7. An expansion of x^m may be secured by Newton's advancing difference formula. The finite integral of this expression may be used to sum the mth powers of the first n natural numbers.

$$\sum_1^n x^m = \Delta^{-1}x^m \Big]_1^{n+1} = \Delta 0^m(n+1)_2 + \Delta^2 0^m(n+1)_3 + \Delta^3 0^m(n+1)_4 + \cdots$$

The terms of this expansion are symmetrical and the coefficients are easily determined. This expression is somewhat easier to use than the expansions usually found.

8. The principal object of this paper was a classification of triads of points by means of invariants of the cubics whose roots are co-ordinates of the points in the Argand diagram. The cases treated included those of coincident and collinear points, and points that are the vertices of equilateral, isosceles, and right triangles. A brief discussion was also given of certain parametric equations of central conics.

9. In an address illustrated by lantern slides, Professor Davis showed how the periodic structure of economic series (for example, stock market averages, pig iron production indices, price averages, etc.) can be studied by means of periodogram analysis, difference equations, and the harmonic analysis of lag-correlations. It was shown that high correlations between such series as the stock market average and the pig iron production index are due in large part to coincident periods in the two series. The analysis of lag-correlations reveal these coincident periods, while the analysis of systems of difference equations reveal the non-coincident periods as elastic interactions. In discussing the dangers of too confident prediction in economic matters, Professor Davis showed that a random series smoothed by a twelve-months' moving average had a period almost exactly equal to the fundamental period of the Bradstreet commodity price index. He also touched on the significance of straightline and logistic trend lines and showed how the present decline in the price index was a phenomenon probably predictable on the basis of long-time trend lines. The computations involved in the paper were furnished by the laboratory of the Cowles Commission for Research in Economics.

RUFUS CRANE, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The seventeenth annual meeting of the Rocky Mountain Section was held at Colorado Agricultural College, Fort Collins, Colorado, on Friday and Saturday, April 14–15, 1933. Professor A. G. Clark presided at each of the three sessions.

The attendance was thirty-five, including the following twenty-one members of the Association: C. F. Barr, Jack Britton, A. G. Clark, I. M. DeLong,

J. C. Fitterer, G. W. Gorrell, I. L. Hebel, C. A. Hutchinson, Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, Mary S. Sabin, C. H. Sisam.

Members and friends of the Association were guests of the College on the evening of April 14. At the business session, Professor C. H. Sisam of Colorado College, was elected Chairman for the coming year, and Professor J. C. Fitterer, of Colorado School of Mines, was elected Vice-Chairman.

The following ten papers were read:

1. "A solution of a system of linear matrix equations in two unknowns," by Miss Rachel Achenbach, University of Wyoming, by invitation.

2. "On foci of algebraic curves with applications to cubic curves" (Thesis presented by Ethel A. Rice for M. A. University of Colorado) by Professor Claribel Kendall, University of Colorado.

3. "Entropy, strain and the Pauli exclusion principle" by Professor Guy Berry, Colorado Agricultural College, by invitation.

4. "Notes on Riccati's differential equations" by E. D. Rainville, University of Colorado.

5. "A theorem on point-wise discontinuous functions" by Professor O. H. Rechard. University of Wyoming.

6. "On Graeffe's method of solution of algebraic equations" by Professor C. A. Hutchinson, University of Colorado.

7. "The use of calculators in solving Kepler's problem" by Professor A. W. Recht, University of Denver.

8. "The 'Zig' function of Wirth" by Professor C. F. Barr, University of Wyoming.

9. "The solution of algebraic equations by infinite series" by Professor A. J. Lewis, University of Denver.

10. "Symmetric functions and resultants" by Professor C. H. Sisam, Colorado College.

Abstracts of the papers follow below, the numbers corresponding to the numbers in the list of titles:

1. The purpose of this paper is to develop methods of solution for the sixteen systems of linear matrix equations in two unknowns which result from the corresponding ordinary algebraic system when the constants and unknowns in these latter equations are replaced by matrices of the nth order. Two of the systems yielded to solution by the methods of elimination by addition and by substitution. The other fourteen systems were reduced to a single standard form which was solved by the use of the Hamilton-Cayley equation.

2. In this paper, Miss Rice was particularly interested in the graphical representation of the location of the real foci of certain cubic curves. For practical purposes in the matter of computation the cubic curves chosen were sym-

metric with respect to a point or an axis. The foci of some twenty-four such cubics of varying types were given. The discussion of these special cubics was preceded by a summary of the known results concerning the number of foci of algebraic curves in general.

3. The author shows, by using the geometrical weight method developed by Kimball for an ideal gas, that the entropy of a real gas is proportional to the strain, and that the equations for maximum entropy are equilibrium equations between stress and strain. The velocity distribution function is the same as that of the Fermi-Dirac statistics. This method offers an explanation of the second law of thermodynamics and the Pauli exclusion principle.

4. In the general Riccati Equation $dy/dx = A_0(x) + A_1(x)y + y^2$, it is most desirable to obtain a simple particular solution. If this is available, the complete solution follows almost at once. When A_0 and A_1 are polynomials, it is reasonable to search for polynomial solutions. In these notes Mr. Rainville shows that never more than one or two polynomials, for which simple formulas are given, need be tested as trial solutions. Some extensions varying the functional form of A_0 and A_1 are treated.

5. In this paper there is presented a statement and proof of the following theorem: Given a function, $f(x)$, continuous over a residual point set on an interval (a, b); if the function is defined at the remaining points of the interval by the "closest approximating function" method, it will be point-wise discontinuous on the interval.

6. This is the paper read by title at the Los Angeles Meeting of the Association. An abstract appeared in this MONTHLY, November 1932, p. 503; and the complete paper will appear in a later issue.

7. A demonstration showing how J. Peter's 7-place table of natural trigonometric functions, in which the argument is given in decimals of a degree, combined with the modern calculator, greatly facilitate the solution of Kepler's well-known equation $M = E - e \sin E$.

8. This paper presents the function

$$\operatorname{zig} u = 2\{[R_{u/2}] + (-1)^{[R_{u/2}]} R_{u/2}\}$$

and its derivative; in which $u = f(x)$, $[s]$ means the greatest integer in s, and R_u means $u - [u]$. Its adaptibility to special configurations was demonstrated by using it in the polar equation

$$\rho = a \cos\frac{\pi}{n} \Big/ \cos\left\{\frac{\pi}{n}\left(1 - \operatorname{zig}\frac{n\theta}{\pi}\right)\right\}$$

to map a regular n-sided polygon in a circle of radius a. The function zig u was developed and named by Mr. Don Wirth.

9. This paper outlines methods of expressing all the roots of an algebraic equation by infinite series and formulates conditions of convergence for these series.

10. This paper deals with a proof of the possibility of representing a given integral symmetric function in terms of the elementary symmetric functions and with a method of determining this representation by the use of resultants.

A. J. LEWIS, *Secretary*

THE TWENTY-SECOND MEETING OF THE IOWA SECTION

The twenty-second meeting of the Iowa Section of the Mathematical Association of America was held with the Iowa Academy of Science at Coe College, Cedar Rapids, Iowa, on Friday and Saturday, April 21 and 22, 1933. The meetings were held in room 117, Science Hall.

The attendance was about fifty, including the following twenty-one members of the Association: R. P. Baker, E. W. Chittenden, L. M. Coffin, N. B. Conkwright, C. W. Emmons, Cornelius Gouwens, M. E. Graber, I. J. Gwinn, Gertrude A. Herr, Dora E. Kearney, O. C. Kreider, F. M. McGaw, Arthur Ollivier, J. F. Reilly, H. L. Rietz, W. J. Rusk, E. R. Smith, C. W. Strom, John Theobald, L. E. Ward, Roscoe Woods.

The Section Chairman, Professor L. M. Coffin, presided at both the Friday afternoon and Saturday morning sessions. Dinner was enjoyed together Friday evening in the Jefferson Room, Hotel Roosevelt. The officers elected for 1933–1934 are as follows: Chairman, J. F. Reilly, University of Iowa; Vice-Chairman, M. E. Graber, Morningside College; Secretary-Treasurer, Cornelius Gouwens, Iowa State College.

A committee consisting of Professors R. P. Baker and Roscoe Woods prepared the following statement relative to the death of Daniel Kreth: "Daniel Kreth, engineer and surveyor, of Wellman, Iowa, a charter member of the Association, died in 1932. From 1914 to 1924 Mr. Kreth was an active contributor to the Monthly of problems and solutions. His interest in mathematics showed itself not only by his activities in the Association but also in the collection of a library. The Iowa Section laments his passing as a member and feels keenly the loss of inspiration which comes from knowing a man who derived a great deal of pleasure from his study of mathematics."

The program consisted of fourteen papers, as follows:

1. "On the resolution of $4X = Y^2 - (-1)^{(p-1)/2}\, pZ^2$ for $p = 67$, 71, and $X = (x^p - 1)/(x - 1)$" by Professor Cornelius Gouwens, Iowa State College.

2. "Some properties of the logarithmic potential" by Professor J. J. Westemeier, Des Moines Catholic College, by invitation.

3. "The teaching of the trigonometric functions of $2x$ and of $x/2$" by Professor Roscoe Woods, University of Iowa.

4. "Sophus Lie's geometry of imaginaries" by Professor M. E. Graber, Morningside College.

5. "A problem in simple harmonic motion of a particle moving in a medium of varying density" by Robert MacAllister, Wartburg College, by invitation.