Zimmerman.

It was decided to hold the fall meeting of the Allegheny Mountain Section at California, Pennsylvania, on October 7, 1939.

Following an opening address by Dean Luther Malmberg of Thiel College the first three speakers of the morning session were introduced by Mr. F. R. Layng, Chief Engineer of the Bessemer and Lake Erie Railroad Company. The following eight papers were read:

1. "Use of mathematics in the study of grade and line for steam railroads" by W. S. McFetridge, Principal Assistant Engineer, Bessemer and Lake Erie Railroad Company, introduced by the Secretary.

2. "The use of mathematical formulas in railroad maintenance and operation" by M. F. Mannion, Office Assistant to Chief Engineer, Bessemer and Lake Erie Railroad Company, introduced by the Secretary.

3. "Railroad tracks without joints; welded rail" by H. H. Harman, Engineer of Track, Bessemer and Lake Erie Railroad Company, introduced by the Secretary.

4. "Types of curvature of curves and surfaces" by Dr. Mary T. Speer, University of Pittsburgh, introduced by Professor Taylor.

5. "Euclid in a present-day workshop" by Professor O. F. H. Bert, Washington and Jefferson College.

6. "A tribute to the memory of Professor N. C. Grimes" by Professor P. N. Carpenter, Grove City College.

7. "Quartic surface invariant under the quaternary collineation group of order 168" by Free Jamison, University of Pittsburgh, introduced by Professor Taylor.

8. "Measures of rank correlation" by Frederick Mosteller, Carnegie Institute of Technology.

Abstracts of the papers follow, numbered in accordance with their place on the program:

1. Mr. McFetridge pointed out that the amount of traffic and the speed at which it can be operated over a given grade and line is affected not only by the power of the locomotives, but by certain resistances to movement that must be overcome. The steps necessary to determine the tonnage that may be handled by a locomotive on a certain grade are: (1) determine the tractive effort of the locomotive at the minimum speed decided on; (2) determine the resistance in pounds per ton for the cars to be handled; (3) determine the resistance due to grade and line in pounds per ton. With these data, the tonnage handled by a given locomotive on the line under consideration, or the necessary power to handle a given tonnage, or the maximum grade over which a given tonnage may be handled, can each be determined.

2. Mr. Mannion showed the methods and results of studies of the renewals that have been made for the past twenty to thirty years in an effort to establish the law of probability governing the rate of failure. Numerical data and graphs showing the estimated renewals and actual installations of railroad ties for the past several years were exhibited, also charts which illustrated the similarity between the typical train hour diagram and the probability curve. From this train hour diagram it is possible to compute wage costs and the cost of unproductive time. A study of the effects on operations and of the estimated savings that would result if present grades were altered was shown on several graphs. From these, the efficiency of operation and the wage cost for the revised grades can be computed and the savings for the different grades estimated.

3. Mr. Harman used motion pictures to illustrate the method of welding rails, and explained the reasons why railroads have been led to this experiment. In the usual type of track, with relatively short pieces, the ends of the pieces are worn due to hammering at the ends. Longer pieces of track have been used but there is a limit to the length of single piece that can be used due to excessive cost of fabrication. Charts were shown depicting a study of the longitudinal movement in the welded track and comparative temperatures of air and rail in an experimental mile of welded rail.

4. The various types of curvature associated with curves and surfaces were developed briefly by Dr. Speer, using vector methods. These types included: (a) circular, spherical, and screw curvature for curves in space, and vector, relative, and geodesic curvature for curves on a surface, and (b) normal curvature, and first and second, or mean and specific, curvature for surfaces. Particular reference was made to values for such special types of curves as geodesics, asymptotes, and lines of curvature, and to minimal and developable surfaces.

5. Professor Bert cited the theorem that the locus of the vertex of a right-angled triangle with constant length of hypotenuse is a circle. He then showed a tool, which he called a core box plane, that is of use to pattern makers in cutting circular cylindrical cores, and whose operation depends on the above theorem. The tool is handled like a plane, and the cutting edge moves along the elements of a circular cylinder.

6. This paper was read as a tribute to the memory of the late Nathan Cesna

Grimes, professor of mathematics in Grove City College from 1926 to 1938, by Professor Carpenter, his student, friend, and colleague. The brief chronological sketch of the education, activities, and interests of Professor Grimes indicated the great breadth and depth of these interests and activities. Before his teaching at Grove City College he had been head of the mathematics department and registrar at the University of Arizona, assistant to the president at the University of Oregon, and field employment manager for the Goodrich Rubber Company over the entire United States during the World War. The paper concluded with the resolutions of respect as recorded in the minutes of the faculty of Grove City College.

7. Mr. Jamison discussed the quartic surface

$$2x_0^4 + x_1x_3^3 + x_2x_1^3 + x_3x_2^3 + 6x_0x_1x_2x_3 = 0,$$

which is invariant under the transformation

$$x_n' = \frac{n^3 - 6n^2 + 11n + 6}{12\sqrt{-7}} \left[2x_0 + (\omega^n + \omega^{6n})x_1 + (\omega^{2n} + \omega^{5n})x_2 + (\omega^{3n} + \omega^{4n})x_3\right]$$

and $x_n' = \omega^{n^2}x_n$, where $\omega^7 = 1$, $n = 0, 1, 2, 3$; the generators of the group. There is a set S_1 of 8 planes, of which $x_0 = 0$ is one, whose 28 intersections form a set of bitangents to the quartic. The other bitangents in S_1 form a set of 168. There is a set of 8 triangular pyramids, whose lateral edges, of which $x_1 = x_2 = 0$ is one, each meet the quartic in 4 coincident points at a base vertex; and whose base edges lie in S_1 and are inflexional tangents. There is a set of 56 triangular pyramids, whose base edges lie in S_1 and are inflexional tangents, and whose lateral faces are tangent planes. The 24 base vertices of pyramids in each plane of S_1 are points of inflexion of a plane quartic and may be grouped in 8 triples, so that for each of 28 possible choices of two triples there is a conic which passes through all six points of the two triples and the points of contact of one of the bitangents to the quartic.

8. For variates not functionally related, it is often necessary to test for the existence of correlation. Mr. Mosteller dealt with several such tests by means of various coefficients of rank correlation. Illustrative examples were provided showing the use of these coefficients both for populations with correlation, and for independent variates. From the behavior of these coefficients in sampling known populations, inferences were drawn about the estimation of correlation by means of samples in populations whose character is unknown.

DAVID MOSKOVITZ, *Secretary*