## THE SECOND ANNUAL MEETING OF THE SOUTHWESTERN SECTION

The second annual meeting of the Southwestern Section of the Mathematical Association of America was held at the University of New Mexico, Albuquerque, on Monday and Tuesday, April 25–26, 1938, in conjunction with the meeting of the Southwestern Division of the American Association for the Advancement of Science.

The attendance was eighty, including the following nineteen members of the Association: C. B. Barker, C. A. Barnhart, L. M. Bauer, J. W. Branson, Gordon Fuller, R. F. Graesser, Elizabeth M. Haskins, E. A. Hazlewood, E. R. Hedrick, C. A. Hutchinson, A. J. Kempner, H. D. Larsen, D. H. Leavens, Roy MacKay, L. E. Mehlenbacher, A. B. Mewborn, C. V. Newsom, P. K. Rees, R. S. Underwood.

On Monday evening there was a dinner for members of the Section and their guests, which was followed by an address by Professor A. J. Kempner on "The rôle of isomorphism in scientific systems." Following this address, the group attended the John Wesley Powell Memorial Lecture of the Southwestern Division of the American Association for the Advancement of Science, which was delivered by Provost E. R. Hedrick, of the University of California at Los Angeles, on "The relations of science to economics and to war." Tea was served on Tuesday afternoon by the New Mexico Alpha Chapter of Kappa Mu Epsilon.

At the business meeting the following officers were elected for next year: Chairman, R. S. Underwood, Texas Technological College; Vice-Chairman, L. E. Mehlenbacher, Arizona State Teachers College, Flagstaff; Secretary-Treasurer, H. D. Larsen, University of New Mexico.

Professor J. F. Branson presided as chairman of the "Symposium on teaching problems in mathematics"; Professor R. F. Graesser, chairman, presided at all other sessions.

The following papers were read, the first seven of which constituted the "Symposium."

1. "The function concept in elementary mathematical instruction and in advanced mathematics" by Provost E. R. Hedrick, University of California at Los Angeles.

2. "Some observations on the teaching of the mathematics of finance" by Professor H. D. Larsen, University of New Mexico.

3. "Report from the Joint Commission on the Place of Mathematics in Secondary Education" by Professor C. A. Hutchinson, University of Colorado.

4. "Teaching students to think on paper" by Professor H. B. Leonard, University of Arizona. (Read by Professor A. B. Mewborn.)

5. "Suggestions for reducing mortality in freshman mathematics" by Professor W. P. Heinzman, New Mexico State College. (Read by title.)

6. "Teaching aids for instructors in mathematics" by President H. C. Gossard, New Mexico Normal University, by invitation.

7. "A one-year college mathematics course" by Professor Bulah A. Liles, Texas College of Mines and Metallurgy, introduced by the Secretary.

8. "The efficiency of approximation formulas for determining the rate of interest in amortization schedules" by Wade Ellis, University of New Mexico, introduced by Professor H. D. Larsen.

9. "$N$-tic residues" by Professor R. S. Underwood, Texas Technological College.

10. "An arrangement of a given triangle and certain of its consecutive cevian triangles in a sequence which forms a convergent geometric progression" by Professor C. A. Barnhart, University of New Mexico.

11. "Invariant differential equations" by Dr. Gordon Fuller, New Mexico State College.

12. "On the behavior of certain entire functions in distant portions of the plane" by Professor C. V. Newsom, University of New Mexico.

13. "A null basis for simple closed curves" by Professor Roy MacKay, Eastern New Mexico Junior College.

14. "The perfect group of order 120, with illustrative model" by Professor J. B. Shaw, University of Illinois. (Read by title.)

15. "Some properties of the numbers 3, 5, 11, 17, and 41" by C. B. Barker, University of New Mexico.

16. "On the roots of algebraic equations with complex coefficients" by Professor A. J. Kempner, University of Colorado.

17. "Moments about the arithmetic mean of a hypergeometric frequency distribution" by Professor H. D. Larsen, University of New Mexico.

18. "A multiple correspondence in space" by Professor E. J. Purcell, University of Arizona. (Read by title.)

Abstracts of some of these papers follow, the numbers corresponding to the numbers in the list of titles:

1. This paper appeared in full in the August-September 1938 number of this MONTHLY.

8. The exact determination of an unknown rate of interest involved in an amortization plan requires the solution of an equation of degree $n+1$. This presents serious practical difficulties, and recourse is therefore made to various approximation formulas. Mr. Ellis discussed the relative efficiency of the more important of these formulas.

9. Professor Underwood generalized, under specified conditions, well known results concerning quadratic residues to apply to $n$-tic residues.

10. Professor Barnhart showed that, by keeping the corresponding cevian fixed, a sequence formed by a triangle and its consecutive cevian triangles, each in counter-clockwise order, when the cevian in question is either a median or an altitude, arranges itself into two alternate infinite sequences of homothetic triangles, and that the areas of all the triangles in the original sequence form a convergent geometric progression. The question whether this arrangement is possible for the other cevians merits further investigation.

11. The system of equations considered by Dr. Fuller is invariant, but their forms do not show their independence of coördinate systems. The author presented the equations in forms whose law of transformation is evident. Members of the new equations, obtained from the original system, were expressed as the components of a tensor.

12. Professor Newsom presented a revision of a previous theorem (C. V. Newsom, *Bulletin of the American Mathematical Society*, vol. 37, page 666, abstract 282). In the present paper, the author placed more liberal restrictions upon the general coefficient of the given series; namely, when the coefficient $g(n)$ is regarded as a function $g(w)$ of the complex variable $w=x+iy$, it must satisfy the following two conditions: (a) it must be single-valued and analytic throughout the finite $w$-plane, and (b) it must be such that for all values of $x$ and $y$ one may write $|g(w)| < Ke^{k\pi|v|}$, where $K$ is a constant independent of $x$ and $y$, and $k$ is any given positive integer.

13. Professor MacKay showed that every simple closed curve of a peanian (continuous image of a line segment) is homologous to a linear combination of a subset of a particular sequence of simple closed curves of the continuum whose diameters form a null sequence. An application of this property is used to find a necessary and sufficient condition that certain classes of cyclic peanian continua have topological images in the plane.

14. Professor Shaw exhibited a model representing the group of order 120 based on Cayley's color representation. In this way the formation of products of the elements is simple. The order of every element is shown by the model.

15. In an expository paper, Mr. Barker discussed particular properties of the five prime numbers 3, 5, 11, 17, and 41 which apparently are possessed by no other numbers.

16. Professor Kempner extended the principle of symmetry of the roots of equations with real coefficients to equations with complex coefficients. As an incidental result, it was found that fifth degree equations with complex coefficients which are not solvable by radicals must either have their roots distributed symmetrically to the axis of reals, or they must have no real roots and no pairs of conjugate complex roots.

17. In a recent paper, Kirkman developed a method of continuation for obtaining the moments about the arithmetic mean of a binomial distribution. Professor Larsen extended the method to obtain the moments about the arithmetic mean of a hypergeometric distribution.

C. A. BARNHART, *Acting Secretary*

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## THE ANNUAL MEETING OF THE KENTUCKY SECTION

The twenty-first annual meeting of the Kentucky Section of the Mathematical Association of America was held at Morehead State Teachers College on Saturday, May 14, 1938, in conjunction with the annual meeting of the Kentucky Academy of Science. Professor D. E. South, chairman of the Section,

presided. In the absence of Professor Fehn, the chairman appointed Professor W. R. Hutcherson as temporary secretary.

There were forty-seven in attendance, including the following fifteen members of the Association: P. P. Boyd, M. C. Brown, H. H. Downing, L. A. Fair, Georgia M. Haswell, Charles Hatfield, W. R. Hutcherson, Fritz John, F. Elizabeth LeStourgeon, W. L. Moore, Sallie E. Pence, D. W. Pugsley, W. F. Smith, D. E. South, Guy Stevenson.

The following papers were presented:

1. "On representation of binary quadratic forms" by Professor N. B. Allison, Kentucky Wesleyan College, introduced by the Secretary.

2. "Maximum principle for elliptic differential equations" by Professor Fritz John, University of Kentucky.

3. "Calculus in biology; history in surveying—two illustrations" by Professor W. R. Hutcherson, Berea College.

4. "Configuration of double points of cubics of a pencil" by Dr. Sallie E. Pence, University of Kentucky.

5. "A rôle for mathematics in the sciences" by Professor W. L. Moore, University of Louisville.

6. "Summation of divergent series" by Mrs. A. S. Howard, University of Kentucky, introduced by Professor Cohen.

7. "The place of astronomy in the training of high school teachers" by Professor W. F. Smith, New River State College, West Virginia.

Abstracts of some of the papers follow, numbered in accordance with their place on the program:

2. Dr. John showed how the maximum principle is proved for potential functions and can be extended to the solutions of more general linear elliptic differential equations of the second order.

3. Professor Hutcherson stated in his first illustration that, in finding the consumption of sugar per protozoan in a medium where reproduction was obeying the law of logarithmic growth, the integral calculus was used. In his second illustration an old deed of a forty-acre farm, which has been a part of Berea College campus for nearly sixty years, was studied. Its corners were unknown, since the college owned the surrounding land. Mathematics relied upon the memory of oldest residents of the neighborhood concerning possible corners. In cross word puzzle fashion the plat was finally oriented.

4. Dr. Pence presented a paper on the configuration of the double points of the twelve nodal cubics of a pencil of cubics on nine associated points. After stating some properties for the general case, she studied the $D_{12}$ configuration by means of the Geiser and Bertini transformations. She discussed certain special configurations. Among these it was shown that in the pencil of equianharmonic cubics there are only three distinct cubics having double points, each cubic being composed of three concurrent lines. It was also shown that the nodal cubics in the pencil of cubics invariant in the quadratic involution consist of six degenerate cubics, each consisting of an invariant line and a conic which cuts

the line in two double points. There are thus twelve double points which lie by sixes on four conics.

5. Professor Moore gave a brief account of a portion of the book *An Introduction to the Philosophy of Science* by A. C. Benjamin. This portion dealt with the use of symbols in scientific explanation. The speaker confined himself to the use of mathematical symbols as a medium for explanation.

6. The sum of an infinite series $\sum_0^\infty a_n$ can be defined as the limit of a sequence, $s_n = a_0 + \cdots + a_n$. To sum series for which the $s_n$ oscillate, we seek another sequence $\sigma_n$ which shall (a) have the same limit as $s_n$ in those cases where $\lim s_n$ exists, (b) exist in cases where $\lim s_n$ does not exist. Mrs. Howard used the method of weighted means to determine $\sigma_n$ and discussed its properties. From them she showed that at least the limit of the means of the partial sums of the Cauchy product $\sum_{n=0}^\infty (\sum_{i=0}^n a_i b_{n-i})$ of two convergent series exists and is equal to the product of the two series.

7. Professor Smith compared the teacher training requirements for certification of science teachers with the contents of ten different textbooks recently published. He found that not more than one-half of the content matter of these texts is covered by certificate requirements in Kentucky and West Virginia. In all the texts considered, with one exception, at least one whole chapter was devoted to the subject of astronomy with other chapters given over to allied subjects, but in neither state is astronomy included in any part of the teacher training program.

W. R. HUTCHERSON, *Secretary pro tem*

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## THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-second annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, Colorado, April 15 and 16, 1938. There were three sessions. Professor C. A. Hutchinson presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Educational Association.

The attendance was seventy-one, including the following twenty-four members of the Association: L. A. Aroian, C. F. Barr, J. R. Britton, I. M. DeLong, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, G. H. Light, S. L. Macdonald, A. E. Mallory, A. S. McMaster, W. K. Nelson, Greta Neubauer, O. H. Rechard, A. W. Recht, C. H. Sisam, H. C. Wiedeman.

At the business meeting the following officers were elected for next year: Chairman, C. F. Barr, University of Wyoming; Vice-Chairman, D. F. Gunder, Colorado State College.

Following a luncheon at the High School on Saturday, Professor A. J. Kempner, president of the Association, gave a very interesting address on "The situation in collegiate and secondary mathematics."

The following papers were presented:

1. "The type B Gram-Charlier series" by Professor L. A. Aroian, Colorado State College.

2. "Mathematics and science—after Keyser" by Professor S. L. Macdonald, Colorado State College.

3. "Teaching large sections in freshman mathematics" by Professor O. H. Rechard, University of Wyoming.

4. "Roots of algebraic equations with complex coefficients" by Professor A. J. Kempner, University of Colorado.

5. "Slide rules for the solution of two problems in spherical trigonometry" by Professor I. L. Hebel, Colorado School of Mines.

6. "Lagrangean multipliers" by Professor C. A. Hutchinson, University of Colorado.

7. "Cooperation between high school and college teachers of mathematics" by Professor A. E. Mallory, Colorado State College of Education, and F. A. St John, South High School, Denver.

8. "The contributions of high school geometry to the goals of education" by Professor A. J. Lewis, University of Denver.

9. "Preparation of teachers of mathematics with reference to the requirements of the North Central Association" by Professor A. E. Mallory, Colorado State College of Education.

10. "Approximate numbers" by Dr. J. R. Britton, University of Colorado.

11. "Report on the work of the Joint Commission" by Professor C. A. Hutchinson, University of Colorado.

12. "The use of placement examinations in pre-college mathematics" by Professor W. J. Hazard, University of Colorado, introduced by the Secretary.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles.

1. Professor Aroian showed how seven terms of the type B Gram-Charlier series may be used in fitting a frequency distribution. (The paper has appeared in the December 1937 issue of the *Annals of Mathematical Statistics*.)

2. Professor Macdonald discussed the boundary lines of science and mathematics—the distinction between the two and what each is and what each is not. The ideas were chiefly derived from two books by C. J. Keyser, namely: *Pastures of Wonder* and *Humanism and Science*.

3. Professor Rechard taught a college algebra class of 66 students and one in trigonometry containing 78 students in the fall and winter quarters respectively of the current year, employing a classroom procedure differing from the usual recitation method. Normal sections of each subject during each quarter, were taught by other members of the department. On the basis of the Ohio College Ability Test both types of sections were found to be comparable. Final examination grades showed some advantage for the large section in college algebra, but no significant differences either way were found for trigonometry.

5. Professor Hebel designed slide rules for the calculation of the time of sunrise (or sunset) at a given place on a given date, and the determination of

the distance between two points on the surface of the earth. The slide rule for the sunrise problem is a single-setting rule from which the time is read directly from the given latitude and the sun's declination, a table of declinations being given on the reverse side. The second slide rule is a two-setting rule based on the cosine law, from which the distance in statute miles is found from the latitudes of the two places and the difference in their longitudes; for greater accuracy distances have been limited to about 6,000 miles.

6. Professor Hutchinson gave a brief exposition of the subject of constrained maxima and minima, with a discussion of the use of multipliers, and applications in the field of adjustment of observations.

8. In this paper Professor Lewis attempted to show that high school geometry is particularly well fitted to contribute to some of the generally recognized goals of education.

9. The standards set forth by the North Central Association for mathematics teachers are minimum. Adequate preparation defined in terms of present conditions and curriculum trends of the high school imply a general and specialized preparation. The most serious result of these trends is the tendency to minimize the importance of long experience. Mathematics has been the victim of this criticism because immediate and final ends have been confused. Professor Mallory believes that the formal-training pattern of the mathematics teacher should include a broad general education, training in related subjects, professional training in general and special courses, and student teaching.

10. Dr. Britton gave a brief exposition of the rules of computation with approximate data.

11. A report was made by Professor Hutchinson of the progress of the work of the Joint Commission on the Place of Mathematics in Secondary Schools, and of the Commission's plans for the completion of their work.

12. Professor Hazard showed a curve of the scores made by 665 freshmen on a placement examination in pre-college mathematics. This curve was compared with the curve of standardized results from 8200 students and showed very much lower scores, varying from 50% to 80% of the standard. A list of suggested questions for such an examination was offered for criticism.

A. J. LEWIS, *Secretary*

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## THE SIXTH ANNUAL MEETING OF THE WISCONSIN SECTION

The sixth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at the Columbus Community Club of St. Norbert College at Green Bay, Wisconsin, on May 14, 1938. The chairman of the Section, Professor Ethelwynn R. Beckwith of Milwaukee-Downer College, presided. The attendance was forty-five including the following twenty members of the Association: R. H. Bardell, Leon Battig, Ethelwynn R. Beckwith, May M. Beenken, W. W. Bigelow, L. A. V. DeCleene, Henry Ericson, R. C. Huffer, G. J. Kalcik, Elizabeth E. Knight, Morris Marden, Sister Mary Felice,

E. A. Nordhaus, R. E. Norris, G. A. Parkinson, H. P. Pettit, Irene Price, W. E. Roth, P. L. Trump, J. I. Vass.

Sessions were held in the morning and afternoon with luncheon in the Community Club at 12:30. At the close of the luncheon the Very Rev. A. M. Keefe of St. Norbert College graciously welcomed the visiting mathematicians and their guests. The chairman, Mrs. Beckwith, responded. There was also a short talk on the activities of the Intercollegiate Mathematical Association of Milwaukee by Miss Norma Fedders of Milwaukee-Downer College introduced by Mrs. Beckwith.

The business meeting was held at 1:30 p.m. at which the following officers were elected for the coming year: Chairman: R. C. Huffer, Beloit College; Secretary: G. A. Parkinson, University of Wisconsin Extension Division; Program Committee: W. E. Roth, University of Wisconsin Extension Division, and P. L. Trump, Wisconsin High School, Madison. An invitation to meet next year at Milwaukee State Teachers College was accepted by unanimous vote. Appreciation for the hospitality extended to the Section by St. Norbert College was expressed by a rising vote. After the afternoon session, tea was served. This was followed by a short visit to points of historic interest in and around Green Bay and West De Pere.

After the business meeting the afternoon session was devoted to a round-table discussion of the topic, "The survey course in college mathematics." The discussion was introduced by E. A. Nordhaus of the University Extension Division in Milwaukee. W. W. Bigelow of Beloit College, Sister Mary Felice of Mount Mary College, and Professor H. P. Pettit of Marquette University participated.

At the morning meeting the following papers were presented:

1. "On certain pythagorean numbers" by Leon Battig, University of Wisconsin Extension Division.

2. "The estimation of the total fish population of a lake" by Zoe E. Schnabel, University of Wisconsin, introduced by the Secretary.

3. "The number of circles covering a given point set" by R. B. Kershner, University of Wisconsin, introduced by the Secretary.

4. "Applications of mathematics to nuclear physics" by Professor Gregory Breit, Department of Physics, University of Wisconsin, by invitation of the Program Committee.

Abstracts of these papers follow, numbered in accordance with their listing above.

1. Series and recursion formulas for sets of pythagorean integers of which the smaller two differ by one can be found in the May 1914 issue of this MONTHLY in an article by George A. Osborne. In the present paper Mr. Battig presented a proof, using only algebraic means, that these series furnish all such integers, exhibited a related series in which the hypotenuse integer was followed by the perimeter of that triangle, and obtained a theorem on sums of squares of two successive terms of this series. Such sums are terms of the same series.

2. Miss Schnabel's paper was published in full in the MONTHLY for June–July 1938.

3. Mr. Kershner consider the following problem: Given a plane point set, how many circles of a given "small" radius $\epsilon$ are required to cover it? By geometrical methods, involving the use of the Euler relation, he obtained the following result: Let $A$ denote the measure of the closure of the given set. Let $N(\epsilon)$ be the minimum number of circles of radius $\epsilon$ which will cover the set. Then $\epsilon^2 N(\epsilon) \to 2\sqrt{3}A/9$ as $\epsilon \to 0$.

4. Professor Breit discussed the mathematical methods which have proved of special value in the study of atomic nuclei. The non-Coulombian forces between two protons were compared with proton-neutron and neutron-neutron forces. Sufficient conditions for energy saturation were discussed; restrictions and conclusions were brought out. Comparison of calculations with the experiments indicates that the observed fine structure is greater than expected on the simplest form of theory. The possibility of still finer structure of nuclear levels was considered.

G. A. PARKINSON, *Secretary*

## THE DIVISION OF ANGLES INTO EQUAL PARTS AND POLYGON CONSTRUCTION

W. B. GIVENS, Fresno State College

Angle trisection has long been of great interest due, primarily, to the restrictions placed on its solution. However, the division of angles into any number of equal parts is of greater importance because of the wider use which may be made of this knowledge. In an article published last year I developed a method for trisecting angles by the use of the strophoid as an auxiliary curve [1]. While investigating this problem, I discovered another method for solving it by the use of an entirely different curve; and the further fact that by its use, angles can also be divided into any number of equal parts.

In the time of the ancient Greeks, Hippias invented a device, known as the quadratrix, by means of which angles can be divided into equal parts. Gino Loria devotes a chapter in one of his books to a consideration of various sectrix curves which may be used for the division of angles into three or more equal parts [2].

This paper is given over, first, to a discussion of the general equation defining an auxiliary curve used for the division of angles; second, to the problem of constructing auxiliary triangles which are used in laying out the curve with compasses and ruler; and third, to the division of angles into equal parts and polygon construction.

In Figure 1, lay off a unit distance $OC$ on the polar axis $OX$. Draw a line $OA$ forming angle $\theta$ with $OC$. Then the problem is to locate a point $D$ on line $OA$ so that angle $ODC = \theta/n$, $n$ being a positive integer. Let $OD$ be the radius vector, $r$. Since angle $OCD = 180° - (n+1)\theta/n$ we see, by the law of sines, that the polar equation of the locus of $D$ is