It is with more than passing pleasure that I recall the fine courtesy, the generosity, the extreme modesty and the enthusiasm exhibited by Frère Gabriel Marie in occasional correspondence during the past decade.

R. C. ARCHIBALD.

BROWN UNIVERSITY,
March 23, 1917.

FIRST REGULAR MEETING OF THE IOWA SECTION.

The first regular meeting of the Iowa Section of The Mathematical Association of America was held at Grinnell College, Grinnell, Iowa, on April 28, 1917, and the following program given:

(1) "A unified course for Freshman mathematics:" by Professor R. B. MCCLENON, Grinnell College. Leader of the discussion: Professor JULIA COLPITTS, Iowa State College.

(2) "The foundation of Freshman mathematics in technical schools:" by Dean E. W. STANTON, Iowa State College. In his absence, the paper was read by Professor. MARIA ROBERTS, Iowa State College. Leaders of the discussion: Professors J. F. REILLY, Iowa State University, and C. W. EMMONS, Simpson College.

(3) "Putting life into dry bones:" by Professor F. M. MCGAW, Cornell College. Leaders of the discussion: Professors W. J. RUSK, Grinnell College, and W. E. BECK, Iowa State University.

The following also took part in the discussions: Professors Weston, Trowbridge, Stewart and Neff. All the papers were good and the discussions were to the point showing a keen interest in the sort of a program offered. The action at the business session in planning two meetings each year also indicates something of the interest taken in the Iowa Section. The attendance included some twenty members of the Association and others who will become members in due course.

The following officers were elected for the ensuing year: I. F. NEFF, Drake University, Chairman; R. B. MCCLENON, Grinnell College, Vice-Chairman; W. E. BECK, Iowa State University, Secretary.

G. A. CHANEY, *Chairman,* I. F. NEFF, *Secretary-Treasurer.*

THE ROCKY MOUNTAIN SECTION OF THE ASSOCIATION.

In September, 1916, it was suggested to Dr. G. H. Light, of the University of Colorado, that a section of The Mathematical Association of America be formed to include the states of Wyoming and Colorado. The suggestion was acted upon and as a result a meeting was called at the University of Colorado on April 7, 1917.

The meeting was a great success and the Rocky Mountain Section of the Association was formed with the following officers: C. B. Ridgaway, Professor of Mathematics, University of Wyoming, Chairman. C. C. VanNuys, Professor of Physics, Colorado School of Mines, Vice-Chairman. G. H. Light, Assistant Professor of Mathematics, University of Colorado, Secretary-Treasurer.

Papers were presented by O. C. Lester, Professor of Physics, University of Colorado, on "The Solid Angle," and Florian Cajori, Professor of Mathematics, Colorado College, on "Fluxions." Discussion of these papers was general.

There were twenty-one present at the meeting, fifteen of whom are already members of the Association and the others will join at once: C. B. RIDGAWAY, Professor of Mathematics, C. E. STROMQUIST, Professor of Mathematics, J. C. FITTERER, Professor of Civil Engineering, University of Wyoming; C. R. BURGER, Professor of Mathematics, G. E. F. SHERWOOD, Assistant Professor of Mathematics, C. C. VANNUYS, Professor of Physics, H. M. SHOWMAN, Assistant Professor of Civil Engineering, F. W. LUCHT, Assistant Professor of Mechanical Engineering, W. J. HAZARD, Assistant Professor of Mechanical Engineering, Colorado School of Mines; S. L. MACDONALD, Professor of Mathematics, Colorado A. & M. College; G. W. FINLEY, Professor of Mathematics, Colorado State Teacher's College; FLORIAN CAJORI, Professor of Mathematics, Colorado College; W. H. HILL, Greeley High School; E. L. BROWN, East Denver High School; I. M. DELONG, Professor of Mathematics, G. H. LIGHT, Assistant Professor of Mathematics, CLARIBEL KENDALL, Instructor in Mathematics, O. C. LESTER, Professor of Physics, J. W. WOODROW, Assistant Professor of Physics, DR. O. A. RANDOLPH, Instructor in Physics, C. E. SPERRY, Assistant Professor of Mathematics, University of Colorado.

G. H. LIGHT,
Secretary-Treasurer.

THE KENTUCKY SECTION OF THE ASSOCIATION.

The Mathematics Section of the Association of Kentucky Colleges and Universities (now the Kentucky Section of the Mathematical Association of America) was organized in April, 1909, and since then has met regularly twice a year.

This organization has directed most of its attention to a consideration of problems peculiar to collegiate work, one result of which has been a tendency toward a standardization of the mathematical courses in the colleges of the state.

Another feature of the work of this organization has been the consistent efforts put forth to strengthen and improve the teaching of mathematics in the high schools of the state. It was at first planned to work out a correlated course in mathematics for the high schools but this was later abandoned. In 1910 it was decided to test the degree of preparation of all students entering the colleges of the state by setting examinations covering algebra and plane and solid geometry.

elements. Hence the midpoint M_k of the inner segment I_iI_j corresponds to the ideal point of the line I_iI_j. From this it follows that *the ideal line of the plane transforms into a conic through each of the midpoints of the six segments* I_iI_j in addition to passing through A, B and C, as noted above.

We may now show that *this conic is a circle.* If P is an ideal point the lines a and b joining it to two of the mirrors are parallel. Then the sum of the angles α, β and γ is four right angles. Now as P moves on the ideal line a change in

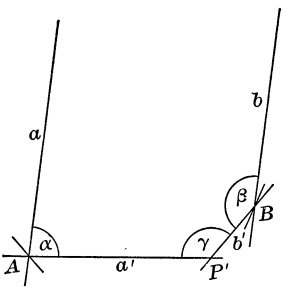

FIG. 3.

the angle α is accompanied by a change in the angle β of the same magnitude but opposite in sign. Then $\alpha + \beta$ is constant and hence $\gamma = 360° - (\alpha + \beta)$ is constant and P' moves on a circle. It follows, then, that *the conic obtained by transforming the ideal line of the plane is the nine-point circle of each of the four triangles* $I_iI_jI_k$.

THE ROCKY MOUNTAIN SECTION.

The second annual meeting of the Rocky Mountain section of the Mathematical Association of America was held at Laramie, Wyoming, under the auspices of the University of Wyoming, March 29 and 30, 1918.

The meeting opened with a dinner in Hoit Hall at 6 P. M., at which the address of welcome by Acting President Nelson of the University of Wyoming and the response by Professor O. C. Lester, of the University of Colorado, were given. After the dinner, an adjournment to the administration building was made and the following program was carried out:

1. Special Courses in Mathematics for Technical Students. PROFESSOR S. L. MACDONALD, Colorado A. & M. College, Ft. Collins.
2. The Theory of the Mercury Arc. PROFESSOR J. W. WOODROW, University of Colorado, Boulder.
3. The Length Integral in the non-Euclidean World of Poincaré. PROFESSOR C. E. STROMQUIST, University of Wyoming, Laramie.

The purpose of this paper was to derive an integral for length in the non-Euclidean world proposed by Poincaré in his "Foundations of Science," English translation by Halsted, page 75. The shortest distance between two points is

assumed to be the circle through these points and perpendicular to the boundary sphere. The general form of the length integral under this assumption is then worked out for the case of the plane. Under the further restriction that transversals are perpendicular to their extremals, the length integral along a curve $y = f(x)$ between two points, $P(x_1, y_1)$ and $P(x_2, y_2)$, reduces to the form

$$\int \frac{\sqrt{1 + p^2}}{x^2 + y^2 - R^2}\, dx,$$

where $p = dy/dx$ and R is the radius of the boundary sphere.

4. The Trend Curve for the Price of Copper. PROFESSOR C. S. SPERRY, University of Colorado, Boulder.

5. A Problem in Geometry. MR. J. Q. MCNATT, Colorado Fuel & Iron Co., Florence.

The author gave a new proof for the relation between the side of a regular inscribed pentagon and the side of a regular inscribed decagon.

6. Some Systems of Coördinates. PROFESSOR G. H. LIGHT, University of Colorado, Boulder.

This paper dealt principally with intrinsic coördinates and showed the extremely simple form that the equations of some well-known curves and their evolutes assume when expressed in terms of these coördinates.

7. Mathematics at the Front. MR. W. H. HILL, Greeley High School, Greeley.

8. Some Functions of Solid Angles. PROFESSOR J. C. FITTERER, University of Wyoming, Laramie.

9. The Origin of the name "Rolle's Curve." The Origin of the name "Mathematical Induction." PROFESSOR FLORIAN CAJORI, Colorado College, Colorado Springs.

The second of these papers by Professor Cajori appeared in the May number of this MONTHLY; the first will appear in a later issue.

10. The Sine and Cosine Integrals $\int \sin x/x\, dx$ and $\int \cos x/x\, dx$ in Electromagnetism. PROFESSOR C. C. VANNUYS, Colorado School of Mines, Golden.

This paper deals with interesting physical applications of the functions known to mathematicians as the sine and cosine integrals. One of the problems dealt with is that of determining the equivalent resistance and inductance due to radiation of electromagnetic waves of a long straight conductor carrying a harmonic alternating current of single frequency such as is employed in the oscillation circuits used in radio telegraphy.

Another problem discussed is that of the electromotive force induced in a straight vertical conductor by an oscillatory current in a parallel conductor at a great distance from it. In each case, the results are obtained in terms of these series. The paper closes with an analysis of the five integrals given below. γ in these series is Euler's constant.

$$Six = \int_0^x \sin x/x\, dx = x - x^3/3!3 + x^5/5!5 - \cdots,$$

$$Cix = \int_{\infty}^{x} \cos x/x\, dx = \gamma + \log x - x^2/2!2 + x^4/4!4 - \cdots,$$

$$Eix = \int_{\infty}^{x} e^{-x}/x\, dx = \gamma + \log x + x + x^2/2!2 + \cdots,$$

$$Shix = \int_{0}^{x} \sinh x/x\, dx = x + x^3/3!3 + x^5/5!5 + \cdots,$$

$$Chix = \int_{\infty}^{x} \cosh x/x\, dx = \gamma + \log x + x^2/2!2 + x^4/4!4 + \cdots.$$

On account of the length of the program and the interest shown in the papers it was found necessary to adjourn at 11 P. M. until 8:30 the next morning, when the program was completed and officers were elected for the ensuing year as follows:

CHAIRMAN, C. C. VANNUYS, Professor of Physics, Colorado School of Mines.

VICE-CHAIRMAN, S. L. MACDONALD, Professor of Mathematics, Colorado A. & M. College.

SECRETARY-TREASURER, G. H. LIGHT, Assistant Professor of Mathematics, University of Colorado.

Five visitors were present and the following fifteen members: C. R. Burger, Colorado School of Mines; I. M. DeLong, University of Colorado; J. C. Fitterer, University of Wyoming; W. H. Hill, Greeley High School; O. C. Lester, University of Colorado; G. H. Light, University of Colorado; S. L. Macdonald, Colorado A. & M. College; J. Q. McNatt, Colorado Fuel & Iron Co.; O. A. Randolph, University of Colorado; C. B. Ridgaway, University of Wyoming; H. M. Showman, Colorado School of Mines; C. S. Sperry, University of Colorado; C. E. Stromquist, University of Wyoming; G. P. Unseld, Westminster High School; C. C. VanNuys, Colorado School of Mines.

G. H. LIGHT, *Secretary.*

THIRD ANNUAL MEETING OF THE OHIO SECTION.

The third annual meeting of the Ohio Section of the Mathematical Association of America was held at the Ohio State University, Columbus, on March 29, 1918, in connection with the meetings of some sections of the Ohio College Association, and the Association of Ohio Teachers of Mathematics and Science. Chairman Forbes B. Wiley occupied the chair, being relieved by Professor R. B. Allen for an interval.

The following thirty persons were registered, all but the last eight being members of the Association:

R. B. Allen, Kenyon College; W. E. Anderson, Wittenberg College; G. N. Armstrong, Ohio Wesleyan University; C. L. Arnold, Ohio State University;

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION.

The Rocky Mountain Section of the Mathematical Association of America held its annual meeting at Ft. Collins, Colorado, April 12, 1919; the members were royally entertained by President Lory and the professors of Colorado Agricultural College. The papers were extremely interesting and were discussed very thoroughly. The twenty-four persons in attendance included the following thirteen members of the Association: I. M. DeLong, University of Colorado, J. C. Fitterer, University of Wyoming, W. H. Hill, High School, Greeley, Col., Claribel Kendall, University of Colorado, O. C. Lester, University of Colorado, G. H. Light, University of Colorado, S. L. Macdonald, Colorado Agricultural College, O. A. Randolph, University of Colorado, C. B. Ridgaway, University of Wyoming, C. H. Sisam, Colorado College, C. S. Sperry, University of Colorado, T. O. Walton, Colorado School of Mines, and J. W. Woodrow, University of Colorado.

The following papers were read at morning and afternoon sessions:

Generalization of the addition formulæ in trigonometry by Professor I. M. DeLong, University of Colorado; New experiments for the laboratory course in general physics, and Positions and properties of the cardinal points of a lens system from the standpoint of the wave theory, by Professor J. W. Woodrow, University of Colorado; Probable errors of Mendelian class frequencies by Mr. Breeze Boyack, Colorado A. and M. College; Calculation of high frequency resistance of wires by Professor O. C. Lester, University of Colorado; Development of empirical formulas for the solution of problems in hydraulics by Mr. L. R. Parshall, Colorado A. and M. College; Electromagnetic waves on wires by Professor O. A. Randolph, University of Colorado; On non-ruled optic surfaces whose plane sections are elliptical by Professor C. H. Sisam, Colorado College; Some characteristics of the mercury arc by Professor J. W. Woodrow, University of Colorado; An application of hyperbolic functions of a complex quantity to the determination of the performance of long distance alternating current transmission lines by Professor L. S. Foetz, Colorado A. and M. College.

An invitation from Colorado College to hold the next meeting there was received and accepted. The following were elected officers for the ensuing year:

Chairman: C. H. Sisam, Colorado College.

Vice-chairman: T. O. Walton, Colorado School of Mines.

Secretary-Treasurer: G. H. Light, University of Colorado.

G. H. Light, *Secretary-Treasurer.*

ELECTION OF MEMBERS.

The Council of the Association has elected to membership the following persons and institutions:

Lantern slides were used to illustrate the apparatus and methods. Incidents of historical interest were related and particular attention was called to a persistent occurrence at the last experiments of a small displacement of the fringes, far less than the theory calls for, which has never been satisfactorily explained.

4. This lecture was fully reported in *Science*, March 26, 1920, pp. 301–311.

5. In the round table discussion, Mr. Beatty summarized the replies to questionnaires mailed out to about fifty of the leading high schools of Ohio. These reveal a tendency to minimize the amount of mathematics required for graduation. One unit each of algebra and geometry is required. In most cases one half unit each of advanced algebra and soild gometry is offered as an elective, but in many cases is not elected by the pupil. There is a tendency for more pupils to enter college deficient in a half-unit or more of mathematics. There was a feeling expressed that the same care in selecting teachers of mathematics was not exercised nor the same respect accorded mathematics as was done in former years. The opinion seemed to prevail that there was no more reason for discouragement over results in mathematics than in other subjects.

G. N. ARMSTRONG, *Secretary-Treasurer.*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION.

The fourth regular meeting of the Rocky Mountain Section was held at Colorado College, Colorado Springs, Colorado, April 2, 3. There were two sessions, presided over by Professor C. H. Sisam.

The attendance was twenty-five, including the following fourteen members of the Association: I. M. DeLong, J. C. Fitterer, W. H. Hill, H. A. Howe, Claribel Kendall, G. H. Light, J. Q. McNatt, S. L. Macdonald, O. A. Randolph, H. E. Russell, C. H. Sisam, C. S. Sperry, C. E. Stromquist, J. W. Woodrow.

The officers appointed for the meeting to be held at Denver in 1921 are: Chairman, H. A. HOWE, Denver University; Vice-chairman, W. H. HILL, Greeley High School; Secretary-Treasurer, G. H. LIGHT, Univ. of Colorado.

The following eight papers were read:

(1) "Some physical correlations in a group of one hundred S. A. T. C. men" by Professor J. C. FITTERER;

(2) "Families of curves whose evolutes are similar curves" by Professor G. H. LIGHT;

(3) "Grades for different placings of ears of corn" by Professor W. V. LOVITT;

(4) "Ionization in the mercury arc" by Professor J. W. WOODROW;

(5) "Discussion of the cycloidal curve" by Mr. J. Q. MCNATT;

(6) "Projective differential geometry in a four space" by Professor W. V. LOVITT;

(7) "The teaching of logarithms and slide rule in the first year of high school" by Professor C. E. STROMQUIST;

(8) "On ruled surfaces whose asymptotic curves are cubics" by Professor C. H. SISAM.

1. Professor Fitterer presented correlation tables which were computed between stature and stride, stride and weight, weight and stature. The correlation coefficient in the first was 0.29, in the second very nearly zero, and in the third 0.55. The average age was 19.3 years, average weight was 136 pounds, average stature was 5′ 8″, average stride was 5.7 ft. A hypsobaric coefficient (weight in pounds per foot of stature) was also found, which averaged 24 pounds per foot.

2. Professor Light's paper appears elsewhere in this issue.

3. Numerical grades were given by Professor Lovitt for different placings of any number of ears. The results are determinate, though arbitrary. The results are in use and are giving satisfaction with competent corn judges.

4. It was assumed by Professor Woodrow that (*a*) An electron, on the average, loses all of its translatory energy at each impact; (*b*) The molecules are capable of storing up energy, *i.e.*, after the energy is received, it is radiated by electromagnetic waves at a rate which is proportional to the instantaneous energy, and additional increments of energy can be added by successive impacts of different electrons with the same molecule; (*c*) The molecule can also receive energy which has been radiated from the surrounding molecules and which is proportional to the fourth power of the temperature of the gas or vapor. From these assumptions, the following equation was obtained

$$X = \frac{H}{I} p^{1/3}(K - p^{2/3}).$$

Where X is the electric force, I is the current, p is the pressure of the gas, and H and K are constants.

5. Mr. McNatt gave methods of constructing the cycloid and its evolute.

6. Given the linear differential equation of order five

$$y^{(5)} + 5p_1y^{(4)} + 10p_2y^{(3)} + 10p_3y^{(2)} + 5p_4y^{(1)} + p_5y = 0,$$

Professor Lovitt found invariants and covariants for the transformation $y = \lambda(x)\bar{y}$, $\xi = \xi(x)$. Some geometric interpretations were given.

7. Professor Stromquist suggested the following course for the first year of high school: (*a*) Tabulation and graphing of functions; (*b*) Meaning of positive and negative exponents, applying the four rules; (*c*) Square root of arithmetical numbers; (*d*) Logarithms, based on exponents; (*e*) Slide rule. The course has been successful in the Laramie High School.

8. Professor Sisam classified completely, and discussed the properties of, the ruled surfaces whose asymptotic curves are gauche cubics.

G. H. LIGHT, *Secretary-Treasurer.*

THE MARCH MEETING OF THE ROCKY MOUNTAIN SECTION.

The fifth regular meeting of the Rocky Mountain Section was held at the University of Denver, Denver, Colorado, on March 25, 26. Sessions were held on Friday afternoon and Saturday morning. The presiding officer was Professor H. E. RUSSELL of the University of Denver.

The attendance was thirty-three, including the following nineteen members of the Association: E. L. Brown, I. M. DeLong, A. R. Fehn, G. W. Finley, J. C. Fitterer, W. H. Hill, C. A. Hutchinson, H. A. Howe, G. H. Light, F. H. Loud, J. Q. McNatt, S. L. Macdonald, D. H. Menzel, H. E. Russell, C. H. Sisam, C. S. Sperry, C. E. Stromquist, O. B. Trout, J. W. Woodrow.

Those in attendance were royally entertained at a six-thirty dinner and welcomed to the University by Chancellor W. D. Engle. The reply for the Association was made by S. L. Macdonald. At the business meeting which followed, W. H. Hill, of the Greeley High School, was chosen Chairman and G. W. Gorrell, of the Colorado School of Mines, Vice-Chairman for the meeting to be held at the State Teacher's College at Greeley next year. Dean Howe then invited the guests to inspect the observatory, which was greatly enjoyed by all.

The following nine papers were read:

(1) "The effect of polarized light on the photographic plate" by Dr. J. W. Woodrow, professor of physics, University of Colorado.

(2) "The effect of translation upon certain dispersion and correlation formulas" by Professor C. E. Stromquist, University of Wyoming.

(3) "Trajectories" by Mr. Philip Fitch, North Denver High School (by invitation).

(4) "Note on extraneous loci" by Professor G. H. Light, University of Colorado.

(5) "On correspondence between curves" by Professor C. H. Sisam, Colorado College.

(6) "Mathematics of the high school as preparation for college" by Mr. B. F. Kitchen, Colorado Agricultural College (by invitation).

(7) "Thermal properties of glass" by W. B. Pietenpol, associate professor of physics, University of Colorado.

(8) "The suspended chain" by J. Q. McNatt, engineer for the Colorado Fuel and Iron Co.

(9) "A proof of a theorem in the adjustment of observations by the use of determinants" by Professor C. S. Sperry, University of Colorado.

Abstracts of the papers follow below, the numbers corresponding to the numbers in the list of titles.

1. It has been suggested by H. S. Allen and others that the action of light on the photographic film is due to a photoelectric effect. If this is true, incident plane polarized light should have different effects in different directions. Professor Woodrow photographed dark lines through a good Nicol's prism and found

that the lines parallel to the plane of vibration were distinctly sharper than those at right angles to the plane.

2. In this paper Professor Stromquist derives certain statistical formulas, in particular those for the standard deviation and the coefficient of correlation, that result from a given correlation, or double entry, table by adding new individuals to the table, by translating individuals in the table, and by superimposing one correlation table upon another.

3. Mr. Fitch discussed the trajectories due to a flow of water from an orifice subject to constant angle and constant kinetic energy.

4. Professor Light gave the geometrical conditions that must be fulfilled when the extraneous loci are cusp-loci, tac-loci, and singular solutions.

5. Professor Sisam discussed some properties of algebraic correspondences between two given algebraic curves of which at least one is rational.

6. Mr. Kitchen brought out, among other good points, the fact that high school students do not know how to draw conclusions from definite statements.

7. Experimental work on the rate of thermal expansion of glass from room temperature to 750 degrees Centigrade has brought out the relation between this and other thermal properties. Professor Pietenpol showed how the expansion of glass is of particular importance in its relation to the annealing of glass, and that a determination of the rate of expansion at high temperatures may be used as an exact method of determining the suitable annealing temperature.

8. Mr. McNatt took up the derivation of the equation of the catenary, and some of the properties of the equation were applied to the solutions of problems arising in connection with cables used in mines.

9. Professor Sperry gave a proof of a well known theorem that the average value of the ratio of the weight of the observed value of an unknown to its adjusted value for a series of unknowns is equal to the number of unknowns divided by the number of observations. Instead of the usual proof by undetermined coefficients, certain transformations were effected by means of determinants. This proof is believed to be superior in directness and simplicity.

G. H. LIGHT, *Secretary-Treasurer.*

RATIONAL TRIANGLES AND QUADRILATERALS.

By L. E. DICKSON, University of Chicago.

1. **The questions treated.** The chief object of this paper is to make a material simplification in Kummer's classic investigation of rational quadrilaterals. Incidentally it is shown that every rational triangle may be formed by juxtaposing two rational right triangles, so that it suffices to know Diophantus's complete solution of $x^2 + y^2 = z^2$ in rational numbers. From the latter will be deduced all solutions in integers, a problem usually treated independently of the former problem of the rational solutions. For most equations the two problems are essentially distinct.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION.

The sixth regular meeting of the Rocky Mountain Section was held at the State Teachers College, Greeley, Colorado, on April 14–15. Sessions were held on Friday afternoon and Saturday morning. The presiding officer was Professor G. W. Finley of the State Teachers College.

There were thirty in attendance, including the following eleven members of the Association:

I. M. DeLong, B. F. Dostal, A. R. Fehn, G. W. Finley, Philip Fitch, J. C. Fitterer, H. A. Howe, Claribel Kendall, G. H. Light, J. Q. McNatt, H. E. Russell.

Those in attendance were royally entertained at a six-thirty dinner in the Club House by Professor and Mrs. Finley. Professor LIGHT was elected chairman and Professor FEHN, vice-chairman, for the meeting to be held at the University of Colorado next year. A committee, consisting of Professor DeLong and Mr. Fitch, was appointed to draw up suitable resolutions upon the death of Dr. G. B. Halsted. It was also voted to extend an invitation to the national Association to hold its next meeting at the University of Colorado in September of this year or as soon thereafter as possible.

The following papers were presented:

(1) "Certain congruences determined by a given surface" by Dr. CLARIBEL KENDALL;

(2) "Kepler's problem for high planetary eccentricities" by Dean H. A. HOWE;

(3) "On the parametric equations of straight lines of which certain polar curves are envelopes" by Mr. PHILIP FITCH;

(4) "Games in mathematics teaching" by Mrs. LAURA C. GRAVES (by invitation);

(5) "A problem in mensuration" by Mr. J. Q. MCNATT;

(6) "The application of hyperbolic functions to transmission line problems in engineering" by Mr. B. F. DOSTAL;

(7) "The content of a course in plane geometry" by Mr. FITCH;

(8) "Population curves" by Professor J. C. FITTERER.

Abstracts of papers follow below, the numbers corresponding to the numbers in the list of titles.

1. Miss Kendall gave formulas which she had developed for obtaining the curves on a surface which would give the developables of any congruence associated with the surface. A line of the congruence was given for every point on the surface. She also gave the formula for obtaining the focal points on any such line. These results were applied to several special congruences and some interesting relations among the lines determining these congruences were obtained. Several of these lines are lines of the osculating quadric at the point and are in harmonic relation to one another.

2. Dean Howe gave a method for determining to a very close approximation the position of asteroids, except in the cases where the eccentricity is very great.

3. Mr. Fitch demonstrated a short method for finding the equations of lines traced by reflected rays of light, applying the same to known caustic curves.

4. Mrs. Graves suggested that all elementary mathematics should be taught after the fashion of the old spelling bee.

5. The problem considered by Mr. McNatt was to find the radius of the sphere which displaces the maximum amount of water contained in a conical vessel.

6. Mr. Dostal reviewed recent developments in the applications of the hyperbolic functions to loaded and balanced telephone lines and cables, and to power transmission lines.

7. This paper dealt with the subject matter, fundamental concepts and elementary principles of plane geometry. Mr. Fitch pointed out the relation of these concepts to those of higher mathematics and their use in other subjects.

8. Professor Fitterer showed that the hyper-tan curve, $y = a \tanh bx + c$, closely graphs population data. Its use in municipal and state problems involving probable future growth constituted an important application.

G. H. LIGHT, *Secretary-Treasurer.*

ORGANIZATION MEETING OF THE SOUTHEASTERN SECTION.

On April 29, 1922, mathematicians of the Southeastern States met in the Main Building of Georgia School of Technology, Atlanta, Georgia. There were sixty-three present at the meeting, of which number the following fifteen are members of the Association:

D. F. Barrow, J. B. Coleman, T. R. Eagles, Floyd Field, Tomlinson Fort, Miss Leslie Gaylord, J. F. Messick, A. B. Morton, M. T. Peed, W. W. Rankin, Jr., H. A. Robinson, Douglas Rumble, W. V. Skiles, D. M. Smith, R. P. Stephens.

At the business meeting it was decided to present a petition to the Trustees of the Association asking permission to form a Southeastern Section of the Association, to include the following states: Alabama, Florida, Georgia, North Carolina, South Carolina, and Tennessee. After the program all present were entertained at lunch by Georgia School of Technology.

The officers elected are Professor FLOYD FIELD, Georgia School of Technology, Chairman; Professor R. P. STEPHENS, University of Georgia, Vice-Chairman; Professor W. W. RANKIN, JR., Agnes Scott College, Secretary-Treasurer. The Program Committee is composed of Professor W. W. Rankin, Jr., Chairman, Professor J. B. Coleman, University of South Carolina, and Professor Tomlinson Fort, University of Alabama.

The following program was carried out, abstracts being given with numbers to correspond to those of the program:

(1) "Some possibilities of the slide rule" by Professor D. M. SMITH;

(2) "Marking systems at the University of Georgia" by Professor D. F. BARROW;

(3) "Zero and infinity in elementary mathematics" by Professor J. F. MESSICK;

(4) "History of mathematics" (illustrated with slides) by Professor W. W. RANKIN, JR;

the Ohio colleges and universities. The main point of interest was that the number of hours required for a mathematical major ranges from fifteen to thirty-eight hours. So wide a range indicates that the colleges are not certain as to the basic values of many courses. The author urges that more interest should be taken in geometrical subjects, such as modern geometry and descriptive geometry.

G. N. ARMSTRONG, *Secretary-Treasurer.*

ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION.

The seventh annual meeting of the Rocky Mountain section was held at the University of Colorado, Boulder, Colorado, on March 30 and 31.

The attendance was twenty-four, including the following fourteen members of the Association: I. M. DeLong, G. W. Finley, P. Fitch, J. C. Fitterer, G. W. Gorrell, C. A. Hutchinson, Claribel Kendall, O. C. Lester, G. H. Light, F. H. Loud, S. L. Macdonald, J. Q. McNatt, W. J. Risley, H. E. Russell.

The section voted to accept the invitation of the Colorado Fuel and Iron Co. to hold the next meeting at the Steel Plant in Pueblo, Colorado. Mr. J. Q. MCNATT was elected chairman for this meeting.

At the close of the Friday session, the section was favored with a talk by Dr. SAUL EPSTEEN on certain phases of the Einstein Theory.

The following nine papers were read:

(1) "Certain associativity conditions in linear algebras" by Assistant Professor CLARIBEL KENDALL. (Asst. Professor G. W. SMITH, Collaborator.)

(2) "The curve of the price of lumber" by Professor S. L. MACDONALD.

(3) "The intrinsic equation for a family of curves possessing a certain property" by Professor G. H. LIGHT.

(4) "The bearing which the work in the grades has upon college mathematics" by Professor G. W. GORRELL.

(5) "Parabolic grouping of pythagorean triangles" by Professor W. J. HAZARD.

(6) "To calculate the radius of a circle inscribed in a concave triangle" by Mr. J. Q. MCNATT.

(7) "A nomographic perpetual calendar" by Mr. W. K. NELSON.

(8) "Graphical methods for complex roots" by Professor W. J. HAZARD.

(9) "A game of solitaire for an arithmetician" by Dr. F. H. LOUD.

In the absence of the author, a paper by Professor C. H. SISAM was read by title only. All the papers led to considerable discussion. Abstracts of the papers follow below, the numbers corresponding to the numbers in the list of titles:

(1) In this paper it was shown that the associativity conditions

$$A_{ijkm} \equiv \Sigma(\gamma_{ijk}\gamma_{kim} - \gamma_{ikm}\gamma_{jik}) = 0 \qquad (i, j, k, m = 1 \cdots n)$$

for a linear algebra of order n are highly redundant. For $n = 2$ eight independent

syzygies among A_{ijkm} have been found. Four of these are linear and four quadratic. In the general case there will be at least $n^4 - n^3$ such relations.

(2) Taking the price of all commodities for the year 1913 as a standard, lumber included, the ratio of the price of lumber to this standard was computed for each year, beginning with 1865. Plotting a curve from the results thus obtained gave as a "most probable" equation that of a straight line. It was found that a more accurate equation was that of a parabola. Upon further investigation it was found that a logarithmic curve was a trifle better.

(3) Professor Light derived the general equation of the curves whose polars with respect to a fixed circle cut off a segment on the normal proportional to the radius of curvature. When the circle becomes a point, the intrinsic equations for logarithmic spirals, cardioids, parabolas, circles, lines and hyperbolas are obtained.

(4) It is the belief of a number of college teachers of mathematics, judged by the expressions at association meetings, that the instruction in high schools is of inferior quality. The writer believed this to be largely unfounded. High school teachers are usually well prepared to do their work. The amount of required work in high school is not sufficient to give to the high school teacher the time necessary to make much impression upon his students.

Grade teachers are not specially prepared in mathematics and are unable to give clear explanations in many instances. They are likely to fall into the practice of mere formal work in arithmetic.

A test was given to college freshmen in which problems were taken from a grade text in arithmetic. Several students did not attempt certain problems and very few attempted any analysis, though explanation was requested. The writer was inclined to believe that the student's customary approach to a problem follows his practice in the grades rather than that required in high school.

(5) In his paper Professor Hazard showed that when such triangles are plotted in the first quadrant with one acute vertex at the origin and with the base and altitude of the triangle as the coördinates of the other acute vertex, then the triangles having a common property have their vertices on a parabola. Triangles with a common unit difference between the hypotenuse and one side lie on one parabola; those with a difference of 9 lie on another parabola, etc. All possible integral differences represent, also, integral sums of the hypotenuse and one side. There are four families of parabolas, confocal at the origin, being the loci of the vertices of triangles having a constant integral odd difference, odd sum, even difference, and even sum of the hypotenuse and one side. Possible odd differences and sums are given by the squares of the odd numbers. Even differences and sums are given by twice the squares of all the integers.

(6) A unique method was shown in this paper for computing the radius of a circle inscribed in the area enclosed by three mutually tangent circles, when the radii of these circles were known.

(7) This paper showed how some equations of three or more variables may be represented very simply by the methods of Nomography, while the cartesian

representation is hard to construct and hard to read. A method was given of constructing a chart for an equation which represents the day of the week as a function of the year, month and day of the month, thus producing a graphical perpetual calendar.

(8) Professor Hazard commented on the fact that most text books of algebra give graphical methods for the real roots of quadratic and other equations, but either state or imply that the graphical method fails in the case of complex roots because the plotted curve does not cross the axis of X. When the roots of a quadratic are represented by $(r \pm ni)$ the vertex of the graph is the point (r, n^2), so it is only necessary to find the square root of the ordinate to have the roots determined. This may be done by the geometrical method. Reference was made to the straight line and circle construction for the real roots of a quadratic, used by L. E. Dickson and attributed by him to D'Ocagne and Lill. It was shown how this construction may be extended to find the complex roots of quadratics. Reference was also made to a construction given by Schultze for complex roots.

(9) This paper dealt with problems in the theory of numbers and considered the examination of primes with a view to the relation as modulus sustained by each to smaller numbers, the exponents to which the latter belong, and similar inquiries bearing upon the properties of repetends in both the decimal and binary systems. The author's examination, beginning with the smallest primes, has reached the prime, 2161.

PHILIP FITCH, *Secretary.*

MARCH MEETING OF THE SOUTHEASTERN SECTION.

The second meeting of the Southeastern Section of the Mathematical Association of America was held at Agnes Scott College, Decatur, Georgia, March 10, 1923. The meeting was held in the college chapel with Professor FIELD presiding.

There were eighty-five present, including the following twenty-four members of the Association:

D. F. Barrow, S. M. Barton, J. B. Coleman, T. R. Eagles, Floyd Field, Tomlinson Fort, Miss Leslie Gaylord, J. P. Hill, A. W. Hobbs, J. W. Hinton, Miss Ruby Hightower, J. F. Messick, Miss Fannie S. Mitchell, R. E. Mitchell, A. B. Morton, I. C. Nichols, M. T. Peed, W. W. Rankin, Jr., D. Rumble, David Eugene Smith, D. M. Smith, R. P. Stephens, A. H. Stevens, Miss Rose Wood.

The following officers were elected: R. P. STEPHENS, chairman; TOMLINSON FORT, vice-chairman; W. W. RANKIN, Jr., secretary-treasurer. The executive committee decided to hold the next meeting at the University of Georgia, Athens, Ga.

Agnes Scott College entertained the Southeastern Section at lunch after the regular program had been completed.

Professor DAVID EUGENE SMITH of Columbia University was the principal speaker. Professor Smith was brought to Agnes Scott College by the College Lecture Association. He addressed the faculty and students and many visitors

ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION.

The eighth annual meeting of the Rocky Mountain section was held at the Steel Works Y. M. C. A., Pueblo, Colorado, on March 28 and 29. There were sixteen present, including the following eight members of the Association: I. M. DeLong, Philip Fitch, G. W. Gorrell, G. H. Light, S. L. Macdonald, J. Q. McNatt, H. E. Russell, C. H. Sisam. The section voted to hold the next meeting at the University of Wyoming. The following officers were elected: J. C. FITTERER, chairman; S. L. MACDONALD, vice-chairman; PHILIP FITCH, secretary; G. H. LIGHT, treasurer.

On Friday evening Mr. F. E. Parks, Manager of the Steel Works, delivered an address of welcome. He pointed out the advantages to all concerned of having the members of the section as guests of the company. Professor S. L. Macdonald responded to this address in a fitting manner, assuring Mr. Parks that the company's problems were also those of the section, and expressed the members' appreciation of the company's generous hospitality. An organ recital, followed by an address by Mr. D. K. Dunton, concluded the evening session. On Saturday morning the members visited the Steel Plant, Mr. Louis Deesz of the company officiating as guide.

The following nine papers were read:

(1) "Report of the Cincinnati meeting" by Professor H. E. RUSSELL.

(2) "The undergraduate mathematics club" by Professor S. L. MACDONALD.

(3) "To compute the radius of the circle inscribed in the area bounded by the arcs of three mutually tangent circles" by Mr. J. Q. MCNATT.

(4) "Misleading definitions of 'f' in the elementary theory for finding the envelope of $f(x, y, c) = 0$" by Professor I. M. DELONG.

(5) "A problem in probability" by Professor G. W. GORRELL.

(6) "Pedal curves and related envelopes" by Mr. PHILIP FITCH.

(7) "On curves whose first polars have a rectilinear component" by Professor C. H. SISAM.

(8) "Times of rising and setting of the planets" by Dean H. A. HOWE.

(9) "Magic squares of the first nine orders" by Professor F. H. LOUD.

In the absence of the authors, the papers by Dean Howe and Professor Loud were read respectively by Professors Russell and Sisam.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. In his report, Professor Russell commented on the attendance and interest of the Cincinnati meeting and dwelt briefly on the salient features of some of the more interesting papers.

2. Professor Macdonald's paper dealt with the advisability of having undergraduate mathematics clubs and showed how interest in mathematics was stimulated by such organizations. The meeting of pupils with a common interest often reveals qualities in them that would otherwise be dormant.

3. The paper by Mr. McNatt demonstrated an interesting method of computing the radius of the circle inscribed in the area bounded by three mutually tangent circles, in terms of the radii of these circles.

4. Professor DeLong pointed out that there were definitions of "f," as applied to the elementary theory for finding the envelope of $f(x, y, c) = 0$, that were misleading and remarked that some authors of works on calculus had made no attempt to clarify the subject.

5. Professor Gorrell compared methods of attacking problems in probability and discussed the advantages of having more than one viewpoint of a problem.

6. In his paper, Mr. Fitch demonstrated a short method for finding the equation of a pedal curve and proved the following properties: (*a*) The pedal of a given curve with respect to a fixed point is the envelope of a family of circles described on the radii vectores from the fixed point to the given curve as diameters. (*b*) The fixed point is a conjugate point of this envelope. (*c*) The caustic of a given curve with respect to a fixed point is a translation of the evolute of its pedal for that point.

7. In this paper, Professor Sisam determined the equations of the non-composite algebraic plane curves which have the property that every line through a fixed point is a component of a first polar with respect to the curve.

8. Dean Howe's paper dealt with the computation of the approximate times of rising and setting of the planets. The object of the method set forth is to render it possible for a student in elementary descriptive astronomy, by using data easily taken from the American Ephemeris, to obtain the time of rising or setting of any planet on any day of the year, with an error not exceeding three minutes. No logarithmic or trigonometric work is needed. The place for which the computation is to be made is supposed to be in the northern hemisphere, and to have a latitude no greater than 60°. Denver was chosen to illustrate the process.

From the Greenwich time when the planet crosses the Greenwich meridian on the given date, the Denver time when it crosses the Denver meridian is obtained by a simple interpolation. Then the problem is quickly finished by using the tables for sunrise and sunset at the end of the Ephemeris, making allowance for the fact that these tables are for the upper limb of the sun, instead of the center. In those infrequent cases where a planet's distance from the celestial equator exceeds twenty-three and a half degrees, an extrapolation is necessary.

9. In this paper, Professor Loud derived several new and interesting ways for forming magic squares.

PHILIP FITCH, *Secretary.*

type by several repetitions of the transformation

$$x = \frac{r^2}{y - b} + a,$$

which is geometrically equivalent to reflecting the roots of the original equation in a circle in such a way as to make the desired root go over into a large root while the sum of the remaining roots is made small.

7. Professor Dustheimer had 300 catalogues examined from every state in the U. S. and every province of Canada. 50% require 2 units of secondary mathematics for entrance. 50% require from 18 to 24 hours for a mathematics major. 50% require 6 hours or less of mathematics for graduation. 30% offer from 20 to 30 hours of mathematics. 80% have no connection between mathematics and physics. 46% offer from 2 to 6 hours of astronomy; 30% have some connection between mathematics and astronomy; 30% teach no astronomy.

Answers to the questionnaire from 55 colleges (33 Ohio colleges) show the following: Only 16% require students majoring in mathematics to take history of mathematics. Sixty-one per cent. give courses in the teaching of mathematics while very few give courses combining the history and teaching of mathematics. Many of these courses are lecture courses, but nearly all of them use a textbook. The courses average 2 hours per semester and are generally open to juniors and seniors.

8. These reports were informal and humorous rather than scientific, dealing mainly with personal experiences.

9. Professor Hancock had Professor Barnett report the result of identical tests given to college freshmen who had had some college mathematics in the University of Cincinnati, Ohio State University and Adelbert College. The results in the three institutions were in fair agreement but decidedly unsatisfactory although the tests were extremely easy. Professor Swartzel explained a plan in use at the University of Pittsburgh whereby they demote students until they find their true level of preparation. Few failures need be recorded by this device, which works beneficially. Professor Holl reported the method used this year in Ohio Wesleyan University in sectioning freshmen entering with only one year of algebra. The discussion extended itself to nearly all phases of present-day education.

Professor Barnett presented the matter of the Carus monographs in an effective talk.

G. N. ARMSTRONG, *Secretary-Treasurer.*

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION.

The ninth annual meeting of the Rocky Mountain Section was held at the University of Wyoming, Laramie, on April 10 and 11. There were thirty-six present, including the following ten members of the Association: I. M. DeLong,

Philip Fitch, J. C. Fitterer, H. C. Gossard, Claribel Kendall, G. H. Light, S. L. Macdonald, Letitia Odell, O. H. Rechard and H. E. Russell.

The section voted to hold the next meeting at the Colorado Agricultural College. The following officers were elected: S. L. MACDONALD, chairman; W. V. LOVITT, vice-chairman; PHILIP FITCH, secretary; G. H. LIGHT, treasurer.

Three committees were appointed, one to draft resolutions on the death of Dr. Carl E. Stromquist, formerly head of the department of mathematics at the University of Wyoming and a member of this section; one to formulate plans for the use of standard tests in connection with the teaching of mathematics in the colleges; and one to consider the advisability of procuring a speaker from outside the section for the next meeting.

On Friday evening, the members of the section were guests at a dinner given by the home economics department of the University. President A. C. Crane delivered an address of welcome on this occasion and expressed his pleasure at having the section meeting held at his institution. Professor S. L. Macdonald responded in a fitting manner in behalf of the guests.

Later in the evening the women members of the section were entertained at a production given by the Coffer-Miller players.

The following nine papers were read:

(1) "An endowment for the publication of the results of mathematical research" by Professor I. M. DELONG.

(2) "Mathematics as an aid in agricultural experimentation" by Professor A. G. CLARK (by invitation).

(3) "The relation of standard tests to the teaching of collegiate mathematics" by Professor H. C. GOSSARD.

(4) "Baade's asteroid" by Dean H. A. HOWE.

(5) "Points of view on the multiplicative axiom" by Professor C. H. RECHARD.

(6) "On the quinquenary cubic expressible as the sum of seven cubes" by Professor C. H. SISAM.

(7) "Integration in series" by Professor G. H. LIGHT.

(8) "Analysis of certain types of composite curves" by Mr. PHILIP FITCH.

(9) "On a tetrahedron" by Professor H. C. GOSSARD.

In the absence of the authors, the paper by Dean Howe was read by Professor Russell and the one by Professor Sisam by title only.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. Professor DeLong pointed out the urgent need of funds for publishing results of mathematical research and made an appeal to all interested in mathematics to assist the American Mathematical Society in establishing an endowment large enough to insure the publication of the fine results that are being achieved by mathematicians in this country.

2. Professor Clark mentioned, in a brief way, various typical problems arising in agricultural experimental work where the need for mathematical treatment

was obvious. An actual problem, dealing with the peculiar results of the cross of two varieties of barley, was solved.

3. This paper was a report of experiments with speed, accuracy, and power tests in freshman and sophomore college mathematics. As a result of this report, the colleges and universities of the Rocky Mountain Section of the Association voted to coöperate in a continuation of this experiment.

4. The discovery of Baade's asteroid last October was made by photography. The computation of a preliminary orbit at the Student's Observatory of the University of California was beset with special difficulties, but when a reasonably correct orbit was finally obtained, a request from Denver brought an ephemeris of the planet, furnished in advance of publication, so that measures of its position with the 20-inch Denver telescope might begin at once. The orbit has been found to be the most eccentric planetary orbit known, with one exception. The inclination of the orbit is among the dozen highest. Only three planetoids come nearer to the earth than it does. Furthermore it is unusual in that it varies regularly in brightness in a period of a few hours. This is probably an indication of axial rotation, perhaps the first shown by any planetoid. Despite the faintness of this planet, a very large number of observations has been made at Denver, but the approaching opposition of the asteroid will cause a termination of the series.

5. Professor Rechard discussed the multiplicative axiom as brought to the fore by Zermelo's Theorem. The discussion centered around the points of view of various mathematicians, especially Baire, Borel, Hadamard, and Lebesgue. A summary of the principal problems and special fields affected by one's point of view on the axiom was included.

7. This paper gives methods for finding the solution of the differential equation

$$(x - a)^2 p_0 \frac{d^2y}{dx^2} + (x - a) p_1 \frac{dy}{dx} + p_2 y = X.$$

It is shown when this equation will have a solution in ascending and descending powers of $(x - a)$ as well as when a logarithmic solution occurs. The particular integral is found in a very simple manner.

8. Mr. Fitch discussed the analysis of composite curves obtained from experimental data arising from observations made on thermoluminescent and similar effects. He pointed out the necessity of being guided by the scientific facts involved while attempting such an analysis.

9. Following a suggestion by Dr. Morley of Johns Hopkins the author of this paper presented twenty-one relations between the edges and faces of a tetrahedron. The equation of the absolute is expressed first so that its coefficients are in terms of the edges of the tetrahedron and second in terms of its faces. The twenty-one resulting coefficients are then equated giving the desired relations.

PHILIP FITCH, *Secretary.*

AFFILIATION OF THE ASSOCIATION OF TEACHERS OF MATHEMATICS IN NEW ENGLAND WITH THE MATHEMATICAL ASSOCIATION OF AMERICA

The council of the Association of Teachers of Mathematics in New England have voted to accept a plan of affiliation originating in 1925 in conferences between President J. L. Coolidge and the officers of that Association. As modified by the Trustees of the Mathematical Association and put into effect by this recent action, it is agreed

1. That the members of the Association of Teachers of Mathematics in New England may become members of the Mathematical Association of America without the payment of the customary initiation fee, the A.T.M.N.E. to supply annually to the Mathematical Association a list of new members whom the Association may invite to their membership under this special condition.

2. That it is understood that whenever the M.A.A. holds meetings in New England, the A.T.M.N.E. engages to aid in every way, and that meetings of the A.T.M.N.E. in connection with those of the M.A.A. as an affiliated organization will be welcomed.

3. That the present agreement may be terminated by either party on six months' notice.

4. That the vote by this council here recorded makes this agreement effective.

W. D. CAIRNS, *Secretary-Treasurer.*

ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The tenth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado Agricultural College at Fort Collins, Colorado, on April 16, 17. There were thirty-six present including the following seventeen members of the association: A. G. Clark, I. M. DeLong, G. W. Finley, Philip Fitch, J. C. Fitterer, H. C. Gossard, A. J. Kempner, Claribel Kendall, G. H. Light, W. V. Lovitt, S. L. Macdonald, J. Q. McNatt, L. R. Odell, O. H. Rechard, W. J. Risley, H. E. Russell, and C. H. Sisam.

The section voted to hold the next meeting at Colorado College, Colorado Springs. The secretary was instructed to invite the Association to hold a summer meeting at the University of Colorado sometime in the near future.

A committee was appointed consisting of the secretary, Miss Odell, Professor Russell, and Professor Risley to compile material on the significance and value of the study of mathematics. This material is to be published by the section.

The following officers were elected: W. V. LOVITT, chairman; W. J. RISLEY, vice-chairman; PHILIP FITCH, secretary and G. H. LIGHT, treasurer.

The section was favored Friday evening with an address "The New Heavens" by Dr. D. W. Morehouse, president of Drake University.

A complimentary dinner was served on Friday to all present, at which time President C. A. Lory of the Colorado Agricultural College delivered an address of welcome to which Professor I. M. DeLong made an appropriate response.

The following sixteen papers were read:

(1) "Report on the experiment with a standardized test in college algebra" by Professor H. C. GOSSARD.

(2) "The interpretation of errors" by Professor R. L. PARSHALL (by invitation).

(3) "Graphic solutions" by Mr. J. Q. MCNATT.

(4) "Mathematical logic" by Professor H. V. CRAIG (by invitation).

(5) "Concerning d'Alembert's principle" by Professor J. C. FITTERER.

(6) "Finite trigonometric series" by Professor A. G. CLARK.

(7) "Mathematics for freshman women" by Mrs. NELLIE LANDBLOM (by invitation).

(8) "On the reliability of the composite score of a battery test" by Mr. PHILIP FITCH.

(9) "Concerning rigorous proofs" by Professor S. L. MACDONALD.

(10) "Note on the limit functions of sequences of functions of certain types" by Professor O. H. RECHARD.

(11) "The use of the discriminant in differential equations" by Professor G. H. LIGHT.

(12) "Index number bias" by Professor W. V. LOVITT.

(13) "A system of vector coordinates" by Professor H. C. GOSSARD.

(14) "On a property of the Hessians of cubic forms" by Professor C. H. SISAM.

(15) "Complex roots of equations" by Professor A. J. KEMPNER.

(16) "Root extraction with the adding machine" by Professor F. H. LOUD.

In the absence of the author, the abstract of Professor Loud's paper was read by the secretary.

Abstracts of the papers follow below, the numbers corresponding to the numbers in the list of titles.

1. A report of the experiment by the colleges and universities of the Rocky Mountain Section with a standard college algebra speed-accuracy test. This experiment under the direction of the University of Wyoming was voted to be continued through the coming year.

2. This paper dealt with the interpretation of the errors arising in results obtained from experiments on the flow of water over weirs.

3. Mr. McNatt gave some examples of the use of graphic solutions of problems in surveying.

4. The paper discussed the primitive ideas and primitive propositions of the system of Whitehead and Russell; analogies among the calculus of propositions, calculus of classes, and calculus of relations; and the definition of the cardinal numbers 1 and 2. It was shown that a class cannot be used as an argument of anyone of its determining functions and that a hierarchy of functions is necessary.

5. A résumé of its statement and presentation in various texts on mechanics and kinetics was given and it was pointed out that its function, particularly for the teacher of dynamical subjects—especially the practical applications in engineering and allied sciences, consists primarily in simplifying and reducing forces involved in accelerated systems to the field of statics. This service is comparable with the idea Monge had in descriptive geometry, of reducing solid problems to the realm of the plane; and also, as another example, with the conception involved in influence lines in the theory of structures whereby the effect of live loads is simplified to the status of dead loads.

6. Professor Clark indicated that the summation of

$$\sum_{i=1}^{n} F(i) \begin{matrix}\sin\\ \cos\end{matrix} \phi(i)\theta$$

may be effected when $F(i)$ is a rational, integral polynomial by successive application of the parts formula for finite integration,

$$\Sigma u_x \Delta v_x = u_x v_x - \Sigma v_{x+1} \Delta u_x + C_1$$

provided the summation of

$$\sum_{i=1}^{n} \begin{matrix}\sin\\ \cos\end{matrix} \phi(i)\theta$$

is possible.

When $F(i)$ is the quotient of two rational integral polynomials, the summation may be effected by setting up and solving a system of two linear differential equations, the order of the system being the same as the degree of the polynomial forming the denominator.

7. The question as presented by Mrs. Landblom was "What should be the content of a course in mathematics for freshman women and why?" Arguments set forth dealt primarily with problems arising in home life, teaching, special subjects required by curricula, as physics and chemistry, extension work, child welfare, and social settlement. The solution was given in an outline of topics to be covered in a fifty hour course.

8. In this paper it was shown that the composite score of a battery test could be reliable only if the respective scores of its elements were weighted

according to their reliability, validity and independence after having been reduced to equal spread.

9. In his paper Professor Macdonald pointed out the necessity of making conclusions definite in rigorous proofs.

10. An example was given to show that quasi-uniform convergence is not necessary in order that the existence of a limit of a sequence of functions which are pointwise discontinuous shall itself be proved. The following theorem was then proved: A sufficient condition that the limit of a sequence of functions with upper continuity shall have upper continuity is that the sequence is quasi-uniformly convergent.

11. This paper was intended to show how the discriminant can be applied to the solution of differential equations of the type $f(x,y,p)=0$ and showed how the extraneous factor, if any exists, can be detected before solving the equation.

12. Four primary systems of weighting have been devised. In the customary notation these are p_0q_0, p_0q_1, p_1q_0, p_1q_1. The speaker noted the absence of strict mathematical proofs of the assertions made as to the upward or downward bias of an index number weighted with the weights specified above. In this paper some proofs were given of bias when such exists. Professor Irving Fisher has given a proof that the unweighted arithmetic average of relative prices has an upward bias. This paper gave a new proof of this fact.

13. Professor Gossard presented a system of coordinates based upon naming the points (x) of the Gaussian plane by rotations (t) on a base circle plus such translations as called for by the given equation $x=f(t)$. The expressions arising are symmetric functions of (t) and for many types of theorems in geometry the analytic work is exceedingly simple.

14. This paper dealt with the determination of a simplified form for the equation of the Hessian of a cubic in any number of variables.

15. The roots (real and complex) of an equation $w=a_0z^n+a_1z^{n-1}+\cdots a_n\neq 0$, $w=u+iv$, $z=x+iy=re^{i\varphi}$, a_i real or complex, may be isolated and determined to any desired degree of accuracy in the following manner: In a rectangular system of coordinates with a φ-axis and an axis which serves at the same time for $u-$axis and $v-$axis plot, for appropriately chosen values r_i of r, the two curves $u=u(r_i,\varphi)$, $v=v(r_i,\varphi)$. From the order in which the points of intersection of the u-curve with the φ-axis and the points of intersection of the v-curve with the φ-axis follow each other, the number of roots of $w=0$ of absolute value $<r_i$ is read off by a very simple rule. The absolute values of the roots are then determined to any desired degree of accuracy.

Professor Kempner next showed how limits for the arguments of the roots may be determined. The value of the method lies in the fact that instruments

are in existence (harmonic analyzers) which will trace mechanically curves of type $u=u(r_i,\varphi)=\sum c_k \cos k(\varphi)$, $v=v(r_i,\varphi)=\sum d_k \sin k(\varphi)$.

16. This paper dealt with the employment of Newton's binomial formula as a method, and of the adding machine as an instrument, in the extraction of roots. Several cube roots were extracted, as illustrations of certain elementary and rather obvious devices for securing rapid convergence in the series, and otherwise minimizing the labor of computation. In the latter part of the paper, some of the actual records taken from the adding machine were inserted, as fuller demonstration of the method employed.

PHILIP FITCH, *Secretary.*

ELEVENTH ANNUAL MEETING OF THE OHIO SECTION

The eleventh annual meeting of the Ohio Section of the Mathematical Association of America was held at the Ohio State University, Columbus, April 2, 1926, in connection with the meetings of the Ohio College Association.

Thirty-four persons registered attendance, among whom were the following twenty-five members of the Association: R. B. Allen, C. L. Arnold, Grace M. Bareis, I. A. Barnett, H. Blumberg, R. D. Bohannan, W. D. Cairns, V. B. Caris, R. Crane, W. Dancer, O. L. Dustheimer, T. M. Focke, B. C. Glover, H. Hancock, H. W. Kuhn, S. E. Rasor, P. L. Rea, Hortense Rickard, S. A. Singer, C. E. Stout, J. H. Weaver, R. B. Wildermuth, F. B. Wiley, J. B. Winslow, B. F. Yanney.

Officers elected for the coming year were: Chairman, H. W. KUHN; Secretary-Treasurer, RUFUS CRANE; Member of Executive Committee, H. M. BEATTY; Member of Program Committee, T. M. FOCKE. A resolution was adopted expressing the appreciation of the members of the section for the life and services of the late Professor G. N. Armstrong, and their regret at his death. A committee was appointed to study the advisability of attempting to organize a new section of the American Mathematical Society, centering in Ohio. A committee was appointed to study ways and means of improving the teaching situation in the secondary schools. It is expected that the next meeting will be held on April 8, 1927.

The following papers were read:

(1) "Alphabetic symbolism applied to some operations on power series" by the Chairman, Professor R. D. BOHANNAN, Ohio State University.

(2) "Euclidean invariants of plane second degree curves" by Professor C. C. MACDUFFEE, Ohio State University.

(3) "Controversial mathematics" by Professor H. BLUMBERG, Ohio State University.

ELEVENTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The eleventh annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado College, Colorado Springs, Colorado, on April 22, 23. There were forty present including the following twenty members of the association: G. H. Albright, A. G. Clark, E. A. Cummings, I. M. DeLong, Philip Fitch, A. J. Kempner, Claribel Kendall, G. H. Light, W. V. Lovitt, S. L. Macdonald, J. Q. McNatt, W. K. Nelson, Letitia Odell, O. H. Rechard, H. L. Rietz, W. J. Risley, H. E. Russell, Mary S. Sabin, C. H. Sisam, C. W. Wray.

The section voted to hold its next annual meeting at the Colorado School of Mines, Golden, Colorado. The following officers were elected: W. J. RISLEY, chairman; G. W. FINLEY, vice-chairman; PHILIP FITCH, secretary; G. H. LIGHT, treasurer.

At the complimentary dinner given by Colorado College on Friday, Dean C. B. Hershey delivered an address of welcome to which Professor W. J. Risley made the response. The section was favored Friday evening with an address "Group Insurance" by Professor H. L. Rietz of the University of Iowa. Professors H. E. Russell and C. H. Sisam read sketches of the lives and work of the late Dean H. A. Howe and the late Professor F. H. Loud respectively.

The following eleven papers were read:

1. "The sectioning of college freshmen in mathematics by means of the Iowa Placement Test" by Professor C. F. BARR.
2. "A grade weigher" by Professor G. H. ALBRIGHT.
3. "A graphic solution of tensions in cables" by Mr. J. Q. MCNATT.
4. "On a type of involutions in space" by Professor C. H. SISAM.
5. "Concerning the Heusler alloys" by Mr. PHILIP FITCH.
6. "An elementary method of solving matricial equations" by Professor O. H. RECHARD.
7. "Applications of elementary divisors" by Mr. D. L. GUNDER (by invitation).
8. "On a geometrical problem from the MONTHLY" by Professor A. J. KEMPNER.
9. "The Carus monograph on statistics" by Professor H. L. Rietz.
10. "Index number bias" by Professor W. V. LOVITT.
11. "A nomograph" by Professor W. K. NELSON.

In the absence of the author, the abstract of Professor Barr's paper was read by Professor Rechard. Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles.

1. The Iowa Training Examination in Mathematics was given at Purdue University to some 800 entering freshmen engineers, fall of 1925. This paper is a study of the degree of correlation between the grades obtained, and the subsequent classroom records in mathematics, of the persons involved. There was found a high degree of correlation. Furthermore, there appeared clearly defined lines for segregation into inferior, normal, and superior groups. The careful analysis of classroom records for the members of each group showed a remarkably high power of selectivity for the Iowa examination, and commends it as a tool for segregation of entering freshmen in mathematics. The conclusions are supported by the results of a similar study, made the preceding year, and ending simultaneously with the present study. While the paper demonstrates the possibility, it does not argue the advisability of classroom segregation of students.

2. In this paper Professor Albright described the construction and manipulation of a device for determining the average grade of a student involving the weighing of his various marks to correspond to the units of credit carried by his course. The device was of the type of a simple lever. Such an instrument was exhibited which would calculate the average with an error not exceeding three-hundredths of one percent.

3. Mr. McNatt gave a graphic solution of the tensions in a cable used to carry movable loads as in the case of aerial trams.

4. In this paper, the author discusses involutions in space such that the locus of the lines joining corresponding points is a congruence of order one defined by a (l, m) between the points on and the planes through a fixed line. Several particular involutions of this type were discussed at some length.

5. This paper dealt with mathematics as applied to data obtained from measurements on the resistance, permeability and thermo-electric affect of the Heusler alloys.

6. Professor Rechard showed that the solution of the general equation of the same order reduces to the problem of solving n^2 equations in n^2 unknowns, each equation being of degree n. A quadratic equation was used to illustrate the method.

7. Mr. Gunder showed that the principles of elementary divisors was directly applicable to the solution of problems of the dynamics of a particle where the particle is executing small vibrations about an equilibrium configuration. The study of this problem is greatly facilitated by the reduction of the equations of motion from the usual coordinates to normal coordinates.

8. Problem 3171(3167; 1926, 104) states that if, in an ellipse, a straight line is drawn through one Focus F_1, its intersection P_1 with the circumference joined with the other Focus F_2, the intersection P_2 of P_1F_2 with the ellipse again joined with F_1, etc. etc., the limiting position of the straight line will be

the straight line joining F_1F_2. It is shown that this property is in no way characteristic of the ellipse. An analogous property exists, for example, for all closed convex curves.

9. Professor Rietz discussed first the question of making statistical theory more available to the general mathematical reader by means of the third Carus Mathematical Monograph. He then divided the problems considered in the monograph into two general classes. In problems of the first class, the main interest centers around the properties of a random sample drawn from a "population." In problems of the second class, the main interest centers around the question of making and checking the validity of statistical inferences about the population from which the sample is drawn. In dealing with the sample we introduce very early the concepts of relative frequency, arithmetic mean, various other averages, and certain measures of dispersion. In considering the population, we introduce the parallel concepts of probability, mathematical expectation of the value of a variable, and of the powers of the deviations of a variable from its expected value. After discussing these concepts, the paper gives a summary of the material of the monograph dealing with the following three topics of dominant interest in recent progress in statistics: Generalization of frequency curves, correlation theory, and random sampling theory.

10. Definite proofs were made as to the presence or absence of bias for the weighted arithmetic average of relatives for the weights I p_0q_1, II p_0q_1, III p_iq_i, III p_iq_0, IV p_iq_i. Definite proofs were given as to the relative size of the index numbers with weights I, II, III, IV.

11. In this paper a description was given of a nomograph of an approximation to the formula for the relation between effective and nominal interest rates.

For given values of j and p, i could be read accurately to at least four significant figures.

PHILIP FITCH, *Secretary*

THE TWELFTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twelfth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado School of Mines, Golden, Colorado, on April 20–21, 1928. There were fifty-five present including the following twenty-five members of the association: Ralph Beatley, A. G. Clark, E. A. Cummings, I. M. DeLong, J. R. Everett, G. W. Finley, Philip Fitch, J. C. Fitterer, G. W. Gorrell, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, G. H. Light, E. P. Martinson, S. L. Macdonald, Margaret McGinley, J. Q. McNatt, W. K. Nelson, O. H. Rechard, W. J. Risley, Mary S. Sabin, C. H. Sisam, E. B. Stouffer, C. W. Wray.

The section voted to hold its next annual meeting at the Colorado Teachers' College, Greeley, Colorado. The following officers were elected: G. W. FINLEY, chairman; G. W. GORRELL, vice-chairman; PHILIP FITCH, secretary, G. H. LIGHT, treasurer.

At a complimentary dinner given by the School of Mines on Friday, President Coolbaugh delivered an address of welcome to which Professor G. W. Finley made the response. The section was favored Friday evening by an address, "Mathematics in Italian Universities" by Professor E. B. Stouffer of the University of Kansas. Professors DeLong and Gorrell gave brief talks on the life and work of the late Professor H. E. Russell.

The following fourteen papers were read:

1. "A problem" by Mr. J. Q. MCNATT.

2. "The intersection of two special ruled cubics" by Professor SAUL POLLOCK (by invitation).

3. (a) "A method of solving quadratics" and (b) "A model for trigonometric functions" by Mr. J. W. HAZARD (by invitation).

4. "Teaching mathematics by subjects or topics" by Professor G. W. GORRELL.

5. "Note on the convergence of an infinite series" by Professor C. A. HUTCHINSON.

6. "Teaching engineering mathematics to the poor student" by Professor J. R. EVERETT.

7. "An interpretation of the imaginary resulting from an application of the principle of continuity" by Mr. PHILIP FITCH.

8. "The use of the discriminant in solving a certain type of differential equation" by Professor G. H. LIGHT.

9. "Mathematics and the Society for the Promotion of Engineering Education" by Professor J. W. RISLEY.

10. "Remarks on a question raised by Hessenberg" by Professor A. J. KEMPNER.

11. "On a special quartic surface with a double line" by Professor C. H. SISAM.

12. "Cocoanuts and congruence" by Professor RALPH BEATLEY.

13. "Some fundamental concepts in differential projective geometry" by Professor E. B. STOUFFER.

14. "Civic values in the study of mathematics" by Professor A. S. ADAMS (by invitation).

At the close of the Friday afternoon session, Dr. C. A. Heiland gave an explanation of the apparatus used in geophysics for the purpose of locating bodies of ore and oil.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles:

In his address on "Mathematics in Italian universities," Professor Stouffer told of various experiences in connection with his study of mathematics in Italy during the year 1926–27. Special mention was made of the mathematics curricula in Italian universities and of the character of the lectures delivered by some of the distinguished Italian geometers.

1. Mr. McNatt gave a method of calculating the volume of liquid contained in a right circular cylindrical tank, when the tank is inclined at any given angle.

2. Professor Saul Pollock exhibited a series of six models illustrating the special curves of intersection of two ruled cubic surfaces each having two pinch points. These curves of the ninth degree were composites and pointed out the results of matching pinch points, rulings, common conics, etc. In addition, a device was demonstrated by means of which it is possible to study space curves of any degree. The principle consisted of intersecting a surface with another surface consisting of light. The resulting curve of intersection is at once visible, and by moving one of the surfaces it is possible to study the variation in the curve.

3. (a) Mr. Hazard described a method of solving the quadratic equation in one unknown by means of a table of quarter-squares. Such tables have been computed for the purpose of finding the product of numbers by means of the relation: the quarter-square of the sum of two numbers minus the quarter-square of their difference equals their product. In the quadratic we have the sum and the product of the roots given, hence we can use the above relation to solve for the difference. The roots are then found from their sum and difference. This method has marked advantages over solving the quadratic equation by logarithms when the coefficients are large.

3. (b) Mr. Hazard demonstrated a rotating model for use in class instruction, in which the signs and values of sines and cosines are shown with their continuous variations throughout the entire circle.

5. Mr. Hutchinson's note dealt with real infinite series in a variable, x, which may be converted into power series in y by a substitution, $y=f(x)$. The interval of convergence of the power series is projected orthogonally onto the curve $y=f(x)$, thence orthogonally onto the x-axis, giving the regions of convergence, and divergence, of the original series.

8. There is always an extraneous factor when a differential equation of the type $f(x, y, p)=0$ has a singular solution. This paper by Professor Light shows how that factor can be found before formally solving the differential equation.

10. In a long article, Grundbegriffe der Mengenlehre, Neue Abhandlungen der Fries' schen Schule, 1906, G. Hessenberg raised the question as to what would happen if it could be proved, for example, that it is impossible to decide whether the number 2^{μ}, where $\mu=\sqrt{2}$ is an algebraic or a transcendental number. In his opinion mathematics would in this case be confronted with a difficulty more serious than any which has so far been encountered. The writer has not had access to the paper for many years and does not recall that any qualifying statements were made concerning the nature of the system of axioms used or concerning the laws of formal logic. In the absence of any such restrictions, it is easy to show that the difficulty would be solved by the introduction of two algebras, in one of which it would be an axiom that 2^{μ} is algebraic, in the other of which 2^{μ} would be transcendental.

11. In this paper, the author discusses some properties of those quartic surfaces with a double line which have the property that the conics on the surface, in pairs, constitute degenerate quartic curves of the first kind.

13. The projective differential properties of many figures may be studied by means of canonical expansions. Professor Stouffer derived such a canonical expansion for the equation of the plane curve, starting from a general expansion for the equation. The method required the proper choice of the triangle of reference. In a similar manner a canonical expansion suitable for the study of curved surfaces was also derived.

14. An answer to the question "What can the average student get out of the study of Mathematics" was given by Professor Adams. The values to be derived from the study were considered from the point of view of the student's social, economic, and individual life. His social life was shown to have been helped in giving him the valuable mental habits of neatness, orderliness, accuracy, persistence, intellectual honesty, and attention. His economic life has been benefited by direct application of mathematical principles to his business or profession, by fundamental training for further study in the sciences, and by the development of his ability to think clearly, rapidly, and accurately. His indi-

vidual life has profited by basic training in appreciation of beauty and by a realization of the exactness and order of the Universe.

PHILIP FITCH, *Secretary*

THE FIFTH MEETING OF THE INDIANA SECTION

The fifth meeting of the Indiana Section of the Mathematical Association of America was held May 11, 12, 1928 at Butler University, Indianapolis, Indiana.

There were sixty-one present at the meeting including the following thirty-one members of the Association: R. J. Aley, W. C. Arnold, Gladys L. Banes, Stanley Bolks, G. E. Carscallen, P. T. Copp, H. T. Davis, S. C. Davisson, J. E. Dotterer, W. E. Edington, P. D. Edwards, E. D. Grant, H. E. H. Greenleaf, Laurence Hadley, Cora B. Hennel, F. H. Hodge, E. N. Johnson, Kathryn M. Kennedy, Florence Long, Juna M. Lutz, T. E. Mason, H. R. Mathias, R. E. Peterson, C. K. Robbins, D. A. Rothrock, J. R. K. Stauffer, C. E. Stout, K. P. Williams, H. E. Wolfe, W. A. Zehring, and H. A. Zinszer.

On Friday evening at 6:30 a banquet was held at the Claypool Hotel which was attended by members of the Association and their guests. President Aley of Butler University presided.

At eight o'clock a public lecture under the auspices of Butler University was given by President D. W. MOOREHOUSE of Drake University, Des Moines, Iowa, on the subject: "The Milky Way." President Moorehouse, by means of lantern slides, traced the ever interesting history of man's expanding knowledge of the universe. The speaker called special attention to the perplexing problem presented by the presence of black patches in the midst of brilliant star clouds and showed how the evidence points to the existence of great dark nebulous masses in the milky way.

At the session on Saturday morning on the Butler campus, presided over by Professor J. E. Dotterer, Manchester college, chairman, the following officers were elected: Professor H. E. H. GREENLEAF, De Pauw University, chairman; Professor H. A. ZINSZER, Hanover College, vice-chairman; Professor H. T. DAVIS, Indiana University, secretary-treasurer.

A chairman's address was made by Professor J. E. DOTTERER on the subject: "The Mathematician as a Salesman." Professor Dotterer pointed out the duty incumbent upon the teacher, in addition to his actual instruction, of showing the fundamental connection between mathematics and actual living. He urged the need of exhibiting mathematics not only as a tool used in solving and explaining the universe in which we live, but also as a discipline, a cultural subject and an art.

Professor R. H. COON of the Latin Department of Indiana University fur-

of introductory matter, 72 pages of free translation and commentary, and 85 pages of a very elaborate and critical bibliography of Egyptian mathematics contributed by Professor Archibald and including references to the literature of over fifty documents dating from 3500 B.C. to about 1000 A.D. This bibliography is further supplemented in the second volume especially in the light of recent remarkable discoveries in the field of Babylonian mathematics.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirteenth regular meeting of the Rocky Mountain Section of the Mathematical Association was held at the State Teachers College, Greeley, Colorado on April 12–13, 1929. There were three sessions, at which Prof. G. W. Finley acted as chairman.

The attendance was thirty-five including the following twenty-three members of the association: C. F. Barr, A. S. Clark, J. R. Everett, G. W. Finley, J. C. Fitterer, G. W. Gorrell, C. A. Hutchinson, H. Karnow, A. J. Kempner, Miss Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. S. McMaster, J. Q. McNatt, R. R. Middlemiss, W. K. Nelson, E. D. Rainville, O. H. Rechard, A. W. Recht, W. J. Risley, L. J. Rote, C. H. Sisam.

The following officers were elected for the coming year: Chairman, G. W. Gorrell, University of Denver; Vice-chairman, A. J. Kempner, University of Colorado; Secretary-treasurer, A. J. Lewis, University of Denver. Following the election of officers a resolution was presented in appreciation of the work of Philip Fitch who passed away last autumn. Mr. Fitch was secretary of the association for many years and rendered a very valuable service to the work of the association in this district.

The following eleven papers were read:

1. "Statistical treatment of the factor of soil heterogeneity in agricultural experimentation," by Professor Andrew G. Clark, Colorado Agriculture College.
2. "Mathematical geography," by Professor Chas. A. Hutchinson, University of Colorado.
3. "The icosahedron," by Mr. A. J. Lewis, University of Denver.
4. "A geometrical approximation of π," by Mr. L. J. Rote, Denver, Colorado.
5. "The hypersurface of bi-secants of a curve in four-way space," by Dr. C. H. Sisam, Colorado College.
6. "A geometrical construction showing the relation between the in-center and circum-center of a triangle," by Mr. J. Q. McNatt, Canyon City, Colorado.
7. "Graphical methods of approximating irrational roots," by Professor James R. Everett, Colorado School of Mines.
8. "Present trend in the organization of subject matter of high school mathematics," by Professor A. E. Mallory, Colorado State Teachers College.
9. "An age sifter," by Professor Walter K. Nelson, University of Colorado.

10. "On the solution of linear equations," by Dr. Aubrey J. Kempner, University of Colorado.

11. "Some problems in elementary research," by Dr. J. L. Gibson, University of Utah, by invitation.

Abstracts of papers follow:

1. Professor Clark showed how an assumption of continuity in the variation of soil fertility would offset the unreliability of results in experimentation plots due to their insufficient number. Using adjoining plots, a correlation is effected which uses a sufficient number of plots to overcome the unreliability of first results.

2. Professor Hutchinson gave a brief outline of the subject of cartography, indicating some of the problems of mathematical interest in that field.

3. This is an outline of the methods used by Felix Klein in developing the icosahedral equation and showing its use in the solution of the quintic equation.

4. Mr. Rote showed a construction for a square which approximated closely the area of a circle.

5. This paper deals with the problem of finding those properties of the hyper-surface of bi-secants to an algebraic curve in four dimensions that can be determined by projecting the given curve from a given line onto a plane.

6. Mr. McNatt developed in a new way the known formula for the distance between the in-center and circum-center of a triangle.

8. Professor Mallory emphasized the fact that the present tendency in the selection of subject matter of high school mathematics was toward the adaptation of material to pupils' ability and toward a more informal treatment of the subject generally.

9. Professor Nelson explained briefly a device consisting of nine cards which may be used to determine the age of a person. When the cards are placed according to instructions the age of the person appears in large type through an opening in the back of the pile of cards.

10. This paper was published in the August-September issue of this "Monthly."

11. The general parametric equations of the space and body centrodes of certain disks and disk-like bodies, and other surfaces and solids supported by and rolling between two intersecting planes are found. The abscissas of the instantaneous axes of rotation give the values of definite integrals whose peculiarities have in these problems specific physical meanings. This makes it possible to use the planes as mechanical analyzers of many integrals, including some elliptic and hyperelliptic integrals. Points rigidly attached to the rolling bodies generate roulettes, the abscissa of each point of which, using the general equations, contains a definite integral. These curves, under certain conditions, degenerate into many well known forms, such as the cycloids. If we assume the equations of the space and body centrodes and study the integrals and equations of curves which follow from them, we find a field in which research has been done by students of limited mathematical attainments. Other problems of

a mechanical nature leading either to new methods or new results were mentioned. It was suggested that more attention be paid to the finding of this type of problem for the purpose of stimulating research earlier in the case of students specializing in mathematics.

The members and friends of the association were guests of the State Teachers College on the evening of April 12 at a banquet. President Finley acted as toastmaster. The address of welcome was given by President Frazier of State Teachers College and the response was given by Professor G. W. Gorrell of Denver University, after which there was a very interesting talk by Dean J. L. Gibson of the University of Utah. Dean Gibson recounted his experiences in visiting Germany and German mathematicians after the war.

A. J. LEWIS, *Secretary*

THE MAY MEETING OF THE MINNESOTA SECTION

The regular spring meeting of the Minnesota Section was held at the College of St. Catherine, St. Paul, Minnesota, on Saturday, May 11, 1929. At the request of the chairman, Sister Alice Irene, Professor Dunham Jackson presided at the morning and afternoon sessions.

The attendance was 60 at the luncheon and 80 at the regular session, and included the following 30 members of the Association: W. O. Beal, R. W. Brink, W. E. Brooke, W. H. Bussey, Elizabeth Carlson, H. H. Dalaker, J. M. Earl, Margaret Eide, Gladys Gibbens, C. H. Gingrich, S. Guttman, D. Jackson, C. M. Jensen, W. H. Kirchner, E. L. Mickelson, Marie Ness, M. A. Nordgaard, G. C. Priester, Inez Rundstrom, R. E. Scammon, R. R. Shumway, Sister Alice Irene, Sister Prudentia Morin, F. J. Taylor, Ella Thorp, A. L. Underhill, M. B. White, H. B. Wilcox, G. L. Winkelmann, F. Wood.

The following officers were elected for the coming year: Chairman, Fredrick Wood, Hamline University, St. Paul, Minnesota; Secretary, A. L. Underhill, University of Minnesota; an Executive Committee consisting of the Chairman, the Secretary, Gladys Gibbens, University of Minnesota, F. J. Taylor, College of St. Thomas, St. Paul, C. M. Jensen, Macalaster College, St. Paul.

A motion was passed expressing the appreciation of the Section for the hospitality of the College of St. Catherine.

The following seven papers were read:

1. "An integrating operator," by Mr. Max Scherberg, University of Minnesota.
2. "Insect populations," by Mr. John Stanley, University of Minnesota.
3. "Newton's method of solving equations," by Professor W. O. Beal, University of Minnesota.
4. "Developmental geometry," by Miss Marie Ness, Department of Anatomy, University of Minnesota.
5. "Approximate solutions of problems in the calculus of variations," by Professor C. G. Priester, University of Minnesota.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The fourteenth annual meeting of the Rocky Mountain Section of the Mathematical Association was held at the University of Denver, Denver, Colorado, on April 11–12, 1930. There were three sessions, Professor G. W. Gorrell acting as chairman at each.

The attendance was forty-two including the following twenty-seven members of the association: C. F. Barr, J. Britton, A. G. Clark, J. R. Everett, J. C. Fitterer, G. W. Gorrell, S. G. Hacker, C. A. Hutchinson, D. Jackson, H. Karnow, A. J. Kempner, Miss Claribel Kendall, A. J. Lewis, G. H. Light, A. S. McMaster, J. Q. McNatt, W. K. Nelson, Miss Greta Neubauer, Miss L. R. Odell, E. J. Purcell, E. D. Rainville, A. W. Recht, W. J. Risley, L. J. Rote, Miss Mary Sabin, C. H. Sisam, Miss Adela M. Thom.

The following officers were elected for the coming year: Professor Claribel Kendall, University of Colorado; Vice-Chairman Professor C. F. Barr, University of Wyoming.

The following papers were read:

1. "Fregier's theorem" by Professor Francis Regan, Colorado Agricultural College, by invitation.
2. "Predicting occultations" by Professor A. W. Recht, University of Denver.
3. "Foci of algebraic curves" by Professor Claribel Kendall, University of Colorado.
4. "On the invariance of certain types of areas" by Professor A. G. Clark, Colorado Agricultural College.
5. "A problem in partial correlation" by Professor G. H. Light, University of Colorado.
6. "Focal surface of a normal congruence of an ellipsoid" by Professor J. R. Everett, Colorado School of Mines.
7. "A formula in terms of greatest integers giving parcel post charges as a function of weight and distance" by Professor W. K. Nelson, University of Colorado.
8. "The elliptic modular group and applications to the theory of functions" by Mr. Earl Rainville, University of Colorado.
9. "Theory of numbers and the multiplication table" by Professor A. J. Kempner, University of Colorado.
10. "Formulas of correlation in several variables" by Professor Dunham Jackson, University of Minnesota.

Abstracts of these papers follow:

1. Mr. Regan presented Fregier's theorem: if a variable chord PQ of a conic subtends a right angle at any fixed point V on the conic it passes through a fixed point F which lies on the normal to the conic at V. The proofs for the parabola, ellipse, and hyperbola were given. The theorems dealing with the locus of the Fregier points of each conic were developed, and several corollaries

growing out of the fundamental theorem were touched upon. All the work was developed from a purely analytic view point.

2. Professor Recht mentioned the value of occultations in determining the position of the moon and described an apparatus for making maps of the United States predicting within a minute the times of occultations.

3. In this expository paper Miss Kendall defined the foci of algebraic curves for the general case and for certain special cases. The four foci, two real and two imaginary, of the central conics were found. The finding of the real foci of a cubic which was the inverse of a hyperbola with respect to one of its vertices illustrated the theorem that foci always invert into foci. Mention was made of the locations of foci for cubics without singularities and for quartics with nodes at the circular points.

4. Professor Clark considered briefly the conditions under which the area cut from the curve $y=f(x)$, a polynomial, would be invariant. As an application of the conclusions, it was proved that the inflexion tangets of a general quartic cut equal areas from the curve.

5. This paper gives formulae for finding the grades that should be expected by a student who is taking mathematics, English, and history in his first year at college. The data were obtained from the actual grades for the first and second quarters.

6. Professor Everett outlined the general theory of linear congruences, and showed how linear congruences might be applied to normals of an ellipsoid. He also discussed the development and nature of the surfaces generated by normals of an ellipsoid.

7. The paper by Professor Nelson presented a formula using greatest integers which gives the parcel post charges in terms of weight and distance. The following refinements make the formula agree closely with the postal laws: (a) All packages of a given weight sent over 1800 miles have the same charges. (b) All packages of half a pound or less have postage charges dependent on the weight only. (c) When a package is sent to a point five miles either side of a zone boundary the charges are uncertain since the zones are not true circles. For such a distance the charges become indeterminate. (d) If the weight is over fifty pounds and the distance over three hundred miles, or the weight over seventy pounds regardless of the distance, the charges become infinite.

8. Mr. Rainville gave an expository account of some of the simpler outstanding properties of the elliptic modular group and the allied functions. Landau's proof of the restricted Picard theorem and some results from the work of Landau and Caratheodory were used as examples of the type of application to the theory of functions. Stress was laid on the fundamental importance of the modular functions in the general theory of analytic functions. The paper was based, in the main part, on Klein's *Theorie der Elliptischen Modulfunktionen* (1890); and L. R. Ford's *Automorphic Functions* (1929).

9. Professor Kempner explained how a large number of the elementary concepts of the theory of numbers (residues, Fermat's theorem, exponent to

which a number belongs, indices, primitive roots, etc.) can in a very simple and satisfactory manner be demonstrated by means of a square table which for a given fixed prime modulus gives both the residues of a^λ for λ fixed, a variable; and for a fixed, λ variable.

Some apparently new results will be presented on another occasion.

10. Professor Jackson's paper discussed applications of the geometrical interpretation of correlation coefficients less simple than those which are treated in papers published in recent volumes of the Monthly. In particular, it gave a geometrical derivation of the regression coefficients for a problem involving three statistical variables.

The members and friends of the association were guests of the University of Denver at a banquet on the evening of April 11. President Gorrell acted as toastmaster. The address of welcome was given by Chancellor Frederick Hunter of the University of Denver. The response was given by Professor A. J. Kempner of the University of Colorado.

Following this a very interesting and instructive address was given by the guest of honor, Professor Dunham Jackson, on "The significance of elementary mathematics in modern statistics."

A. J. LEWIS, *Secretary*

THE SEVENTH ANNUAL MEETING OF THE INDIANA SECTION

The seventh annual meeting of the Indiana section of the Mathematical Association of America was held on May 2–3, 1930 at Earlham College, Richmond, Indiana.

There were forty-five present at the meeting including the following twenty-three members of the Association: W. C. Arnold, R. W. Babcock, Gladys L. Banes, G. E. Carscallen, P. T. Copp, C. S. Doan, J. E. Dotterer, W. E. Edington, P. D. Edwards, E. D. Grant, G. H. Graves, H. E. H. Greenleaf, C. T. Hazard, D. F. Heath, Cora B. Hennel, Florence Long, Juna M. Lutz, T. E. Mason, J. A. Reising, C. K. Robbins, L. S. Shively, K. P. Williams, W. A. Zehring.

On Friday at 5:30 P.M. a reception was given to the visiting members and their guests. At 6:30 P.M. a complimentary banquet which was held in the dining room of the college was attended by sixty guests of the college. Professor E. D. Grant presided at the banquet and introduced President Denny of Earlham College, who made a brief address of welcome. Music was provided during the banquet by a trio of students of the college.

At eight o'clock a short pipe organ recital was presented in Stoddard Auditorium. The public lecture of the evening, under the auspices of Earlham College, was given by Professor Louis C. Karpinski of the University of Michigan

THE FIFTEENTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The fifteenth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, Colo., on Friday and Saturday, April 17 and 18, 1931. There were three sessions, Professor Claribel Kendall presiding at each.

The attendance was forty-eight, including the following twenty-three members of the Association: C. F. Barr, Jack Britton, A. G. Clark, J. R. Everett, J. C. Fitterer, G. W. Gorrell, Sidney Hacker, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, O. C. Lester, A. J. Lewis, G. H. Light, S. L. Macdonald, A. S. McMaster, J. Q. McNatt, W. K. Nelson, Greta Neubauer, E. J. Purcell, E. D. Rainville, O. H. Rechard, Mary S. Sabin, C. H. Sisam.

At the business session the following officers were elected for the next year: Chairman, Professor O. H. Rechard, University of Wyoming; Vice-chairman, Professor A. G. Clark, Colorado Agricultural College. Plans were made to meet at the University of Wyoming in the spring of 1932.

The following papers were presented:

1. "On a biological application of the Poisson series" by Professor A. G. Clark, Colorado Agricultural College.
2. "A theorem on foci" by Mr. E. J. Purcell, University of Colorado.
3. "Projective geometry and some of its relations to other courses in mathematics and mathematical physics" by Professor J. R. Everett, Colorado School of Mines.
4. "Solution of a problem in dynamics" by Professor D. F. Gunder, Colorado Agricultural College, by invitation.
5. "On Riccati equations" by Professor C. A. Hutchinson, University of Colorado.
6. "Geometry as an avocation" by Professor A. J. Kempner, University of Colorado.
7. "The practical experiences of engineering in the mountain districts of Colorado" by Mr. J. Q. McNatt, Colorado Fuel and Iron Company.
8. "Linear systems of curves on an algebraic surface" by Professor C. H. Sisam, Colorado College.
9. "On the application of equations to the solution of congruences with prime moduli" by Mr. E. D. Rainville, University of Colorado.

Abstracts of some of the papers follow, the numbers corresponding to the numbers of the titles:

1. Professor Clark showed how the Poisson distribution could be used to simplify the work of the seed analyst in handling the problem of noxious weed seeds in certification work.

2. Mr. Purcell presented and proved the following theorem on foci, which he believes to be new: If a real algebraic curve of class M has one or more real

axes of symmetry, then in general exactly M foci lie on each axis of symmetry. Any such set of M collinear foci completely determines the remaining foci.

4. The problem of a particle flying off a fixed curve while sliding under the force of gravity is a familiar one to students of elementary mechanics. The corresponding problem when the force of friction is considered had been solved for various particular curves. Professor Gunder gave the solution of the problem for any curve considering friction and the particle sliding from any given position. The solution in the case of the circle was given as an illustrative example.

5. Professor Hutchinson's paper gave an exposition of the elementary theory of the Riccati equation; discussed a few points of contact with the general theory of differential equations; and exhibited graphically the continuity of the integral curves with respect to the constant of integration.

7. Mr. McNatt discussed the unusual situations met in surveying in mountain districts. He showed the application of mathematics to the solution of various problems met in such work.

8. This expository paper deals with the definition of a birational transformation of an algebraic surface; the invariant properties, under birational transformations, of linear systems of curves on an algebraic surface; the significance and invariance of the genera of the surface; the simple and double integrals of the first kind on the surface, and the meaning of irregularity and its connection with non-linear systems of curves on the surface.

9. Mr. Rainville's paper was an expository account of some of the results obtained by extending the congruence notation of the theory of numbers. Some simplifications in solution of certain congruences is effected. A few of the results are to be found in Gauss's works. The most important contribution is probably Poinsot's "Sur l'application de l'algèbre à la théorie des nombres."

A. J. LEWIS, *Secretary*

THIRTEENTH ANNUAL MEETING OF THE ILLINOIS SECTION

The thirteenth annual meeting of the Illinois Section of the Mathematical Association of America was held at the Bradley Polytechnic Institute, Peoria, Illinois, on May 1 and 2, 1931. The chairman, Professor H. B. Curtis, presided at all the sessions.

Forty-three persons registered attendance. Among these were the following twenty-nine members of the Association: Beulah Armstrong, Edith I. Atkin, H. W. Bailey, R. W. Barnard, Walter Bartky, C. E. Comstock, H. B. Curtis, Edna M. Feltges, Elinor B. Flagg, A. E. Gault, L. M. Graves, Martha Hildebrandt, Mildred Hunt, J. M. Kinney, W. C. Krathwohl, Luise Lange, Mayme I. Logsdon, W. D. MacMillan, C. N. Mills, G. E. Moore, E. J. Moulton, Mary W. Newson, W. A. Richards, Mina S. Rees, Mary B. Rumsey, E. W. Schreiber, H. E. Slaught, W. A. Spencer, C. A. VanVelzer, F. E. Wood.

The following officers were elected for the coming year: Chairman, R. W.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The sixteenth regular meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, Wyoming, on Friday and Saturday, April 15 and 16, 1932. There were three sessions, Professor O. H. Rechard, chairman of the Section, presiding at each.

The attendance was thirty-five, including the following thirteen members of the Association: Jack Britton, Pauline F. Folk, G. W. Gorrell, C. A. Hutchinson, M. H. Ingraham, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. S. McMaster, O. H. Rechard.

At the business meeting the following officers were elected for the ensuing year: Chairman, A. G. Clark, Colorado Agricultural College; Vice-Chairman, W. V. Lovitt, Colorado College; Secretary, A. J. Lewis, University of Denver.

Members of the Association and friends were guests of the University of Wyoming at a dinner on the evening of April 15. The principal speakers at the dinner were President A. G. Crane, of the University of Wyoming, and Professor S. L. Macdonald, of the Colorado Agricultural College. The Section was fortunate in having Professor M. H. Ingraham, of the University of Wisconsin, present as guest speaker.

The following seven papers were read:

1. "Operational calculus" by Professor C. A. Hutchinson, University of Colorado.

2. "Contributions of mathematics to life insurance" by Professor G. W. Gorrell, University of Denver.

3. "The Baire classification of functions" by Professor O. H. Rechard, University of Wyoming.

4. "The development of the postulational method in mathematics" by Professor M. H. Ingraham, University of Wisconsin.

5. "The teaching of calculus" by Professor S. L. Macdonald, Colorado Agricultural College.

6. "Geometric progressions" by Professor A. J. Kempner, University of Colorado.

7. "Controversial topics in mathematical logic" by Professor M. H. Ingraham, University of Wisconsin.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. This paper is expository in character, and presents the salient features of the operational calculus, as applied to the solution of problems in electrical engineering.

2. This paper gives a brief history of the problem of life insurance and shows the role mathematics has played in its development.

3. In this paper, Professor Rechard presents the Baire method of classifying functions as given by Baire in his dissertation "*Sur les functions de variables*

réelles" published in "Annali di Matematica Pura et Applicata" in 1899. Functions of classes one, two, and three are exhibited and the classical proofs are given that the classification can be correct to any number α of the first or second class, but is not exhaustive. Comparison is made between the Baire, Young, and Sierpinski methods of classification. Some of the questions which need to be answered before a necessary and sufficient condition can be found for a function to be of class two, Baire, are suggested.

4. This paper discusses the historical development of the postulational method, and the major characteristics and uses of this method. Especial attention is paid to the use of the method for generalization, in which case the postulates should be non-categorical, and for establishing isomorphisms in which case they should be categorical. As illustrations, the postulates for a field, for Euclidean geometry, and Huntington postulates for an arithmetic mean are used. It is pointed out that at least logic is generally assumed as a background for sets of mathematical postulates.

5. It is the belief of the writer that in an elementary course in calculus, definitions, principles and descriptive matter should be reduced to a minimum consistent with clearness and rigor. It is maintained in this paper that most text books are at fault in this particular. The paper maintains that the derivative is not a rate, it is not a slope. The derivative is the limit of a ratio and is an entirely abstract concept. Rate and slope are merely properties of the derivative. By making clear that in certain cases a distinction is necessary between the limit of a ratio and the ratio of the limits the writer holds that the definition of the derivative may be clarified, which is seldom done by text book writers.

7. This paper discusses some of the current attempts to examine and explain the relation of logic to mathematics. In particular three schools of thought are considered. 1) The school led by Russell which attempts to define all mathematics in logical terms. 2) The school led by Brouwer which makes mathematics prior to logic and places stringent limitations on the use of classical logic. 3) The school led by Hilbert which is interested in the formal structure of mathematics and the questions of formal consistency, and studies mathematics as a set of marks on paper which are made in accordance with certain rules. The attempt is made to give a sympathetic discussion of each of these three points of view.

A. J. LEWIS, *Secretary*

ERNEST JULIUS WILCZYNSKI

Ernest Julius Wilczynski was born in Hamburg, Germany, on November 13, 1876, and died in Denver, Colorado, on September 14, 1932, after a lingering illness of about ten years. With respect to his original contributions to existing mathematical knowledge, his influence on the development of mathematical institutions in the United States, his interest in the promotion of good teaching,

7. An expansion of x^m may be secured by Newton's advancing difference formula. The finite integral of this expression may be used to sum the mth powers of the first n natural numbers.

$$\sum_1^n x^m = \Delta^{-1}x^m \Big]_1^{n+1} = \Delta 0^m(n+1)_2 + \Delta^2 0^m(n+1)_3 + \Delta^3 0^m(n+1)_4 + \cdots$$

The terms of this expansion are symmetrical and the coefficients are easily determined. This expression is somewhat easier to use than the expansions usually found.

8. The principal object of this paper was a classification of triads of points by means of invariants of the cubics whose roots are co-ordinates of the points in the Argand diagram. The cases treated included those of coincident and collinear points, and points that are the vertices of equilateral, isosceles, and right triangles. A brief discussion was also given of certain parametric equations of central conics.

9. In an address illustrated by lantern slides, Professor Davis showed how the periodic structure of economic series (for example, stock market averages, pig iron production indices, price averages, etc.) can be studied by means of periodogram analysis, difference equations, and the harmonic analysis of lag-correlations. It was shown that high correlations between such series as the stock market average and the pig iron production index are due in large part to coincident periods in the two series. The analysis of lag-correlations reveal these coincident periods, while the analysis of systems of difference equations reveal the non-coincident periods as elastic interactions. In discussing the dangers of too confident prediction in economic matters, Professor Davis showed that a random series smoothed by a twelve-months' moving average had a period almost exactly equal to the fundamental period of the Bradstreet commodity price index. He also touched on the significance of straightline and logistic trend lines and showed how the present decline in the price index was a phenomenon probably predictable on the basis of long-time trend lines. The computations involved in the paper were furnished by the laboratory of the Cowles Commission for Research in Economics.

RUFUS CRANE, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The seventeenth annual meeting of the Rocky Mountain Section was held at Colorado Agricultural College, Fort Collins, Colorado, on Friday and Saturday, April 14–15, 1933. Professor A. G. Clark presided at each of the three sessions.

The attendance was thirty-five, including the following twenty-one members of the Association: C. F. Barr, Jack Britton, A. G. Clark, I. M. DeLong,

J. C. Fitterer, G. W. Gorrell, I. L. Hebel, C. A. Hutchinson, Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, Mary S. Sabin, C. H. Sisam.

Members and friends of the Association were guests of the College on the evening of April 14. At the business session, Professor C. H. Sisam of Colorado College, was elected Chairman for the coming year, and Professor J. C. Fitterer, of Colorado School of Mines, was elected Vice-Chairman.

The following ten papers were read:

1. "A solution of a system of linear matrix equations in two unknowns," by Miss Rachel Achenbach, University of Wyoming, by invitation.

2. "On foci of algebraic curves with applications to cubic curves" (Thesis presented by Ethel A. Rice for M. A. University of Colorado) by Professor Claribel Kendall, University of Colorado.

3. "Entropy, strain and the Pauli exclusion principle" by Professor Guy Berry, Colorado Agricultural College, by invitation.

4. "Notes on Riccati's differential equations" by E. D. Rainville, University of Colorado.

5. "A theorem on point-wise discontinuous functions" by Professor O. H. Rechard. University of Wyoming.

6. "On Graeffe's method of solution of algebraic equations" by Professor C. A. Hutchinson, University of Colorado.

7. "The use of calculators in solving Kepler's problem" by Professor A. W. Recht, University of Denver.

8. "The 'Zig' function of Wirth" by Professor C. F. Barr, University of Wyoming.

9. "The solution of algebraic equations by infinite series" by Professor A. J. Lewis, University of Denver.

10. "Symmetric functions and resultants" by Professor C. H. Sisam, Colorado College.

Abstracts of the papers follow below, the numbers corresponding to the numbers in the list of titles:

1. The purpose of this paper is to develop methods of solution for the sixteen systems of linear matrix equations in two unknowns which result from the corresponding ordinary algebraic system when the constants and unknowns in these latter equations are replaced by matrices of the nth order. Two of the systems yielded to solution by the methods of elimination by addition and by substitution. The other fourteen systems were reduced to a single standard form which was solved by the use of the Hamilton-Cayley equation.

2. In this paper, Miss Rice was particularly interested in the graphical representation of the location of the real foci of certain cubic curves. For practical purposes in the matter of computation the cubic curves chosen were sym-

metric with respect to a point or an axis. The foci of some twenty-four such cubics of varying types were given. The discussion of these special cubics was preceded by a summary of the known results concerning the number of foci of algebraic curves in general.

3. The author shows, by using the geometrical weight method developed by Kimball for an ideal gas, that the entropy of a real gas is proportional to the strain, and that the equations for maximum entropy are equilibrium equations between stress and strain. The velocity distribution function is the same as that of the Fermi-Dirac statistics. This method offers an explanation of the second law of thermodynamics and the Pauli exclusion principle.

4. In the general Riccati Equation $dy/dx = A_0(x) + A_1(x)y + y^2$, it is most desirable to obtain a simple particular solution. If this is available, the complete solution follows almost at once. When A_0 and A_1 are polynomials, it is reasonable to search for polynomial solutions. In these notes Mr. Rainville shows that never more than one or two polynomials, for which simple formulas are given, need be tested as trial solutions. Some extensions varying the functional form of A_0 and A_1 are treated.

5. In this paper there is presented a statement and proof of the following theorem: Given a function, $f(x)$, continuous over a residual point set on an interval (a, b); if the function is defined at the remaining points of the interval by the "closest approximating function" method, it will be point-wise discontinuous on the interval.

6. This is the paper read by title at the Los Angeles Meeting of the Association. An abstract appeared in this MONTHLY, November 1932, p. 503; and the complete paper will appear in a later issue.

7. A demonstration showing how J. Peter's 7-place table of natural trigonometric functions, in which the argument is given in decimals of a degree, combined with the modern calculator, greatly facilitate the solution of Kepler's well-known equation $M = E - e \sin E$.

8. This paper presents the function

$$\operatorname{zig} u = 2\{[R_{u/2}] + (-1)^{[R_{u/2}]} R_{u/2}\}$$

and its derivative; in which $u = f(x)$, $[s]$ means the greatest integer in s, and R_u means $u - [u]$. Its adaptibility to special configurations was demonstrated by using it in the polar equation

$$\rho = a \cos\frac{\pi}{n} \Big/ \cos\left\{\frac{\pi}{n}\left(1 - \operatorname{zig}\frac{n\theta}{\pi}\right)\right\}$$

to map a regular n-sided polygon in a circle of radius a. The function zig u was developed and named by Mr. Don Wirth.

9. This paper outlines methods of expressing all the roots of an algebraic equation by infinite series and formulates conditions of convergence for these series.

10. This paper deals with a proof of the possibility of representing a given integral symmetric function in terms of the elementary symmetric functions and with a method of determining this representation by the use of resultants.

A. J. LEWIS, *Secretary*

THE TWENTY-SECOND MEETING OF THE IOWA SECTION

The twenty-second meeting of the Iowa Section of the Mathematical Association of America was held with the Iowa Academy of Science at Coe College, Cedar Rapids, Iowa, on Friday and Saturday, April 21 and 22, 1933. The meetings were held in room 117, Science Hall.

The attendance was about fifty, including the following twenty-one members of the Association: R. P. Baker, E. W. Chittenden, L. M. Coffin, N. B. Conkwright, C. W. Emmons, Cornelius Gouwens, M. E. Graber, I. J. Gwinn, Gertrude A. Herr, Dora E. Kearney, O. C. Kreider, F. M. McGaw, Arthur Ollivier, J. F. Reilly, H. L. Rietz, W. J. Rusk, E. R. Smith, C. W. Strom, John Theobald, L. E. Ward, Roscoe Woods.

The Section Chairman, Professor L. M. Coffin, presided at both the Friday afternoon and Saturday morning sessions. Dinner was enjoyed together Friday evening in the Jefferson Room, Hotel Roosevelt. The officers elected for 1933–1934 are as follows: Chairman, J. F. Reilly, University of Iowa; Vice-Chairman, M. E. Graber, Morningside College; Secretary-Treasurer, Cornelius Gouwens, Iowa State College.

A committee consisting of Professors R. P. Baker and Roscoe Woods prepared the following statement relative to the death of Daniel Kreth: "Daniel Kreth, engineer and surveyor, of Wellman, Iowa, a charter member of the Association, died in 1932. From 1914 to 1924 Mr. Kreth was an active contributor to the Monthly of problems and solutions. His interest in mathematics showed itself not only by his activities in the Association but also in the collection of a library. The Iowa Section laments his passing as a member and feels keenly the loss of inspiration which comes from knowing a man who derived a great deal of pleasure from his study of mathematics."

The program consisted of fourteen papers, as follows:

1. "On the resolution of $4X = Y^2 - (-1)^{(p-1)/2}\, pZ^2$ for $p = 67$, 71, and $X = (x^p - 1)/(x - 1)$" by Professor Cornelius Gouwens, Iowa State College.

2. "Some properties of the logarithmic potential" by Professor J. J. Westemeier, Des Moines Catholic College, by invitation.

3. "The teaching of the trigonometric functions of $2x$ and of $x/2$" by Professor Roscoe Woods, University of Iowa.

4. "Sophus Lie's geometry of imaginaries" by Professor M. E. Graber, Morningside College.

5. "A problem in simple harmonic motion of a particle moving in a medium of varying density" by Robert MacAllister, Wartburg College, by invitation.

R. G. STURM, M.S. (Illinois) Research Engineer, Physicist, Aluminum Research Labs., New Kensington, Pa.

V. B. TEMPLE, A.M. (Texas) Head of Dept., Louisiana Coll., Pineville, La.

H. M. TENNEY, B.S. (Greenville Coll.) Instr., Chem. and Math., Greenville Coll., Greenville, Ill.

S. B. TOWNES, A.M. (Oklahoma) Asst. Prof., Univ. of Oklahoma, Norman, Okla.

J. F. WARDWELL, A.B. (Hamilton) Jr. Instr., Johns Hopkins Univ., Baltimore, Md.

MARGARET C. WEEBER, A.M. (Chicago) Asst. Prof., Hood Coll., Frederick, Md.

E. V. WHITE, A.M. (Baylor) Dean, Head of Dept., Texas State Coll. for Women, Denton, Texas.

G. H. WILSON, A.M. (Pennsylvania) Instr., Physics, Univ. of Delaware, Newark, Del.

W. F. WINN, Radio Operator, Sabine Transportation Co., Port Arthur, Texas.

Reports of progress were made on the two commissions of the Mathematical Association and a plan which might provide adequately for funds needed to carry out their plans as they are being developed.

W. D. CAIRNS, *Secretary-Treasurer*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The eighteenth regular meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado College, Colorado Springs, Colorado, on Friday and Saturday, April 20–21, 1934.

There were three sessions. Professor C. H. Sisam of Colorado College presided at each.

The attendance was forty-five, including the following twenty-four members of the Association: C. F. Barr, Jack Britton, A. G. Clark, I. M. DeLong, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, Mary S. Sabin, C. H. Sisam.

The members and friends of the Section were guests of the College at a banquet on the evening of April 20. At the business session on Saturday, Professor J. C. Fitterer of Colorado School of Mines was elected Chairman for the coming year. Professor A. W. Recht of the University of Denver was elected Vice-Chairman.

The following ten papers were read:

1. "A note on polynomial curves" by Jack Britton, University of Colorado.

2. "A mathematical analysis of the hardening of copper" by J. D. Keyes, Montana School of Mines. (Read by the secretary in the absence of the author.)

3. "The Schwarz-Christoffel transformation as applied to the solution of certain problems in elasticity" by Professor D. F. Gunder, Colorado Agricultural College.

4. "A problem in magic squares" by Professor W. K. Nelson, University of Colorado.

5. "Fundamentals of the trial load method for stress analysis of arched dams" by R. E. Glover, United States Reclamation Bureau, by invitation.

6. "On the Schwarz-Christoffel transformation" by Professor C. A. Hutchinson, University of Colorado.

7. "Solution of a problem in heat conduction" by E. D. Rainville, Junior Engineer, United States Reclamation Bureau.

8. "A survey course in mathematics" by Professor O. H. Rechard, University of Wyoming.

9. "Complex numbers" by Professor A. J. Kempner, University of Colorado.

10. "Evaluation of certain expectancies" by Professor A. G. Clark, Colorado Agricultural College.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. The paper given by Mr. Britton is concerned with the following problem: Under what restrictions may we assign all the abscissas and two of the ordinates of the extremes (maxima and minima) of the polynomial curve $y=a_0x^n+\cdots+a_n$, which is to have the maximum number of extremes? This paper will appear in an early issue of this MONTHLY.

2. Recent investigations at Montana School of Mines by J. D. Keyes, assisted by C. L. Wilson, indicate that the natural law governing the precipitation hardening of copper may be expressed by the equation

$$R = a(T + b)^{-1} + c,$$

where R is the resistivity of the copper being hardened, T is the time of annealing, and a, b, and c are arbitrary constants. Theoretical approach to the problem resulted in failure, but an empirical approach based upon correspondence between certain arithmetical and geometrical series, resulted in the above equation. No effort was made to derive a formula that would fit data on the problem; the effort was made, however, to satisfy a fundamental requirement of a natural law: if corresponding values of the variables within a very limited range are known, and if the natural law is also known, then all other corresponding values of the variables may be found by extrapolation. Tests of extrapolation made with the above equation gave satisfactory results.

3. Professor Gunder gave a brief expository discussion of the value of the Schwarz-Christoffel transformation as applied to the solution of boundary value problems in which the area for which the solution is to be obtained is a polygon. He followed this with an example which gave the solution of the flexure problem for an elastic beam of rectangular cross section but with two symmetrical horizontal slits extending inward along the central line of the section.

4. Professor Nelson discussed a 3×3 magic square which has 9 as its center number. The remaining eight positions are to be filled so that the sum of the rows, columns and diagonals will be 27, with no element greater than 15. The solutions, four in number, were obtained by a graphical interpretation, transformations and inequalities.

5. In this paper, R. E. Glover described the methods devised by engineers of the U. S. Reclamation Bureau for calculating the stresses in such structures. The process is one of successive approximation involving the use of trial loads of various types which are applied for the purpose of satisfying the boundary conditions, and meeting the equilibrium and continuity conditions throughout the structure. The requirements for a satisfactory solution were examined with the aid of Kirchhoff's uniqueness theorem in the theory of Elasticity.

6. A brief outline of the application of the conformal transformation of Schwarz and Christoffel to problems of electric machine design was given.

7. Mr. Rainville considered the problem of the conduction of heat in a plane wedge with special reference to placing the formulas in a form suitable for computation of the temperature history of large dams. The boundary conditions were taken in such a manner that use can be made of the known fact that bedrock is at mean annual air temperature.

8. In this paper Professor Rechard suggested the need for a course in mathematics for students who do not wish to specialize in mathematics but who do wish to know something about the rôle this science has played and is playing in the history of the race. As a basis for discussion he outlined a course given during the winter quarter at the University of Wyoming. The two main points in the outline were the development of elementary mathematics as a human interest story, and the presentation of the foundations of mathematics in such a way as to emphasize the fact that mathematics is an invention of the human intellect.

9. Professor Kempner made some remarks, partly of a pedagogical character, concerning the distinction between "absolute value" and "distance" in the theory of complex numbers.

10. In this discussion, Professor Clark developed some of the most important methods for finding bounds for the expectancies of various functions with reference to distribution functions to which but very mild restrictions are applied. The methods were extended to cover cases of more than one variable and various examples of their application and interpretation were given.

A. J. LEWIS, *Secretary*

THE SECOND ANNUAL MEETING OF THE WISCONSIN SECTION

The second annual meeting of the Wisconsin Section of the Mathematical Association of America was held at the Oshkosh State Teachers College on Saturday, May 5, 1934. The Chairman of the Section, Professor G. A. Parkinson, presided.

The attendance was sixty-eight, including the following twenty-five members of the Association: Leon Battig, Ethelwynn R. Beckwith, May M. Beenken, Theodore Bennett, H. H. Conwell, L. A. V. DeCleene, Margaret C. Eide, H. P. Evans, M. L. Hartung, R. C. Huffer, M. H. Ingraham, Elizabeth E.

of d gives a set to go with k. Using the formula, $(n+d)^2-n^2=(2k)^2$, where $2k$ is any even integer, we see that d is an even divisor of $4k^2$, less than $2k$, and the quotient of $4k$ by d must also be even.

2. Professor Townes gave a geometric description of a problem in number theory. The integers were represented by the vertices of a lattice work.

3. In his paper Professor Heimann pointed out that the greatest problem facing colleges today is that of working out a closer correlation between high school and college mathematics, so that a student may be able to continue his mathematical training in college without any loss of time. College standards must be maintained by stating definitely where college mathematics should begin and by refusing to give credit for work below that level. Finally, the methods of teaching have changed as well as the emphasis on teaching skill.

4. Professor Johnson gave a historical sketch of the problem of constructing a square so each side should pass through a given point. All published solutions to this problem are essentially of two types. It was shown how to construct a particular one of the six solutions which this problem has in general. When there is an infinite number of solutions order may be so introduced into the problem that four of the solutions differ from the remainder, also the locus of the centers of the squares is a circle. Interesting properties of this circle were discussed. If the four given points constitute an orthocentric group the circle is the nine-point circle of the orthocentric group.

E. F. ALLEN, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The nineteenth regular meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado School of Mines, Golden, Colorado, Friday and Saturday, April 19 and 20, 1935.

There were three sessions, Professor J. C. Fitterer presiding at each.The attendance was fifty-two including the following twenty members of the Association: L. A. Aroian, C. F. Barr, Jack Britton, J. R. Everett, J. C. Fitterer, G. W. Gorrell, I. L. Hebel, Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, C. H. Sisam.

Professor A. W. Recht of the University of Denver was elected chairman for next year. The next meeting was scheduled for some time in April 1936 at the University of Denver. Members and friends of the Association were guests of the School of Mines at a banquet held the evening of April 19. The following eight papers were read:

1. "Teaching science in mathematics" by Professor J. C. Stearns, University of Denver, introduced by Professor Lewis.

2. "The Pearsonian system of frequency curves" by Professor L. A. Aroian, Colorado State College.

3. "Remarks on a boundary value problem of the heat equation" by E. D. Rainville, Junior Engineer, U. S. Bureau of Reclamation.

4. "The use of convergent series in the evaluation of integrals with infinite limits" by Professor J. R. Everett, Colorado School of Mines.

5. "Greatest integers" by Professor W. K. Nelson, University of Colorado.

6. "A graphic solution of complex roots" by Professor C. F. Barr, University of Wyoming.

7. "Conformal mapping in hydrodynamics" by Professor C. A. Hutchinson, University of Colorado.

8. "Waring's problem and Diophantine equations with inequality conditions" by Professor A. J. Kempner, University of Colorado.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles:

2. Professor Aroian developed the differential equation of the Pearsonian system on the usual assumptions. The main types were discussed and examples of them from actual practise were given.

4. Mr. Rainville's paper discussed two distinct functions, apparently both of them solutions to the same mixed boundary value problem of the one dimensional heat equation. The need for a careful definition of what is meant by a solution to such a problem was noted. A tentative definition was adopted, without, however, any existence or uniqueness theorems to substantiate the conjecture.

5. Professor Nelson discussed greatest integers of bracketed numbers with regard to the interchanging of the operation of applying the bracket with the operations of adding, multiplying and raising to integral powers. The definite integral of a function of x and the definite integral of the bracket of the same function of x were compared. Formulas in terms of greatest integers were given for the Colorado sales tax and for service charges for checking accounts in Denver banks.

6. A brief section of Professor Barr's paper presented a circle method for representing the complex roots of a quadratic equation. Its purpose was to extend the construction given in Dickson's Theory of Equations to include complex roots. The body of the paper was devoted to a graphic construction for the complex roots of cubic equations with but one real root. The method involved only the graph of the cubic and the drawing of two straight lines. One of these lines was a properly selected member of the pencil of lines through the intersection of the curve with the x-axis. The other was the vertical locus of the midpoints of the chord formed by the curve and its other real intersections with the lines of this pencil. The real part of the complex roots is given by the x-position of this vertical line, and the coefficient of the imaginary part by the horizontal distance from this line to the intersection of the curve with this selected line of the pencil. The selection of the line is made by doubling the slope of the tangent to the curve from its real x-intercept.

The major part of the Saturday morning session was given over to a sym-

posium on "The secondary mathematics situation." Professor C. A. Hutchinson of the University of Colorado acted as chairman for the symposium. The leaders were Professor C. H. Sisam, Colorado College; Professor A. E. Mallory, Colorado State College of Education, by invitation; and Mr. H. W. Charlesworth, East High School, Denver, by invitation.

A. J. LEWIS, *Secretary*

THE ANNUAL MEETING OF THE TEXAS SECTION

The annual meeting of the Texas Section was held at the Texas Technological College, Lubbock, Texas, on April 20, 1935.

Among the forty persons attending the meeting were the following twenty-two members of the Association: J. H. Binney, E. O. Box, H. E. Bray, J. E. Burnam, Nat Edmonson, E. L. Harp, Jr., J. A. Hurry, Roy MacKay, Lida B. May, J. N. Michie, E. D. Mouzon, Jr., C. A. Murray, W. L. Porter, P. K. Rees, C. R. Sherer, F. W. Sparks, Ruth W. Stokes, Jennie L. Tate, E. L. Thompson, F. E. Ulrich, R. S. Underwood, C. N. Wunder.

At noon those attending the meeting were, jointly with the Texas Section of the Society for the Promotion of Engineering Education, the guests of the Lubbock Chamber of Commerce at an old-fashioned barbecue. The Texas Technological College entertained those attending the meeting at a dinner in the evening following the meeting. President Bradford Knapp of the College was the principal speaker.

At the business session following the presentation of papers the following officers were elected for the coming year: Chairman, F. W. Sparks, Texas Technological College; Vice-Chairman, L. R. Ford, Rice Institute; and it was voted to make the office of Secretary-Treasurer a permanent one, with Nat Edmonson, Agricultural and Mechanical College of Texas, continuing in the office. It was also voted to hold the 1936 meeting of the Section at the A. and M. College, College Station, in conjunction with the meeting of the Texas Section of the S.P.E.E.

The following papers were read:

1. "Non-unique solutions of ordinary differential equations" by O. H. Hamilton, San Antonio Junior College, introduced by Professor Sparks.
2. "An expansion for an n-dimensional determinant" by Professor F. W. Sparks, Texas Technological College.
3. "Roots of the derivative of a polynomial" by Professor H. E. Bray, Rice Institute.
4. "Generalized Vandermonde determinants," second paper, by E. R.Heineman, Texas Technological College, introduced by Professor Porter.
5. "Symbolic cubic forms in six variables" by Professor Ruth W. Stokes, North Texas State Teachers College.
6. "Rational fractions" by W. L. Scott, Rice Institute, introduced by Professor Bray.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The twentieth regular meeting of the Rocky Mountain Section was held at the University of Denver, Denver, Colorado, on April 17–18, 1936. There were three sessions. Professor A. W. Recht presided at each.

The attendance was forty-one, including the following twenty-five members of the Association: L. A. Aroian, Jack Britton, Sister Rose Margaret Cook, W. D. Dickinson, Jr., J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. E. Mallory, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, C. H. Sisam, W. M. Stewart.

Professor A. E. Mallory, Colorado State College of Education, was elected chairman for the coming year. Professor C. A. Hutchinson, University of Colorado, was elected vice-chairman.

The following papers were read:

1. "Thiele's semi-invariants and their application to problems in statistics" by Professor L. A. Aroian, Colorado State College.

2. "The separation of the roots of the trinomial equation" by Professor A. J. Lewis, University of Denver.

3. "Theorems on a classical heat problem" by E. D. Rainville, United States Bureau of Reclamation.

4. "On the determination of the coefficients of the Kreisteilungs-Gleichung" by Professor A. J. Kempner, University of Colorado.

5. "Four notes on the solution of systems of linear matrix equations in two and three unknowns" by Professor O. H. Rechard, University of Wyoming.

6. "The mathematics of ancient China" by Professor I. L. Hebel, Colorado School of Mines.

7. "Pohlke's theorem in four dimensions" by Professor C. H. Sisam, Colorado College.

8. "The analytic discussion of the locus of the radical center of three circles related to a triangle" by W. M. Stewart, University of Wyoming.

9. "The projective generation of curves and surfaces" by Professor Claribel Kendall, University of Colorado.

Abstracts of the papers and discussions follow below, the numbers corresponding to the numbers in the list of titles:

1. The main properties of semi-invariants were derived by Professor Aroian and applied to the following types of distributions: normal, binomial, Poisson, and Pearson type III. The results were further applied to the Gram-Charlier distribution, and to the problem of the distribution of means in samples of N from an infinite population. The exposition was based on class notes of Professor C. C. Craig.

2. In this paper Professor Lewis shows a method of separating the roots of

a trinomial equation by dividing concentric rings, having the origin of the complex plane as center, into equal sections each containing just one root of the given equation.

3. Mr. Rainville considered what may be a new solution for the problem of the conduction of heat in an infinite slab initially at a constant temperature and with its surfaces held thereafter at a different constant temperature. The newer solution presents marked advantages for purposes of extensive computations as compared to the classical solution of Fourier.

4. A very simple mechanical rule was established by Professor Kempner for the determination of the coefficients of the irreducible Kreisteilungs-Gleichung.

5. In 1933 Miss Achenbach presented a method for finding the unique solution of each of the sixteen systems of linear matrix equations which may be obtained from the system $A_iX+B_iY=C_i(i=1, 2)$, by permitting the coefficients A_i, $B_i(i=1, 2)$ to be transferred from left to right-hand multipliers of their respective unknowns in all possible combinations one, two, three, and four at a time. In solving the simple system in which all the coefficients are on the left, she included among the restrictions on the system that the matrices A_i, B_i $(i=1, 2)$ be non-singular. Professor Rechard presented four notes concerning this system of equations.

Note I shows that a solution of the system can be found even if one of the four coefficients is singular. Note II establishes in general a solution, other than the trivial one in case C_1 and C_2 are both zero. Note III points out the fact that the sixteen systems treated are all special cases of the general one, $A_iXB_i+C_iYD_i=E_i(i=1, 2)$. This inclusive system yields to the method employed to solve the fourteen systems in which all the coefficients are not on the same side of the unknowns. In Note IV the solution for a system in three unknowns, in which the coefficients are all on the same side of their respective unknowns, is developed under suitable restrictions on the singularity of the matrices involved.

6. To compare the mathematical development of early China with that of other nations, the ancient "Arithmetic in Nine Sections" was reviewed by Professor Hebel. He gave the mensuration rules of the early Chinese and their processes for the manipulation of fractions and the solution of certain systems of equations in more detail than is given in the usual mathematical histories, thus establishing the Chinese among the pioneers in mathematical science.

7. Pohlke's theorem, in four dimensions, was discussed by Professor Sisam. This theorem can be stated as follows: Let OP_i $(i=1, 2, 3, 4)$ be any four given line segments originating at 0 and lying in a space π of three dimensions. There exist four equal, mutually orthogonal segments $O^*P_i^*$, lying in a four dimensional space that contains π, from which the given segments may be obtained by a sequence of (at most) two parallel projections of which the first projects the four orthogonal segments on a three-space π' and the second is orthogonal to π'.

8. Mr. Stewart discussed the following locus problem: Let a general triangle with the vertices A, B and C, and the opposite sides a, b and c respectively, be considered. Let any straight line l intersect the sides, or the sides extended, in the finite points a', b' and c' respectively. Then let three circles be drawn; the first with center at A and radius Aa'; the second with center at B and radius Bb'; and the third with center at C and radius Cc'. Determine the locus of the radical center of these three circles, as the line l rotates about a finite fixed point (x_j, y_j), and the asymptotes of this locus and find the locus of this radical center when the point (x_j, y_j) moves to infinity in various directions. Results were obtained by elementary methods for the general situation and certain limiting cases.

9. In this expository paper Professor Kendall discussed in some detail the projective generation of point-rows of the second order from two non-concentric, non-perspective pencils of lines lying in the same plane, and the dual problem concerning sheaves or pencils of rays of the second class, these loci being identified with our ordinary curves of the second order. Following this she discussed the generation of curves of the third order formed from the points of intersection of corresponding rays of a sheaf of rays of the first order projective to a sheaf of rays of the second order. Brief mention was made of the extension of these methods of generating curves of higher order and of generating surfaces.

A. J. LEWIS, *Secretary*

THE SPRING MEETING OF THE MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA SECTION

The spring meeting of the Maryland-District of Columbia-Virginia Section of the Mathematical Association of America was held at the United States Naval Academy, Annapolis, Maryland, on Saturday, May 9, 1936. The Chairman, Professor G. T. Whyburn, of the University of Virginia, presided over both sessions, morning and afternoon. Commander J. A. Logan, executive officer and acting head of the Post-graduate School of the U. S. Naval Academy, officially welcomed the members and their guests. Five papers were presented at the morning session, while in the afternoon, at the invitation of the Section, Professor Tobias Dantzig of the University of Maryland delivered a lecture entitled "Some curious aspects of mathematical history."

The attendance was sixty-nine including the following forty-three members of the Association: O. S. Adams, N. H. Ball, C. C. Bramble, Paul Capron, Randolph Church, G. R. Clements, Abraham Cohen, A. E. Currier, J. H. Curtiss, Alexander Dillingham, J. A. Duerksen, P. J. Federico, Michael Goldberg, Harry Gwinner, F. E. Johnston, L. M. Kells, W. D. Lambert, A. E. Landry, C. L. Leiper, S. B. Littauer, G. A. Lyle, Carol V. McCamman, E. J. McShane, Ruth G. Mason, Florence M. Mears, T. W. Moore, F. D. Murnaghan, J.L. Nagle, Walter Penney, C.H. Rawlins, A.W. Richeson, R.E. Root,

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-first annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College of Education, Greeley, Colorado, on Friday and Saturday, April 16 and 17, 1937.

There were three sessions. Professor A. E. Mallory presided at each of the Friday sessions and at the business meeting Saturday morning. Professor C. A. Hutchinson presided during the Saturday morning program. Professor W. L. Hart of the University of Minnesota was the guest speaker.

The attendance was sixty-six, including the following twenty-four members of the Association: L. A. Aroian, C. F. Barr, J. R. Britton, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, W. L. Hart, I. L. Hebel, C. A. Hutchinson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. E. Mallory, A. S. McMaster, W. K. Nelson, Greta Neubauer, E. D. Rainville, O. H. Rechard, A. W. Recht, C. H. Sisam, and W. M. Stewart.

The following officers were elected for the coming year: Chairman, C. A. Hutchinson, University of Colorado; Vice-Chairman, C. F. Barr, University of Wyoming.

The members and friends of the Association were guests of Colorado State College of Education at a dinner Friday evening. The Saturday morning session was a joint meeting with the National Council of Teachers of Mathematics.

The following thirteen papers were read:

1. "A method of finding a solution of a system of three linear matrix equations in three unknowns" by Professor O. H. Rechard, University of Wyoming.
2. "Mechanical and graphical calendars" by Professor W. K. Nelson, University of Colorado.
3. "Note on an operational formula" by Professor C. A. Hutchinson, University of Colorado.
4. "Concyclic points and some allied configurations" by Professor C. H. Sisam, Colorado College.
5. "Certain distributions with binomial series as components" by Professor A. G. Clark, read by Professor L. A. Aroian, Colorado State College.
6. "Elementary remarks on Moebius's barycentric calculus" by Professor A. J. Kempner, University of Colorado.
7. "Inexact mathematics" by Professor W. L. Hart, University of Minnesota, by invitation of the Section.
8. "Statement of problems regarding high school mathematics—a report" by Professor C. A. Hutchinson, University of Colorado, and H. W. Charlesworth, East High School, Denver.
9. "Can mathematical values be measured?" by Professor O. H. Rechard, University of Wyoming.
10. "Who should study mathematics? What should be studied?" by Professor A. E. Mallory, Colorado State College of Education, and Ethelyn W. Rhiner, Greeley Public Schools.

11. "Some curriculum questions" by Professor C. H. Sisam, Colorado College.

12. "Remarks on the calculation of π" by Marjorie H. Beaty, University of Colorado, introduced by Professor Hutchinson.

13. "The trend in secondary mathematics and associated collegiate phenomena" by Professor W. L. Hart, University of Minnesota.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. In considering the system $A_iXB+C_iYD+E_iZF=C_i(i=1, 2, 3)$ in which all the matrices are square and the constant matrices are non-singular, it is found that a solution in terms of the constant matrices involves the solution of a k-termed linear matrix equation in a single unknown, k, a function of the order of the matrices. Since, so far as Professor Rechard has been able to determine, the value of the unknown in such an equation has been described in terms of the constant matrices in the equation only for the cases $k=1, 2$, it is necessary in order to solve the system under consideration to transform the problem of solving this latter equation to that of solving an ordinary algebraic system of n^2 linear equations in n^2 unknowns. Professor Rechard showed how this could be accomplished and illustrated the method for a system in which the matrices were all of second order.

2. Any mechanical or graphical device for adding numbers might be used in making a perpetual calendar. Two calendars designed by Professor Nelson, one a desk calendar and the other a slide rule calendar of form to carry in a notebook, were demonstrated. The desk calendar may be set to show the year, name of month, and calendar for the month for any month in the twenty-four hundred year period beginning with 1 A.D. The notebook calendar may be set to show the calendar for a year at a time for any year of the same period.

3. Professor Hutchinson's note appears in this MONTHLY, pages 371–2.

4. Professor Sisam gave a simple method of deriving a series of formulas and identities arising from certain configurations of points in the plane and in space.

5. Professor Clark considered the distribution of total successful occurrences of a set of trials of an event, where, although the individual trials have an equal constant probability of successful outcome, such an outcome in an individual trial is counted as of a certain multiplicity preassociated with that trial. It was found that the resulting distribution of total successes could be broken into components which are simple binomial series. Relations were derived for the movements of such a distribution in terms of p, the constant probability of success, and r_j $(j=1, 2, \cdots, s)$, the multipicities associated with the s trials of the event.

7. Professor Hart emphasized the fact that, although mathematics has important uses for formulas and equalities, on the other hand there is a major section of analysis where, in contrast, features such as inequalities and successive approximations are met, and he suggested the importance of this contrast

in overcoming the restricted formal viewpoint of more elementary mathematics.

12. Miss Beaty gave a comparison between the method of isoperimeters and that of equal areas.

13. In this paper Professor Hart discussed proposals about secondary mathematics connected with existing tendencies to socialize the secondary curriculum. He labeled any approach to this problem as illogical if it involves a simultaneous attack on college entrance requirements. Regardless of the pertinence of such an attack, he advanced criticisms of the technique of some of its supporting educational research and suggested that it merely establishes the consistency of established entrance requirements and course prerequisites. Finally, he rested the case for secondary mathematics largely on the existing prerequisites, either tangible or intangible, for college courses and curricula, rather than on inflexible entrance requirements.

A. J. LEWIS, *Secretary*

THE APRIL MEETING OF THE SOUTHEASTERN SECTION

The fifteenth annual meeting of the Southeastern Section of the Mathematical Association of America was held in Nashville, Tennessee, on Friday and Saturday, April 16–17, 1937. There were in attendance about one hundred fifty persons from thirty-six institutions, including the following forty-eight members of the Association: H. G. Ayre, D. H. Ballou, W. S. Beckwith, R. V. Blair, Iris Callaway, M. G. Carman, R. D. Carmichael, T. C. Carson, Edna J. Cofield, W. A. Cordrey, H. M. Cox, Forrest Cumming, L. A. Dye, W. W. Elliott, D. C. Harkin, M. A. Hill, Jr., P. R. Hill, P. M. Hummel, W. R. Hutcherson, R. O. Hutchinson, J. A. Hyden, J. B. Jackson, Rosa L. Jackson, H. T. Karnes, G. B. Lang, F. A. Lewis, J. F. Locke, A. N. McPherson, Nellie P. Miser, W. L. Miser, W. A. Moore, J. S. Morrel, Mabel I. Nowlan, W. P. Ott, K. B. Patterson, D. D. Peele, W. W. Rankin, H. A. Robinson, J. A. L. Saunders, W. E. Sewell, A. R. Sloan, F. H. Steen, R. P. Stephens, Ruth W. Stokes, D. L. Webb, W. L. Williams, F. L. Wren, J. T. C. Wright, E. Kathryn Wyant.

Sessions were held the afternoon of the 16th at George Peabody College, and the evening of the 16th and the morning of the 17th at Vanderbilt University. Chairman W. W. Rankin presided, except Friday evening and part of Saturday morning when the Section was divided into subgroups according to the nature of the papers presented. Subgroups were presided over by Vice-Chairman J. B. Jackson, Dean C. M. Sarratt and Professor W. P. Ott. On the evening of the 16th a dinner was held in honor of the visiting speaker, Dean R. D. Carmichael of the University of Illinois. At this time Professor F. L. Wren presided.

At the business session on the 17th the following officers were chosen for 1937–38: Chairman, J. B. Jackson, University of South Carolina; Vice-Chair-

the line in two double points. There are thus twelve double points which lie by sixes on four conics.

5. Professor Moore gave a brief account of a portion of the book *An Introduction to the Philosophy of Science* by A. C. Benjamin. This portion dealt with the use of symbols in scientific explanation. The speaker confined himself to the use of mathematical symbols as a medium for explanation.

6. The sum of an infinite series $\sum_0^\infty a_n$ can be defined as the limit of a sequence, $s_n = a_0 + \cdots + a_n$. To sum series for which the s_n oscillate, we seek another sequence σ_n which shall (a) have the same limit as s_n in those cases where $\lim s_n$ exists, (b) exist in cases where $\lim s_n$ does not exist. Mrs. Howard used the method of weighted means to determine σ_n and discussed its properties. From them she showed that at least the limit of the means of the partial sums of the Cauchy product $\sum_{n=0}^\infty (\sum_{i=0}^n a_i b_{n-i})$ of two convergent series exists and is equal to the product of the two series.

7. Professor Smith compared the teacher training requirements for certification of science teachers with the contents of ten different textbooks recently published. He found that not more than one-half of the content matter of these texts is covered by certificate requirements in Kentucky and West Virginia. In all the texts considered, with one exception, at least one whole chapter was devoted to the subject of astronomy with other chapters given over to allied subjects, but in neither state is astronomy included in any part of the teacher training program.

W. R. HUTCHERSON, *Secretary pro tem*

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-second annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, Colorado, April 15 and 16, 1938. There were three sessions. Professor C. A. Hutchinson presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Educational Association.

The attendance was seventy-one, including the following twenty-four members of the Association: L. A. Aroian, C. F. Barr, J. R. Britton, I. M. DeLong, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, G. H. Light, S. L. Macdonald, A. E. Mallory, A. S. McMaster, W. K. Nelson, Greta Neubauer, O. H. Rechard, A. W. Recht, C. H. Sisam, H. C. Wiedeman.

At the business meeting the following officers were elected for next year: Chairman, C. F. Barr, University of Wyoming; Vice-Chairman, D. F. Gunder, Colorado State College.

Following a luncheon at the High School on Saturday, Professor A. J. Kempner, president of the Association, gave a very interesting address on "The situation in collegiate and secondary mathematics."

The following papers were presented:

1. "The type B Gram-Charlier series" by Professor L. A. Aroian, Colorado State College.

2. "Mathematics and science—after Keyser" by Professor S. L. Macdonald, Colorado State College.

3. "Teaching large sections in freshman mathematics" by Professor O. H. Rechard, University of Wyoming.

4. "Roots of algebraic equations with complex coefficients" by Professor A. J. Kempner, University of Colorado.

5. "Slide rules for the solution of two problems in spherical trigonometry" by Professor I. L. Hebel, Colorado School of Mines.

6. "Lagrangean multipliers" by Professor C. A. Hutchinson, University of Colorado.

7. "Cooperation between high school and college teachers of mathematics" by Professor A. E. Mallory, Colorado State College of Education, and F. A. St John, South High School, Denver.

8. "The contributions of high school geometry to the goals of education" by Professor A. J. Lewis, University of Denver.

9. "Preparation of teachers of mathematics with reference to the requirements of the North Central Association" by Professor A. E. Mallory, Colorado State College of Education.

10. "Approximate numbers" by Dr. J. R. Britton, University of Colorado.

11. "Report on the work of the Joint Commission" by Professor C. A. Hutchinson, University of Colorado.

12. "The use of placement examinations in pre-college mathematics" by Professor W. J. Hazard, University of Colorado, introduced by the Secretary.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles.

1. Professor Aroian showed how seven terms of the type B Gram-Charlier series may be used in fitting a frequency distribution. (The paper has appeared in the December 1937 issue of the *Annals of Mathematical Statistics*.)

2. Professor Macdonald discussed the boundary lines of science and mathematics—the distinction between the two and what each is and what each is not. The ideas were chiefly derived from two books by C. J. Keyser, namely: *Pastures of Wonder* and *Humanism and Science*.

3. Professor Rechard taught a college algebra class of 66 students and one in trigonometry containing 78 students in the fall and winter quarters respectively of the current year, employing a classroom procedure differing from the usual recitation method. Normal sections of each subject during each quarter, were taught by other members of the department. On the basis of the Ohio College Ability Test both types of sections were found to be comparable. Final examination grades showed some advantage for the large section in college algebra, but no significant differences either way were found for trigonometry.

5. Professor Hebel designed slide rules for the calculation of the time of sunrise (or sunset) at a given place on a given date, and the determination of

the distance between two points on the surface of the earth. The slide rule for the sunrise problem is a single-setting rule from which the time is read directly from the given latitude and the sun's declination, a table of declinations being given on the reverse side. The second slide rule is a two-setting rule based on the cosine law, from which the distance in statute miles is found from the latitudes of the two places and the difference in their longitudes; for greater accuracy distances have been limited to about 6,000 miles.

6. Professor Hutchinson gave a brief exposition of the subject of constrained maxima and minima, with a discussion of the use of multipliers, and applications in the field of adjustment of observations.

8. In this paper Professor Lewis attempted to show that high school geometry is particularly well fitted to contribute to some of the generally recognized goals of education.

9. The standards set forth by the North Central Association for mathematics teachers are minimum. Adequate preparation defined in terms of present conditions and curriculum trends of the high school imply a general and specialized preparation. The most serious result of these trends is the tendency to minimize the importance of long experience. Mathematics has been the victim of this criticism because immediate and final ends have been confused. Professor Mallory believes that the formal-training pattern of the mathematics teacher should include a broad general education, training in related subjects, professional training in general and special courses, and student teaching.

10. Dr. Britton gave a brief exposition of the rules of computation with approximate data.

11. A report was made by Professor Hutchinson of the progress of the work of the Joint Commission on the Place of Mathematics in Secondary Schools, and of the Commission's plans for the completion of their work.

12. Professor Hazard showed a curve of the scores made by 665 freshmen on a placement examination in pre-college mathematics. This curve was compared with the curve of standardized results from 8200 students and showed very much lower scores, varying from 50% to 80% of the standard. A list of suggested questions for such an examination was offered for criticism.

A. J. LEWIS, *Secretary*

THE SIXTH ANNUAL MEETING OF THE WISCONSIN SECTION

The sixth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at the Columbus Community Club of St. Norbert College at Green Bay, Wisconsin, on May 14, 1938. The chairman of the Section, Professor Ethelwynn R. Beckwith of Milwaukee-Downer College, presided. The attendance was forty-five including the following twenty members of the Association: R. H. Bardell, Leon Battig, Ethelwynn R. Beckwith, May M. Beenken, W. W. Bigelow, L. A. V. DeCleene, Henry Ericson, R. C. Huffer, G. J. Kalcik, Elizabeth E. Knight, Morris Marden, Sister Mary Felice,

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-third annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, on Friday and Saturday, April 28–29, 1939. There were three sessions. Professor C. F. Barr, chairman of the Section, presided at each.

The attendance was forty-one, including the following eighteen members of the Association: C. F. Barr, J. R. Britton, I. M. DeLong, G. W. Gorrell, D. F. Gunder, C. A. Hutchinson, Dunham Jackson, L. Louise Johnson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, W. K. Nelson, Greta Neubauer, O. H. Rechard, A. W. Recht, W. M. Stewart.

The following officers were elected for the coming year Chairman, D. F. Gunder, Colorado State College; Vice-Chairman, W. V. Lovitt, Colorado College; Secretary-Treasurer, A. J. Lewis, University of Denver. The 1940 meeting is to be held at Colorado State College, Fort Collins, Colorado.

The Section was honored in having Professor Dunham Jackson of the University of Minnesota as its guest speaker. The Saturday morning session was a joint meeting with the Mathematics Section of the Eastern Division of the Colorado Educational Association.

The following papers were presented:

1. "Teaching college mathematics to large classes" by Professor O. H. Rechard, University of Wyoming.

2. "On the Diophantine equation $x(x+1)\cdots(x+n-1)=y^k$" by Dr. L. Louise Johnson, University of Colorado.

3. "Leading differences for backward interpolation" by Professor W. V. Lovitt, Colorado College.

4. "Operational calculus and the theory of numbers" by Professor C. A. Hutchinson, University of Colorado.

5. "Evaluating shear stresses without finding flexure function" by Professor D. F. Gunder, Colorado State College.

6. "On foci and branch points" by Professor A. J. Kempner, University of Colorado.

7. "Orthogonality" by Professor Dunham Jackson, University of Minnesota.

8. "Morley triangles" by Professor Claribel Kendall, University of Colorado.

9. "A new class of orthogonal polynomials" by Professor Dunham Jackson, University of Minnesota.

10. Report: "College-secondary mathematics coordinating committee" by Professor D. F. Gunder, Colorado State College.

11. Discussion: "The report of the Joint Commission" by Professor C. A. Hutchinson, University of Colorado; Professor G. W. Gorrell, University of Denver; and Miss Glennie Bacon, University High School, Laramie.

12. "Are we ready?" by Wendall Wolf, Morey Junior High School, Denver, introduced by the Secretary.

Abstracts of some of the papers follow, numbered in accordance with their place on the program:

1. Professor Rechard continued the report given the Section at its last meeting on the teaching of large classes in mathematics at the University of Wyoming. In addition to the two reported on last year, three other classes were divided into large (over 50) and small (under 30) sections. On the basis of percentile rank in the Ohio College Ability Test two pairs of sections, those in analytic geometry and college algebra, were found to be not comparable. In the third sections which were comparable, no significant difference was found in their mathematical attainments as measured by three one-hour tests and the final two-hour examination. Thus of five classes divided into large and small groups, the three in which comparability on the P.R. basis was established have shown no significant difference in mathematical attainment between the large and small groups.

2. Let the product of n consecutive positive integers be $P_n=x(x+1)\cdots(x+n-1)$. The equation $P_n=y^k$, y and k integers greater than 1, is known to be impossible if $n=2$, 3, or k; if $n\leqq 203$, $k=2$; if $n\leqq 13$, $k=3$, or 5; if $n=4$, k one of a certain infinite set of primes; if one of the integers is ap, p prime, $a\leqq 8$; or if $x<n^2$. Dr. Johnson showed that the equation is impossible also if $n=2k$; if $x\leqq(n+1)^k-n$; if one of the integers is $p_1^{\alpha_1}$, $p_2^{\alpha_2}$, $\cdots$, $p_\mu^{\alpha_\mu}$, the p's prime, $\sum_{i=1}^{\mu}\alpha_i<k$; or if one of the integers is a^k, $a<p$, p the greatest prime which divides P_n; and that the equation has at most one solution for $n=4$, 5, or 6, k a prime $\geqq 7$. There are a few other closely related results.

3. Leading difference formulas for subtabulation are derived from Newton's formula for negative interpolation. These leading differences at the end of a table being computed, the subtabulations can be made by a series of subtractions. Professor Lovitt has hence exposed the leading differences for the subtabulation in terms of the leading differences at the end of a table of differences instead of the leading differences at the beginning of the table.

4. Professor Hutchinson gave a critique of an article by B. van den Pol, in the December 1938 *Philosophical Magazine*.

5. The complete solution of the well known flexure problem requires the determination of a function x satisfying $\nabla^2 x=0$ within the section and the condition that the derivative shall be a specified function on the boundary. However, for the actual determination of the stresses within the beam only the partial derivatives of x are needed. Professor Gunder found an accurate and relatively simple method of finding these derivatives by transforming the given boundary onto the unit circle and setting up the solution for x in terms of the usual integral involving the transformed value of the normal derivative. This expression is then differentiated under the integral sign and the resulting integral evaluated mechanically to give the required stresses.

7. Professor Jackson emphasized the fact that mathematical study is largely concerned with the acquisition of fundamental general ideas which recur in varied and progressively more complicated situations. Familiarity with such

fundamental principles is often vitally reinforced, and their significance clarified, by acquaintance with less elementary applications in which they are viewed under diverse aspects. This is illustrated by an outline of the development of the notion of orthogonality from the perpendicularity of elementary geometry through the algebraic and analytical formulations which appear in statistics, mathematical physics, and current research in pure mathematics.

8. Professor Kendall reported on an article, *Morley's Triangle*, given in the Mathematical Gazette of February, 1938. There W. J. Dobbs defines the Morley Triangle as one obtained by the intersections of certain trisectors of the angles of a given triangle. He shows that this triangle is isosceles, and extends the discussion to show that there are 27 such triangles, 18 of which are equilateral.

9. Professor Jackson's paper appeared in full in the October issue of the MONTHLY.

A. J. LEWIS, *Secretary*

THE ANNUAL MEETING OF THE MINNESOTA SECTION

The annual meeting of the Minnesota Section of the Mathematical Association of America was held at Carleton College, Northfield, Minnesota, on Saturday, May 13, 1939. A morning session was held at 10:30 o'clock and was followed by luncheon and an afternoon session at 2:15 o'clock. Professor W. H. Bussey of the University of Minnesota presided at each session.

Seventy-three persons attended the meeting, including the following twenty-eight members of the Association: C. J. Blackall, L. E. Bush, W. H. Bussey, E. J. Camp, C. S. Carlson, S. Elizabeth Carlson, Sister M. Claudette, H. H. Dalaker, J. H. Daoust, Brother Louis De La Salle, Margaret C. Eide, C. H. Gingrich, H. E. Hartig, J. S. Hickman, Dunham Jackson, Margaret P. Martin, W. R. McEwen, Sigurd Mundhjeld, F. J. Polansky, Inez Rundstrom, M. G. Scherberg, C. Grace Shover, F. J. Taylor, H. P. Thielman, Ella Thorp, A. L. Underhill, K. W. Wegner, Marion A. Wilder; and Sister Thomas à Kempis, institutional member representative.

At the business session officers were elected for the coming year as follows: Chairman, C. S. Carlson, St. Olaf College; Secretary, A. L. Underhill, University of Minnesota; Executive Committee, H. P. Thielman, College of St. Thomas, and Sister Thomas à Kempis, College of St. Teresa.

The following nine papers were presented:

1. "The stability of an unsymmetric top" by Professor E. J. Camp, Macalester College.

2. "A number problem" by Professor L. E. Bush, College of St. Thomas.

3. "An appreciation of Sophie Germain" by Sister M. Thomas à Kempis, College of St. Teresa.

4. "On the use of the complex exponential in the solution of differential equations" by Professor H. E. Hartig, University of Minnesota.

The proof was based on the Poisson integral theorem. First the existence of the limit defining $\psi(\lambda)$ was established, then the representation theorem was proved by a reasoning adapted from a paper of R. Nevanlinna.

7. When four fixed points and two variable points on a conic are considered as the vertices of an ordered hexagon, under certain restrictions the Pascal line of the hexagon envelops an algebraic curve. Professor Ott obtained parametric equations for certain of these curves, located their singularities, and obtained their Plücker characteristics.

8. Professor Agnew gave a general discussion of convergence and other methods of summability of series. Emphasis was placed upon relations between different methods of summability and upon Tauberian theorems.

C. W. MUNSHOWER, *Secretary pro tempore*

THE TWENTY-FOURTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-fourth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College of Agriculture and Mechanic Arts, Fort Collins, Colorado, April 19 and 20, 1940. There were three sessions. Professor D. F. Gunder presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Educational Association.

The attendance was seventy-six, including the following twenty members of the Association: C. F. Barr, J. R. Britton, I. M. DeLong, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, W. V. Lovitt, S. L. Macdonald, A. E. Mallory, W. K. Nelson, Greta Neubauer, M. G. Pawley, G. B. Price, O. H. Rechard, A. W. Recht, C. H. Sisam.

At the business meeting the following officers were elected for next year: Chairman, W. V. Lovitt, Colorado College; Vice-Chairman, J. C. Fitterer, Colorado School of Mines.

The following papers were presented:

1. "The line integral of curvature as a measure of its associated central angle" by Professor C. F. Barr, University of Wyoming.

2. "Determination of the differential equation and the equation of the orbit of a central force when the law of the force is known" by Professor J. R. Everett, Colorado School of Mines.

3. "Expansions of determinants of order four and five" by Professor W. V. Lovitt, Colorado College.

4. "The Gaussian solution of the trinomial equation" by Professor A. J. Lewis, University of Denver, read by A. M. Kahan.

5. "Projective representation of an affinely connected space" by T. C. Doyle, University of Wyoming, introduced by Professor Rechard.

6. "Some famous problems of modern mathematics" by Professor G. B. Price, University of Kansas.

7. "Mathematical Reviews" by Professor G. B. Price, University of Kansas.

8. "Report of the college-secondary mathematics coördinating committee" by Professor D. F. Gunder, Colorado State College.

9. "Discussion of trends in the teaching of mathematics in the junior high school" by Dr. G. S. Willey, Director of Instruction, Denver Public Schools; Professor L. Edwards, Colorado State College of Education; Superintendent M. W. Jessup, Bennett, Colorado, by invitation of the program committee.

Abstracts of some of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. The condition that a line integral of curvature along a polar curve be a constant multiple of its associated central angle is expressed by an ordinary differential equation of the second order, which is completely solvable. Embedded in this family of solutions are some of the best known angle-measurement theorems of elementary geometry. Professor Barr advanced the suggestion that the line integral under consideration may be used to unify and greatly extend these theorems. (A preliminary report.)

3. Professor Lovitt obtained the expansion of a determinant of order four by means of a schematic diagram comparable to that used for a determinant of order three.

4. Professor Lewis outlined Gauss's method of finding the real roots of trinomial equations by the use of addition and subtraction logarithms. He showed how the method could be simplified by modern computing machines.

5. Given an affinely connected space of N dimensions with connection components $\Gamma^i_{jk}(x)$ and curvature tensor $\Gamma^i_{jkl}(x)$, the partial differential equations

$$\frac{\partial^2 y}{\partial x^i \partial x^k} = \Gamma^r_{ik}\frac{\partial y}{\partial x^r} - \frac{1}{N-1}\Gamma^r_{ikr}y \tag{1}$$

will admit a system of $N+1$ fundamental solutions $y^\alpha(x)$ determined to within a projective transformation with constant coefficients providing the integrability conditions are satisfied. These solutions $y^\alpha(x)$ serve to define homogeneous coordinates of the point (x^i) and the geometry of the resulting projective representation will find its analytical expression in the invariant theory of (1) under the combined transformations $x^{-i}=x^{-i}(x)$ and $y=\phi(x)y$, where ϕ is an arbitrary non-vanishing factor.

6. Professor Price gave the history and present status of Waring's Problem, the Four-Color Problem, the Jordan Curve Theorem, and the Problem of Plateau. The talk was illustrated with various models and demonstrations, including paper and rubber models of surfaces, a map on a wooden torus which required seven colors for its coloring, and soap film models for the Problem of Plateau. These problems were used: (1) to emphasize the great progress that has taken place in mathematics in recent times; (2) to illustrate the nature and source of problems in mathematics; (3) to point out the difference between a proof in mathematics and a proof in physics. The oldest of the four problems was first studied in 1636; although great progress has been made in recent years,

all four of them are still the subject of research. Problems in mathematics arise from (a) conundrums dealing with the positive integers, (b) a study of the physical world, and (c) from generalizations of simpler problems. Proof in mathematics consists of logical deduction; proof in physics consists of an induction from a large number of experiments.

8. The coördinating committee found that there was little discrepancy between the material presented in high school courses in mathematics and that required or expected of entering college freshmen. As a consequence, the present lack of preparation of college freshmen was attributed to lack of retention. Six suggestions for improving the general situation were offered by the committee. These were:

(1) Teach with emphasis on understanding rather than mechanical manipulation.

(2) Teach the correct terminology to further promote understanding.

(3) Teach the material in larger units with frequent repetition to prevent loss of sight of the subject as a unified whole.

(4) Induce other departments to use mathematics understandingly.

(5) Supplement or replace formal college entrance requirements by placement examinations.

(6) Improve the quality of teachers by requiring of them more and broader education in both mathematics and other fields.

9. The main purpose of education in the junior high school is the furthering of wholesome growth and development of the whole child through broad, meaningful experiences. Mathematics teachers, concerned with the total development of boys and girls, must consider how their subject will contribute to the concerns and problems of youth. The trend for teaching mathematics in the junior high school was summarized by this discussion as follows:

(1) Teaching all children only that which we know all children will use. Children interested in vocational phases may pursue mathematics materials in those fields.

(2) Choosing units of subject-matter through pupil-teacher planning.

(3) Permitting the "natural" method of learning, which is the only good teaching technique.

(4) Not carrying drill beyond the limits of its use by average adults in the community.

(5) Giving ample opportunity for experiences employing the four fundamental processes, simple fractions, percentage, and interest, stressing accuracy in each case.

(6) Permitting many informational problem-solving experiences that are meaningful to the pupils.

(7) Providing a program of diagnostic checking and remedial teaching.

(8) The placing of general, or social, mathematics through the ninth grade, with opportunity for electing algebra in the ninth grade for the present at least.

A. J. LEWIS, *Secretary*

conditions, and to find the elevation corresponding to the corrected range, is also mathematically simple. The mastery and rapid coördination of all these operations in the field is the fine art of the fire control officer. Perhaps the most serious mathematical problem connected with artillery fire is the construction of range tables. Various methods of computation of trajectories, by the approximation of Siacci, by short arc methods of computation, and mechanically by means of the differential analyzer of Bush, together with methods of computing differential corrections, were discussed.

12. "If the trisectors of the interior angles of a triangle A, B, C be drawn, and if those trisectors adjacent to BC meet at P, those adjacent to AC meet at Q, and those adjacent to AB meet at R, then P, Q, R is an equilateral triangle." This is known as the theorem of Morley. Dr. Peters discussed the history of this theorem from its discovery by Frank Morley until the present.

13. Professor Reid was concerned with fundamental relationships and analogs that exist between certain problems for pencils of quadratic forms and the principal results for definitely self-adjoint and H-definitely self-adjoint differential systems as developed by Bliss and the speaker.

14. Professor Wood discussed the desirable properties which a classification should have, and gave illustrations of classifications with those properties and of classifications which did not possess certain stated properties.

C. N. MILLS, *Secretary*

THE TWENTY-FIFTH ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-fifth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado College, Colorado Springs, Colorado, April 18–19, 1941. There were three sessions. Professor W. V. Lovitt, chairman of the Section, presided at each. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Education Association.

There were thirty-four present, including the following twenty-five members of the Association: C. F. Barr, M. T. Bird, Jack Britton, I. M. DeLong, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, W. V. Lovitt, S. L. Macdonald, A. E. Mallory, W. K. Nelson, Greta Neubauer, M. G. Pawley, G. B. Price, O. H. Rechard, A. W. Recht, C. H. Sisam, V. J. Varineau, G. A. Whetstone.

At the business meeting the following officers were elected for next year: Chairman, J. C. Fitterer, Colorado School of Mines; Vice-Chairman, A. E. Mallory, Colorado State College of Education; Regional Governor for Region 12, 1942–43, O. H. Rechard, University of Wyoming.

The joint session held on Saturday morning consisted of a discussion of the two following reports: (1) "The place of mathematics in secondary education"

by the Joint Commission of the M. A. A. and the N. C. T. M.; (2) "Mathematics in general education" by a commission of the Progressive Education Association. The discussion was led by Dr. H. R. Douglass, Director of the College of Education, University of Colorado.

The following papers were presented:

1. "Various types of singular points of differential equations of the first order" by Professor J. R. Everett, Colorado School of Mines.

2. "A note on Klein's determinant approach to the line integral of area" by Professor C. F. Barr, University of Wyoming.

3. "Generalized euclidean rings" by Dr. V. J. Varineau, University of Wyoming.

4. "Specification of elastic strain" by Dr. G. A. Whetstone, Amarillo College.

5. "Interpolation with the calculating machine" by Professor A. W. Recht, University of Denver.

6. "Excursions from the beaten path of undergraduate mathematics" by Professor M. T. Bird, Utah State Agricultural College.

7. "The inherent error in extension of Newton's method for approximating real roots" by Professor M. G. Pawley, Colorado School of Mines.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. Professor Everett discussed (a) nodal points, (b) vortex points, (c) spiral points, and (d) saddle points, arising in differential equations of the first order.

2. Klein develops a special statement of Green's theorem in a plane. His approach is by a determinant of triangular area. The suggestion of generality is obvious. Professor Barr extended this approach to a surface integral of volume.

3. Dr. Varineau defined a class of generalized euclidean rings. He showed that the class of euclidean rings, as defined in the literature, is included in this class of generalized euclidean rings. He also demonstrated that the ring of matrices with elements in a proper euclidean ring is a generalized euclidean ring.

4. In an elastic body in space account must be taken of six components of strain, not all of which can be independent since they are defined as linear combinations of the first order partial derivatives of the three displacements. Dr. Whetstone proved that with the exception of the three sets (e_x, e_y, e_{xy}), (e_y, e_z, e_{yz}), and (e_z, e_x, e_{zx}) we may select any three strains arbitrarily and may then determine which of the coefficients in the Taylor expansions of the other three are arbitrary. These results were obtained by the methods of Riquier.

5. Professor Recht demonstrated the use of the calculator in ordinary interpolation using first and second differences; also, a special method of subdivision of tables to fifths of intervals using up to fourth differences.

6. The results of these excursions as given by Professor Bird are probably not new, but they may be found in novel settings. The values of $\log_{10}2$, $\log_{10}3$, and $\log_{10}7$ were found to four digits directly from the definition of logarithm. The relations between the law of sines, law of cosines, *etc.*, were made explicit. A construction for the axes of the ellipse $Ax^2-2Bxy+Cy^2=D$ was related to the

lines $Ax=By$ and $Bx=Cy$. A construction for the hyperbola $b^2x^2-a^2y^2=a^2b^2$ was related to the parametric form $x=a(1+t^2)/(1-t^2)$, $y=2bt/(1-t^2)$. The series for $\log_e N$ was exhibited as the result of certain rearrangements of the series $1-1/2+1/3-1/4+\cdots$. Unusual weighted sums were considered as approximations for a definite integral and contrasted with the usual "rules." Finally, π was computed by the use of the inverse sine.

7. Professor Pawley described extensions of Newton's method for approximating real roots in which the desired root is approximated by x intercepts of curves of higher order of contact than the tangent. He derived an upper bound to the error involved in these approximations. In particular, he simplified the well known parabolic approximation by expanding an x intercept of the parabola into a convergent alternating series. An upper limit to the error involved in this approximation was derived and illustrated by an example.

A. J. LEWIS, *Secretary*

EQUATIONS IN QUATERNIONS

IVAN NIVEN, University of Illinois

1. Introduction. We prove the existence of a quaternion root of the equation

$$(1) \qquad a(x) = x^m + a_1x^{m-1} + a_2x^{m-2} + \cdots + a_m = 0, \qquad a_m \neq 0,$$

with coefficients from the algebra of real quaternions. The writer had proved this result when m is odd, but the proof was rendered obsolete when Nathan Jacobson pointed out that the result (without restriction on m) can be obtained as a simple consequence of some work of Ore [1]. This is given in detail in §2.

In §3 we give a method for obtaining the roots of (1), which is not very practical in the sense that it involves the simultaneous solving of two real equations of degree $2m-1$. The method used is a generalization of Sylvester's treatment [2] of the quadratic equation corresponding to (1). Sylvester's conclusion that a quadratic equation has six roots is incorrect because he neglects to show that they exist, and also overlooks the possibility of an infinite number of roots; a complete analysis is given in §4 (Theorem 2). The number of roots of (1) is discussed in §5 (Theorem 3), necessary and sufficient conditions being given for an infinite number of roots.

The proof given here of the existence of a root of (1) is stated for the general case where the coefficients of the equation are quaternions over any real-closed field R (*i.e.*, no sum of squares in R is equal to -1, and no algebraic extension of R has this property).

Reinhold Baer, on hearing of this existence proof, proved the converse, so that we have the following strong result:

THEOREM 1. *Let D be a non-commutative division algebra with centrum C. Then every equation* (1) *with coefficients from D has a solution in D if and only if C is a real-closed field, and D is the algebra of real quaternions over C.*

nature of proof and on a postulational system demands a simple synthetic introduction with a gradual transition to analytic geometry and other methods of proofs. Integrated mathematics results in a rich set of concepts arranged spirally in a syllabus.

7. Dr. Kramer-Lasser showed how integrated mathematics is used to develop general space concepts. Even as early as the seventh year, elementary procedures form a foundation for more advanced work. For example, in working with paper patterns, proper numbering of vertices and edges eventually leads pupils to think of the surfaces they construct as sets of triangles with suitable identification of vertices and edges. In the ninth and tenth years, space geometry is studied along with plane. By rotating and translating 3, 4, $\cdots$, $n-1$ dimensional forms, pupils obtain an abstract realization of 4, 5, $\cdots$, n dimensional figures. Combinatorial methods are also used to obtain the concept of higher dimensions. In the eleventh and twelfth years, elementary analytic geometry of three dimensions is studied and analytic generalization to higher dimensions is made.

8. Brother Anselm stated that, in general, the Catholic High Schools in and around New York City adhere to the traditional courses in mathematics. They do this because they think that their present curriculum is the best for their particular needs. They are reluctant to change to the integrated course because most of the leading educators in this field are not certain that integrated mathematics courses are better than the traditional courses. Moreover, they fear that lack of drill and understanding of the reasons for manipulative processes may be brought about by the integrated program. They are convinced also that the much desired purpose of an integrated program can be achieved with the traditional subjects. Hence the Catholic schools look, not to a change of courses to improve secondary mathematics, but rather to better teaching of the present courses.

H. E. WAHLERT, *Secretary*

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-sixth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado School of Mines, Golden, Colorado, April 17 and 18, 1942. There were three sessions. Professor J. C. Fitterer presided at the first two and at the business session of the third. The Saturday morning session was a joint meeting with the mathematics section of the Eastern Division of the Colorado Education Association. Mr. H. W. Charlesworth of East Denver High School presided at this meeting.

The attendance was forty-four, including the following fifteen members of the Association: C. F. Barr, J. R. Everett, J. C. Fitterer, I. L. Hebel, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. E. Mallory, W. K.

Nelson, Greta Neubauer, M. G. Pawley, A. W. Recht, C. H. Sisam, and W. E. Wilson.

At the business meeting the following officers were elected for the coming year: Chairman, Professor A. E. Mallory, Colorado State College of Education; Vice-Chairman, Professor A. W. Recht, University of Denver.

The following papers were presented:

1. "An improved cosine-law slide rule" by Professor I. L. Hebel, Colorado School of Mines.

2. "On determining the 'best' critical region for testing the null hypothesis when the parent populations follow the Poisson law" by H. T. Guard, Colorado State College, introduced by Professor Macdonald.

3. "The use of Cauchy's integral formula in evaluating certain improper integrals" by Professor A. J. Lewis, University of Denver.

4. "A study of roulettes with the aid of the cathode-ray oscillograph" by Professor M. G. Pawley, Colorado School of Mines.

5. "Geometrical demonstration of a theorem on envelopes and its application to solve a maximum and minimum problem" by G. E. Uhrich, University of Colorado, introduced by Professor Kempner.

6. "The mathematical approach to the fundamentals of hydraulics" by C. P. Vetter, Senior Engineer, Bureau of Reclamation, introduced by the Secretary.

7. "Remarks on repeating decimal fractions" by Professor Emeritus I. M. De Long, University of Colorado. Read by Professor A. J. Kempner.

8. "Periodic decimal fractions, primitive roots and quadratic residues" by Professor A. J. Kempner, University of Colorado.

9. "A report of instruction and learning activities as observed in geometry class rooms" by Professor A. E. Mallory, Colorado State College of Education.

10. "The handmaiden spurned" by Professor O. H. Rechard and Professor C. F. Barr, the University of Wyoming.

11. "Some simple proofs of the addition law in trigonometry" by G. E. Uhrich, University of Colorado, introduced by Professor Kempner.

12. "The use of mathematics in industry" by L. A. McElroy, Denver public Schools, introduced by the Secretary.

13. "Constructions by means of a marked ruler and other instruments" by Professor Claribel Kendall, University of Colorado.

Abstracts of the papers follow.

1. Professor Hebel gave an extension of an earlier paper concerning a slide rule for the solution of the cosine law of spherical trigonometry, particularly as applied to distances on the earth. By properly manipulating the equation, an improved slide rule has been devised which gives a direct solution by purely mechanical means, as contrasted to the original model where it was necessary to "take out" intermediate answers to reach the final results. The operation was demonstrated on a specially constructed two meter long classroom model slide rule.

2. This problem was first considered by Przyborowski and Wilenski in *Biometrica*, 1939.

Given the random variables x_i where $\rho(x_i|\lambda_i)=\lambda_i^x e^{-x_i}/x_i!$, $(i=1, 2)$. To determine the best critical region for testing the composite hypothesis, H_0, that $\lambda_1=\lambda_2$ with respect to the alternative hypothesis $\lambda_1<\lambda_2$. The test consists in finding a region, w_0, in the sample space, W, such that, if E is the observed sample point,

1. $P\{E\epsilon w_0 \mid H_0\} \leqq \epsilon$, the desired level of significance.
2. $P\{E\epsilon w_0 \mid H_1\} > P\{E\epsilon w \mid H_1\}$

where H_1 is any alternative hypothesis and w is any other region that satifies 1.

The test reduces to rejection of H_0 when $x_1 \leqq x_\epsilon$ where x_ϵ is chosen so that

$$\sum_{x_1=0}^{x_\epsilon} \binom{s}{x_1} \left(\frac{1}{1+A}\right)^{x_1} \left(\frac{A}{1+A}\right)^{s-x_1} \leqq \epsilon$$

is satisfied. A is the ratio of the sample sizes and $s=x_1+x_2$.

The power function is

$$B(\theta \mid s) = \sum_{x_1=0}^{x_\epsilon} \binom{s}{x_1} \left(\frac{1}{1+A\theta}\right)^{x_1} \left(\frac{A\theta}{1+A\theta}\right)^{s-x} \quad \text{where} \quad \theta > 1.$$

The calculations of tables given the critical region and values of the power function for specified values of s and θ and also with s unspecified were demonstrated by Mr. Guard.

3. Professor Lewis outlined methods of using Cauchy's integral formula and Cauchy's residue formula in evaluation of certain improper integrals with real integrands. He illustrated the method by applying it to two examples.

4. Professor Pawley made a study of the parametric equation

$$x = a_1 \cos n_1 t + a_2 \cos n_2 t + a_3 \cos n_3 t,$$
$$y = a_1 \sin n_1 t \pm a_2 \sin n_2 t + a_3 \sin n_3 t.$$

The equation was shown to represent a group of roulettes, including epicycloids, hypocycloids, epitrochoids, and hypotrochoids, depending upon the relations between the a's and n's. The various curves were pictured on the flourescent screen of a cathode-ray oscillograph. Simple rules were given for anticipating the form of roulette obtained when the a's and n's in the equation are varied. An equation was given for a curve approximating the N-sided regular polygon and these curves were displayed on the screen of the oscillograph.

5. Mr. Uhrich dealt mainly with a geometrical proof of the theorem: The envelope of the family of chords which cut segments of equal area from a given curve is tangent to each chord at its midpoint. The converse of this theorem is proved in Salmon's *Analytische Geometrie der Kegelschnitte*, Kap. 13, with the purpose of application to conic sections; however the method seems to be general.

Besides some special examples, Mr. Uhrich used this theorem to prove: Of all the chords which can be drawn through a fixed point within a closed convex curve, that chord which is bisected by the point cuts off a segment of minimum area from the curve. This theorem was given as a problem in the *Analyst*, vol. 5.

6. Mr. Vetter outlined the various methods of mathematical approach to the solution of problems connected with fluid flow. These methods permit of practical solutions and of solutions that may be verified experimentally only in comparatively few and comparatively simple cases. He demonstrated further, that as an alternative it is possible to attack the problems from purely dimensional considerations without the necessity of simplifying assumptions. The latter method, without leading to final answers, nevertheless gives precise information as to the mathematical form of the functional relationship between the physical quantities involved and, in many instances, permits experimental verification or experimental evaluation of specific functional constants. He analyzed the relationship of the method to the theory of models.

7. Professor Emeritus Ira M. De Long of the University of Colorado is eighty-seven years old, and has been for four months in a Boulder hospital. He has maintained his life-long interest in the properties of repeating decimals, and never tires of telling me interesting and amusing relations which he has discovered for himself. Without claims of priority I present two examples.

(1) If we expand a fraction a/p, p a prime, say $3/7$, we have $10\cdot 3=4\cdot 7+2$, $10\cdot 2=2\cdot 7+6$, $10\cdot 6=8\cdot 7+4$, $10\cdot 4=5\cdot 7+5$, $10\cdot 5=7\cdot 7+1$, $10\cdot 1=1\cdot 7+3$, with the digit period $428571'$ and the remainder period 264513. The $7(4+2+8+5+7+1)=9(2+6+4+5+1+3)$. For a/p, $p\cdot\sum(\text{digits})=9\cdot\sum(\text{remainder})$. For a base g instead of 10, $g\neq 1$, $\not\equiv 0 \bmod p$, 9 is replaced by $g-1$.

(2) To obtain the period of $1/49$ from the period 142857 or $1/7$, proceed as follows: Write

.020408	020408 142857	020408 285714	020408 428571	020408 571428	020408 714285	020408 857142
020408	163265	306122	448979	591836	734693	877550

which is (except for the last 0 which has to be replaced by 1) the correct period.

8. The examination of the period length and digit properties, etc., of an ordinary repeating decimal have a particularly special character, on account of the orientation with respect to the base 10. By considering simultaneously the representation of all fractions a/p, $(a/p)=1$, for a given p, with respect to *all* bases $g>1$, and by fixing attention on the periodic *remainder* sequences rather than upon the periodic *digit* sequences, the general pattern or relations appears clearly. For a/p, $a=1, 2, \cdots, p-1$ we have for each base $g>1$, $(p/g)=1$, the following theorems:

I. Let $d_1=1, \cdots, d_\nu, \cdots, d_\mu=p-1$ be the divisors of $p-1$. Then, denoting

by $L(a/p)_g$ the period length in the remainder sequence, there are exactly $\phi(d_\nu)$ remainder sequences for which $L(a/p)_g = d_\nu$. (For $p=7$, $a=4$, $g=2, 3, \cdots, 6, 8$ we have remainder sequences 124, 513264, 214, 623154, 34, 4, respectively.) The same holds of course for the digit periods.

II. For a given g the remainders $1, 2, \cdots, p-1$, of a given a/p will either form exactly a remainder period, or the $p-1$ remainders break up into α sets of β each, $\alpha\beta = p-1$. In each of these $L(a/p)_g$ one of the three cases occur: Either each number of the period is quadratic residue of p, or each number is a non-residue; or the numbers are alternately residues or non-residues.

III. If $(g/p)=1$, $L(a/p)_g$, is a divisor of $(p-1)/2$, 1 and $(p-1)/2$ inclusive. But if $(g/p)=-1$, $L(a/p)_g$, which must divide $p-1$, does not divide $(p-1)/2$.

9. Professor Mallory gave a report of the type of work being done in certain geometry classes. The classes visited showed a high degree of understanding of the subject matter as revealed by the procedures.

10. The paper by Professor Rechard and Professor Barr grew out of the rejection by a geologist of a simple formula involving a few trigonometric functions on the basis that geologists would not use it. Instead, in his published paper, he described and illustrated a graphic method for solving a chain of right triangles. Where the fault lies for such a situation is the question raised by this paper, in the hope that out of it might come some clue to the old problem of how to make usable mathematics used.

11. In this paper Mr. Uhrich gave a presentation of three of the four proofs with slight variations for the addition theorem for the sine of the sum or difference of two angles which are given in Professor Gerhard Hessenberg's *Ebene und sphärische Trigonometrie*, Kap. IV.

12. The successful person in industry is the one who can separate essential and non-essential factors, get the job done and show a profit. Mathematical training is important to the extent that it applies to the job in hand. Pupils should be offered basic knowledge of mathematics with the problems that are *real to them*. This should help them to develop the habit of applying mathematical knowledge to practical situations. Industry specializes; mathematics essential to one industry has no value to another. But, since no one can predict with 100% accuracy the potentialities of an individual, it would seem wise to recommend training in mathematics for each pupil, at least until a choice of occupation has been made.

13. We are all familiar with constructions by means of an unmarked ruler and compasses, the euclidean instruments. Some or all of these constructions can be carried out by means of other instruments such as an angle ruler, a marked ruler, an unmarked ruler without compasses or by compasses alone. Professor Kendall paid particular attention to constructions by means of a marked ruler, including the trisection of the angle and the finding of a length which is the cube root of a given line segment.

A. J. LEWIS, *Secretary*

4. *A mathematical peculiarity of the plastic stress-strain relations*, by E. A. Davis, Westinghouse Research Laboratories, introduced by Dr. Sturm.

Mr. Davis stated that when a ductile metal flows under the influence of combined stresses, three mutually perpendicular directions can be found along which the deformations are pure extensions. There are two theories at present which deal with the values of the three principal strain rates at any given instant. The older is based upon the law of viscous flow which states that the shear rate on any plane of principal shear stress is proportional to the stress acting on that plane. The newer theory claims that the rates are not proportional to the stresses, but that they may be expressed by relations involving power functions of the stresses. This theory reduces to the older one when a certain exponent n has the value 1. The peculiarity pointed out is that in the newer theory the distribution of strain rates when $n=1$ is the same as the distribution when $n=3$. This is due to the fact that the expression $[a^n-b^n]/[(a+b)^n+b^n]$ has the same value for $n=3$ and for $n=1$.

H. L. DORWART, *Secretary*

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-seventh annual meeting of the Rocky Mountain Section of the Mathematical Association of American was held on Friday and Saturday, April 16–17, 1943, at the University of Denver, Denver, Colorado. It was a joint meeting with the National Council of Teachers of Mathematics and the Eastern Division of the Colorado Education Association. Section meetings were held Friday afternoon and evening, at both of which the Vice-Chairman of the Section, Professor A. W. Recht, presided. Three additional sessions were held on Saturday in conjunction with the other organizations participating in the meeting.

The attendance was one hundred and thirty-five, including the following fifteen members of the Association: A. G. Clark, Sister Rose Margaret Cook, J. R. Everett, J. C. Fitterer, G. W. Gorrell, D. F. Gunder, J. O. Hassler, A. J. Kempner, Claribel Kendall, W. J. LeVeque, A. J. Lewis, A. E. Mallory, A. W. Recht, C. H. Sisam, H. W. Williams.

At the business meeting the following officers were elected for the coming year: Chairman, Professor A. E. Mallory, Colorado State College of Education; Vice-Chairman, Professor A. J. Kempner, University of Colorado.

The following papers were presented:

1. *On the place of mechanics in the system of sciences, and the training of mathematicians for work in applied mechanics*, by Dr. Paul Nemenyi, University of Colorado, introduced by Professor Kempner.

The speaker discussed the unity of the sciences and the place of mechanics in the scheme of scientific studies. He classified mechanics as a part of physics, and considered the relation of mechanics to other parts of physics and to mathe-

matics. In particular, the relation between mechanics and probability was brought out. A program for the training of mathematicians for research and teaching in mechanics was then outlined.

2. *A method of measuring effectiveness in the teaching of college mathematics*, by Professor J. O. Hassler, University of Oklahoma.

Professor Hassler investigated the grades in Calculus II of the students of eleven teachers of Calculus I. The records of the "A," "B," "C" and "D" students were examined separately, and the successes (by separate groups) of the students of the various teachers compared. This was a measure of the effectiveness of the teaching in the first course. The students in each of the four grade classifications were also divided into two groups, namely those who remained with the same teacher and those who had the second course with a different teacher. In this way was obtained an evaluation of the teachers' grading scales.

3. *The integral* $\int x^{-1}dx = \log x$ *as a limiting case of* $\int x^{n-1}dx = x^n/n$, by W. J. LeVeque, University of Colorado.

In this paper it was shown that the integral $\int x^{-1}dx$ can be studied by considering the limit of $\int x^{n-1}dx$ as n approaches zero. The geometric properties of the approximation curves were also investigated.

4. *On a continuous stochastic process*, by Professor A. G. Clark, Colorado State College.

Professor Clark stated that, in ballistics research, it has been the practice to measure the variability of an ordered succession of random variables $x_1, x_2, \cdots, x_n$ by using the mean square successive difference $\delta^2 = (N-1)^{-1}\sum_{i=1}^{N-1}(x_{i+1}-x_i)^2$ as a criterion for measuring variability, rather than the quantity $s^2 = N^{-1}\sum_{i=1}^{N}(x_i-\bar{x})^2$. He considered the problem of testing for determination of trend, and showed that δ/s is the proper criterion to use for this purpose.

5. *On the introduction of coördinates in an affine plane geometry*, by Mrs. Margaret S. Matchett, University of Denver, introduced by Professor Recht.

Using only axioms of connection for points and lines, the parallel axiom, and the configuration of Desargues, it is possible to introduce a system of coördinates into a geometry. These coördinates satisfy all the field properties except that of commutative multiplication. In order to define these coördinates one considers the group of those one-to-one transformations of the plane which map parallel lines into parallel lines. The translations form an invariant sub-group of this group. The inner automorphisms of this sub-group, with addition and multiplication suitably defined, form the division algebra from which the coördinates are taken.

6. *Graphical methods for representation of various types of functional relations*, by Professor A. J. Kempner, University of Colorado.

This paper dealt with a method of plotting the graph of an equation of the type $F[f(x, y), g(x, y)] = C$. The details are as follows: Let $X = f(x, y)$ and

$Y=g(x, y)$. Plot the curve $F(X, Y)=C$ with reference to X and Y axes, and plot on a separate chart the two families of curves $f(x, y)=\alpha$ and $g(x, y)=\beta$ where α and β are parameters. If (α, β) is any point on the curve $F(X, Y)=C$, a point of intersection of the two curves $f(x, y)=\alpha$ and $g(x, y)=\beta$ is a point of the curve $F[f(x, y), g(x, y)]=C$. The method can be extended to the representation of the equation $F[f(x, y), g(x, y)]=H(z)$, leading to a one parameter family of curves in the xy-plane.

7. *Teaching mathematics effectively for war or peace*, by Professor J. O. Hassler, University of Oklahoma.

Professor Hassler reviewed briefly the controversy over the transfer of training, and reported that eighty per cent of the psychological experiments up to date show clear evidence of transfer of training. He remarked that to teach consciously for transfer of training is a prime goal of effective teaching. It was also stated that this object can be achieved by relating subject-matter in every possible way to practical applications, by cultivating habits of independent thinking and generalization, and by exploiting the spirit of discovery in the pupil.

8. *Mathematics abridged has gone to war*, by Professor J. O. Hassler, University of Oklahoma.

The speaker reviewed the present situation wherein frantic efforts (by means of concentrated courses) are being made to make amends for deficient training in mathematics among the youths in the army or about to go into the army. He gave some facts concerning dilution of mathematics courses in the recent past which has contributed to this delinquency. He made a plea for teachers to equip themselves to fight against having a denatured, abridged, and diluted mathematics in the post-war curriculum of the high schools.

A. J. Lewis, *Secretary*

THE MARCH MEETING OF THE SOUTHERN CALIFORNIA SECTION

The twenty-third regular meeting of the Southern California Section of the Mathematical Association of America was held at the University of Southern California, Los Angeles, California, on Saturday, March 13, 1943. Professor Morgan Ward, chairman of the Section, presided.

The attendance was fifty-five, including the following twenty-six members of the Association: O. W. Albert, C. K. Alexander, L. D. Ames, Clifford Bell, L. T. Black, Myrtie Collier, P. H. Daus, D. C. Duncan, W. H. Glenn, Jr., Frances C. Hinds, P. G. Hoel, C. G. Jaeger, G. R. Kaelin, Ada A. McClellan, G. F. McEwen, P. M. Niersbach, W. T. Puckett, Jr., H. R. Pyle, J. M. Robb, G. E. F. Sherwood, D. V. Steed, A. E. Taylor, S. E. Urner, Morgan Ward, W. M. Whyburn, Euphemia R. Worthington.

THE MATHEMATICAL ASSOCIATION OF AMERICA

THE ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-eighth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College of Education, Greeley, Colorado, on April 14 and 15, 1944. There were three sessions, the final session being a joint meeting with the Mathematics Section of the Eastern Division of the Colorado Education Association. Professor A. E. Mallory, Chairman of the Section, presided at each of the sessions.

The attendance was thirty-two, including the following twelve members of the Association: C. F. Barr, A. G. Clark, J. C. Fitterer, Leota C. Hayward, A. J. Kempner, Claribel Kendall, W. J. LeVeque, A. J. Lewis, A. E. Mallory, Greta Neubauer, A. W. Recht, E. C. Varnum.

At the business meeting the following officers were elected for the coming year: Chairman, Jack Britton, University of Colorado; Vice-Chairman, C. F. Barr, University of Wyoming.

The following papers were presented:

1. *Mathematics in the A. S. T. P.*, by Professor A. G. Clark, Colorado State College of Agriculture and Mechanic Arts.

2. *Vibrating membranes*, by August Newlander, University of Denver, introduced by the Secretary.

It was pointed out that the vibrations of a circular membrane are in many respects similar to the vibrations of a string. The displacement of a particular point of the membrane can be obtained by solving a partial differential equation of the second order. The solution of the differential equation can be expressed by means of an infinite series involving sines, cosines, and Bessel functions. The determination of certain constants dependent upon the boundary conditions involves the use of Fourier series and the Fourier-Bessel expansion of a function. After all constants have been determined, the result is an expression giving the displacement of any point of the membrane in terms of its position and the time after releasing the membrane from rest.

3. *Reduction of inverse tangents to integral arguments*, by Professor E. C. Varnum, University of Wyoming.

By a study of the operation $(a-b)/(1-ab)$ the speaker developed formulas by which inverse tangents of rational arguments may be reduced to those having integral arguments, the latter having been well tabulated in recent projects.

4. *A new definition of the Gamma function*, by Mrs. Margaret Matchett, University of Denver, introduced by the Secretary.

The speaker remarked that the Gamma function is uniquely defined by its

functional equation and the condition of logarithmic convexity. It was also stated that this definition yields an explicit expression for the Gamma function as an infinite product.

5. *Suggested changes in the content of high school mathematics*, by Ruth Hoffman, Denver Public Schools, introduced by the Secretary.

6. *Trends in grade placement of arithmetic fundamentals*, by L. B. Garner, Cameron School, Greeley, Colorado, introduced by Professor Mallory.

7. *I know better than I teach, now*, by Professor A. W. Recht, University of Denver.

In this address it was suggested that every teacher constantly fails to reach standards of teaching which he knows to be better. Twelve points for good teaching were submitted with the suggestion that they be used for periodic check-ups. One of these points, that of keeping the student informed of his standing day by day, was explained in detail. A method was shown by which the daily running averages of students in a whole class could be written down in two or three minutes from one setting of a slide rule.

8. *Early computation with Hindoo-Arabic numbers*, by Professor A. E. Mallory, Colorado State College of Education.

A. J. LEWIS, *Secretary*

CALENDAR OF FUTURE MEETINGS

Twenty-Eighth Summer Meeting, Montreal, Canada, June 23–25, 1945.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Secretary.

ALLEGHENY MOUNTAIN
ILLINOIS
INDIANA
IOWA
KANSAS
KENTUCKY
LOUISIANA-MISSISSIPPI
MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Washington, D. C., May, 1945
METROPOLITAN NEW YORK, Brooklyn, April 21, 1945
MICHIGAN
MINNESOTA
MISSOURI
NEBRASKA
NORTHERN CALIFORNIA, Berkeley, January 26, 1946
OHIO, Columbus, April 5, 1945
OKLAHOMA
PHILADELPHIA, Philadelphia, December 1, 1945
ROCKY MOUNTAIN
SOUTHEASTERN
SOUTHERN CALIFORNIA, Los Angeles
SOUTHWESTERN
TEXAS
UPPER NEW YORK STATE
WISCONSIN, Milwaukee, May, 1945

$$x^2\frac{d^2\phi}{dx^2} - (n+m-1)x\frac{d\phi}{dx} + nm\phi = rx^q\phi^{1+p}.$$

Such equations occur in the study of potential distributions in the presence of space charge. Particular cases are the equation of Thomas-Fermi,

$$\frac{d^2\phi}{dx^2} = \phi^{3/2}x^{-1/2},$$

and the equation of Langmuir,

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} = x^{-1}\phi^{-1/2}.$$

11. *Second solutions of certain differential equations associated with the theory of orthogonal polynomials*, by Professor L. W. Swanson, Coe College.

In solving a certain differential equation associated with orthogonal polynomials, a second solution had been omitted. The paper dealt with this second solution of the differential equation.

12. *Mathematics teaching procedure in the light of our experience with the army and navy schools*, by Professor O. C. Kreider, Iowa State College, Professor T. A. Bancroft, Iowa State College, and Professor W. M. Davis, Cornell College.

The speakers discussed the purpose, organization, difficulties, and successes of the army university centers and the navy educational programs overseas.

FRED ROBERTSON, *Secretary*

ANNUAL MEETING OF THE ROCKY MOUNTAIN SECTION

The twenty-ninth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, Colorado, on April 19 and 20, 1946.

The attendance was one hundred and twenty, including the following twenty-three members of the Association: H. H. Alden, C. F. Barr, William Betz, J. R. Britton, A. G. Clark, J. R. Everett, J. C. Fitterer, H. T. Guard, D. F. Gunder, Marian S. Gysland, Leota C. Hayward, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, M. L. Madison, A. E. Mallory, W. K. Nelson, Greta Neubauer, O. H. Rechard, A. W. Recht, G. A. Whetstone.

The following papers were presented:

1. *Spherical trigonometry by projection on a plane*, by Professor I. L. Hebel, Colorado School of Mines.

2. *A compatibility relation in the flow of an incompressible ideal fluid*, by Dr. G. A. Whetstone, Amarillo College.

By applying the procedures developed by Riquier for the study of partial differential equations to the usual four equations

$$\frac{\partial u_i}{\partial t} + u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + u_3 \frac{\partial u_i}{\partial x_3} = g_{x_i} - \frac{1}{\rho} \frac{\partial P}{\partial x_i}, \quad (i = 1, 2, 3)$$

and

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0,$$

under the assumption ρ = constant, the author was lead to a necessary and sufficient compatibility condition.

3. *Teaching mathematics in the army*, by Professor H. T. Guard, Colorado State College.

This paper consisted of a description of the curriculum and the methods of instruction in the United States Military Academy.

4. *On the definition of functions of a complex variable*, by Professor A. J. Kempner, University of Colorado.

5. *Tables for the power function for tests of hypotheses relating to Poisson distributions*, by Professor A. G. Clark, Colorado A. and M. College.

The speaker discussed devices, including recursion formulas, which serve to reduce the labor of computation in constructing tables for the function specified in the title. Such tables are useful in the construction of efficient sampling experiments.

6. *The present educational situation and the crisis in mathematics*, by William Betz, Public Schools of Rochester, N. Y.

The speaker rehearsed the role of mathematics in the recent war effort. He referred to the mathematical deficiencies of millions of our young men, first pointed out by Admiral Nimitz, and later substantiated by selective service tests. It was suggested that there be a re-examination of the controversy between "education" and mathematics. On the basis of significant quotations it was shown that the educational scene is one of confusion bordering on chaos. Mathematics cannot be adjusted to the prevailing educational philosophies without giving up its real purposes. Fortunately, a healthy reaction against the destructive forces in our educational policies is now in the making. In conclusion, the speaker outlined the remedial steps that seem to be necessary if we wish to help improve the situation.

7. *The Laplace transformation*, by Professor J. R. Britton, University of Colorado.

Professor Britton gave an expository talk on the Laplace transformation and its applications to the solution of boundary value and initial value problems. Some of the simpler transforms were derived, and application was made to the problem of a two mass, two spring vibrating system. A mechanical model

served to demonstrate the types of behavior indicated by the previously obtained solution.

8. *The necessary reconstruction of mathematics in the light of war experiences*, by William Betz.

This was an invited address, delivered at a joint session with the National Council of Teachers of Mathematics and the Mathematics Section of the Colorado Education Association. The speaker holds the position of specialist in mathematics for the public schools of Rochester, N. Y. The address dealt with the reports of various committees which have issued pronouncements on the problem of mathematical instruction in the post-war period. He presented a check list of mathematical objectives, and suggested methods for attaining these objectives.

J. R. BRITTON, *Secretary*

ANNUAL MEETING OF THE LOUISIANA-MISSISSIPPI SECTION

The twenty-third annual meeting of the Louisiana-Mississippi Section of the Mathematical Association of America was held at Louisiana Polytechnic Institute, Ruston, Louisiana, on Friday and Saturday, March 22 and 23, 1946. Professor I. C. Nichols was elected temporary chairman and presided at the Friday afternoon and Saturday morning sessions. Professor P. K. Smith presided at the dinner meeting.

There were fifty in attendance, including the following twenty-one members of the association: W. G. Banks, N. A. Court, J. C. Currie, W. L. Duren, Jr., L. M. Garrison, F. C. Gentry, R. V. Guthrie, Jr., J. A. Hardin, W. L. Johnson, H. T. Karnes, C. G. Killen, A. C. Maddox, Dorothy McCoy, B. E. Mitchell, I. C. Nichols, W. V. Parker, P. K. Rees, F. A. Rickey, H. F. Schroeder, C. D. Smith, H. L. Smith, P. K. Smith, V. B. Temple, J. F. Thomson, B. A. Tucker, Marelena White.

At the business meeting the following officers were elected for the coming year: Chairman, W. V. Parker, Louisiana State University; Vice-Chairmen, W. L. Johnson, Mississippi Southern College, Z. L. Loflin, Southwestern Louisiana Institute; Secretary-Treasurer, F. C. Gentry, Louisiana Polytechnic Institute. Invitations to meet at Mississippi Southern College in 1947, and at Southwestern Louisiana Institute in 1948 were accepted.

The following papers were presented at the Friday afternoon program:

1. *Esthetic and moral implications of the ark of the covenant*, by Professor B. E. Mitchell, Millsaps College.

The purport of the speaker's remarks was that if the golden rectangle (length/width = 1.618) and the Platonic rectangle (length/width = 1.732) have esthetic value as polygonal forms, then the Mosaic rectangle (length/width = 1.667) has also, since it differs from the arithmetic, geometric, and harmonic means of the other two by 0.008, 0.007, and 0.006 of a part, respectively.

APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirtieth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, Wyoming, on April 18 and 19, 1947. There were three sessions, with Professor Greta Neubauer of the University of Wyoming presiding at each.

There were sixty-four persons in attendance, including the following twenty-five members of the Association: C. F. Barr, D. L. Barrick, J. R. Britton, A. G. Clark, G. S. Cook, A. T. Craig, A. B. Farnell, H. T. Guard, Mrs. Leota C. Hayward, I. L. Hebel, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, A. E. Mallory, W. K. Nelson, K. L. Noble, O. H. Rechard, A. W. Recht, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, L. C. Snively, V. J. Varineau, Mrs. Lillie C. Walters.

At the business meeting the following officers were elected for the coming year: Chairman, H. T. Guard, Colorado State College of A. and M. A.; Vice-Chairman, I. L. Hebel, Colorado School of Mines; Secretary-Treasurer, J. R. Britton, University of Colorado. Invitations to meet at Colorado State College of A. and M. A. in 1948, and at Colorado School of Mines in 1949 were accepted.

The following papers were presented:

1. *Expansion of an arbitrary function in series of functions associated with Bessel functions*, by Professor Leonard Bristow, University of Wyoming, introduced by Professor C. F. Barr.

The author defined a set of functions by generalizing the Poisson integrals for Bessel and for Struve functions. For a suitable arbitrary function there was obtained an expansion resembling the generalized Schlomilch series.

2. *The solution of an integral equation*, by Professor W. H. Jurney, Colorado School of Mines, introduced by Professor I. L. Hebel.

3. *Note on functions of a matrix*, by Professor Clarence Ross, University of Denver, introduced by A. J. Lewis.

The matrix e^{kt} was expanded into a polynomial in k of degree not greater than $n-1$, where k is an $n \times n$ matrix. An application to the solution of linear homogeneous differential equations was explained.

4. *Bounds for the characteristic roots of a matrix*, by Professor A. B. Farnell, University of Colorado.

A brief history of this subject and related topics was presented. Let $\mathbf{A} = (a_{rs})$ be a square matrix of order n with complex numbers as elements. The equation $|\lambda\mathbf{I} - \mathbf{A}| = 0$, where $\mathbf{I}$ is the unit matrix and λ is a scalar, is called the characteristic equation of the matrix $\mathbf{A}$, and the roots λ_i, the characteristic roots. Several

new bounds for the characteristic roots were given. Let

$$\sum_s |a_{rs}| = R_r, \quad \sum_r |a_{rs}| = T_s, \quad \sum_s |a_{rs}| R_r = U_r, \quad \sum_r |a_{rs}| T_s = V_r.$$

Then $|\lambda|$ is not larger than any of the three numbers $\max_r (U_r)^{1/2}$, $\max_r (V_r)^{1/2}$, $\max_r (U_r V_r)^{1/4}$.

5. *A new method of approximating Fourier coefficients*, by G. L. Collins, Colorado School of Mines, introduced by Professor I. L. Hebel.

This speaker presented a simple method for evaluating the Fourier coefficients of a curve plotted to a predetermined scale. The essential idea of the method consisted of the use of a series of specially ruled transparent plastic sheets.

6. *Wallis' product for* π, by W. W. Mitchell, Jr., University of Colorado, introduced by Professor A. J. Kempner.

It was shown how Wallis determined the value of π between ever narrowing upper and lower bounds by a process of interpolation in a sequence of numbers related to the first quadrant areas under the curves $y=(1-x^2)^n$, $n=0, 1, 2, \cdots$.

7. *On complex roots of algebraic equations*, by Professor A. J. Kempner, University of Colorado.

Given an equation $f(z)=a_0z^n+\cdots+a^n=0$ with real coefficients and roots $z_k=x_k+iy_k$, $k=1, \cdots, n$, one knows how to establish by rational operations equations $G(x)=0$, and $H(y)=0$, each of degree n, such that each x_k is among the roots of the first, each y_k among the roots of the second equation. However, this leaves in each equation n^2-n roots unaccounted for. The location of these roots is determined by the theorem: The n roots of $G(x)=0$ are $x_j=\frac{1}{2}(z_k+z_l)$, $k, l=1, 2, \cdots, n$; the n roots of $H(y)=0$ are $y_j=\frac{1}{2}(z_k-z_l)$. A striking geometrical interpretation in the plane of complex numbers is possible.

Results are extended in toto to equations with complex coefficients without raising the degrees of $f(z)$, $G(x)$, $H(y)$ by letting $z=u+v$ with the restriction that with $u+v$, the number $u-v$ is also a root of $f(z)=0$. The function $G(x)$ is of the form $f(x)\cdot K^2(x)$, K being of degree $(n^2-n)/2$; $H(y)$ is of the form $y^nL(y^2)$, L being of degree $(n^2-n)/2$ in y^2. Similar results hold for the equation for r and for $e^{i\phi}$, $z=re^{i\phi}$.

8. *Statistical inference*, by Professor A. T. Craig, University of Iowa.

This paper was devoted to an exposition of the construction of a mathematical system adequate to furnish methods for drawing inferences from statistical data. The paper included an introduction to the Neyman-Pearson theory of testing statistical hypotheses.

9. *Is mathematics out of this world?* by Professor A. W. Recht, University of Denver.

The main thesis of this paper is that mathematics as presented in high schools and in colleges of liberal arts is out of this world in the sense that the principles of mathematics are set up in the classical and traditional way instead of in the

way in which they occur in real life. The suggestion is made that textbooks be written with the psychological approach by mathematicians who are also experts in fields of real application of mathematics. Problems should be presented as they occur in real life. It is only in this way that mathematics will be able to maintain the high reputation it has acquired in the atomic age; it is only in this way that students in the high schools and colleges will be kept interested in mathematics of reality, and not dazed by operations in a world of unreality.

10. *General mathematics*, by Professor Fred McCune, Colorado State College of Education.

In this paper the author asks why courses in "general" mathematics should duplicate training given in standard algebra and geometry courses. He believes that training in the fundamental skills of arithmetic is more important for the average secondary school student.

11. *The training of mathematics teachers*, by Professor K. H. Stahl, University of Colorado, introduced by the Secretary.

The attitude developed by students in mathematics has great influence not only on them, but also on us as teachers of mathematics. The teacher controls to a great extent the attitudes developed by members of the class, and it is therefore important that all teachers have a proper influence on their students. If the teacher himself is not well grounded in the material to be presented, it is quite unlikely that his influence will be wholesome. In all probability many persons become certified to teach in the elementary schools with very poor backgrounds in arithmetic. It is recommended that college teachers concern themselves with the mathematical preparation of elementary teachers.

12. *Report on the entrance requirement changes at the University of Colorado*, by Professor A. J. Kempner, University of Colorado.

Professor Kempner reported on the recent changes in entrance requirements for the Colleges of Arts and Sciences at the University of Colorado. All students must now offer three units of high school English, besides nine other units in "academic" subjects. These may not be selected arbitrarily; but students may enter the College without any high school work in any chosen one of the four large fields: foreign language, mathematics, natural sciences, social sciences. Under some arrangements students may even enter without any high school work in any chosen two of these fields.

There is opposition within the faculty to this scheme. Departments were not properly consulted.

In mathematics the situation is aggravated by the fact that a student who offers mathematics on his entrance requirements may substitute "high school arithmetic" and "general mathematics" for high school algebra and high school geometry which were required under the old rule.

Criticism of this last regulation centers around the fact that "general mathematics," as the term is understood in our part of the country, represents mathe-

matics courses which were introduced specifically for students who were either admittedly incapable of carrying the standard algebra and geometry courses, or who did not intend to go on to college training, but who wanted vocational courses in mathematics with a minimum of emphasis on theory and logical development. The department of mathematics refuses to recognize these courses as adequate prerequisites for college mathematics. These courses must not be confused with "unified mathematics" courses, which in some parts of the country go under the name of "general mathematics." For these, a strong case can be made out.

The mathematics department consulted groups of Colorado high school teachers, particularly mathematics teachers. The results were revealing. Over a hundred mathematics teachers of the Denver Section of C. E. A. protested unanimously against the changes. The Grand Junction Section, one of the other two sections in the state, sent a similar protest. The mathematics departments of two of the large Denver high schools, Denver North and Denver East, sent unanimous petitions to the president of the University, and so forth.

High school administrators generally favor the new rules, and regret that they do not go farther than they do. There exists scattered disapproval among them, but it has so far not become organized.

Our experience in Colorado proves that we have powerful allies among the high school teachers; they suffer and chafe under the steady deterioration of the standards and are, at least in Colorado, as a group more aware of the dangers and implications of the situation, and far more willing to fight for its improvement, than are college and university faculties.

In the lively discussion which followed the speaker's remarks, sentiment was opposed overwhelmingly to the elimination of mathematics as an entrance requirement, and as bitterly opposed to the admission of high school arithmetic and "general mathematics" in place of algebra and geometry.

J. R. BRITTON, *Secretary*

APRIL MEETING OF THE LOUISIANA-MISSISSIPPI SECTION

The twenty-fourth annual meeting of the Louisiana-Mississippi Section of the Mathematical Association of America was held at Mississippi Southern College, Hattiesburg, Mississippi, on Friday and Saturday, April 25 and 26, 1947. Professor W. V. Parker, Chairman of the Section, presided at the Friday afternoon and Saturday morning sessions. Professor W. L. Johnson, Vice-Chairman for Mississippi, presided at the joint dinner with the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

The attendance was sixty-five including the following thirty members of the Association: T. A. Bickerstaff, H. E. Buchanan, Margaret R. Davis, W. L. Duren, Jr., L. M. Garrison, F. C. Gentry, A. Gilmore, W. C. Griffith, W. L. Johnson, H. T. Karnes, C. G. Killen, Z. L. Loflin, Dorothy McCoy, A. C. Maddox, B. E. Mitchell, S. B. Murray, I. C. Nichols, W. V. Parker, P. K. Rees, F. A. Rickey,

Professor Doyle conducted a panel discussion on ways and means to encourage high schools to strengthen their mathematics programs. It was remarked that colleges should better coordinate what they expect of freshmen, and make better use of placement tests.

P. R. RIDER, *Secretary*

APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-first annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado A. and M. College, Fort Collins, Colorado, April 23 and 24, 1948. Professor H. T. Guard presided at all the sessions.

Among the eighty-one persons who registered were the following thirty members of the Association: C. F. Barr, D. L. Barrick, W. G. Brady, J. R. Britton, F. M. Carpenter, A. G. Clark, G. S. Cook, A. B. Farnell, F. N. Fisch, H. T. Guard, Leota C. Hayward, I. L. Hebel, Ruth I. Hoffman, LeRoy Holubar, Burrowes Hunt, J. A. Hurry, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, A. J. Lewis, M. L. Madison, A. E. Mallory, W. K. Nelson, Greta Neubauer, K. L. Noble, Nathan Schwid, S. R. Smith, L. C. Snively, V. J. Varineau, Lillie C. Walters.

At the business meeting, the officers elected for the coming year were: Chairman, Professor I. L. Hebel, Colorado School of Mines; Vice-Chairman, Professor A. J. Lewis, University of Denver; Secretary-Treasurer, Professor J. R. Britton, University of Colorado. Professor A. J. Lewis was also elected Sectional Governor for a term of three years. A resolution commending Professor Abraham Wald for the excellence of his invited addresses was unanimously adopted.

The program of papers presented was as follows:

1. *A method of defining the real number system*, by Robert Howerton, University of Denver, introduced by A. J. Lewis.

2. *A slow-motion algorithm*, by Burrowes Hunt, University of Colorado.

The euclidean algorith for two relatively prime integers $a>b$ which leads to the equations

$$a = q_1 b + r_1, \qquad b = q_2 r_1 + r_2, \cdots$$

is modified by taking each $q_i = 1$. This algorithm terminates if and only if a and b are successive integers of the Fibronacci sequence. The least positive remainder is 1 if and only if, as a regular continued fraction, $a/b = (1; 1, \cdots, 1, k)$, k being an arbitrary positive integer. If $a/b = (1; 1, \cdots, 1_n, a_0, a_1, \cdots, a_k)$, and $(a_0; a_1, \cdots, a_k) = q/r$, the algorithm gives a least positive remainder r on the nth step.

3. *A note on expansion of determinants*, by Professor W. R. Eikelberger, University of Denver, introduced by A. J. Lewis.

4. *An approximation to the solution of a non-linear partial differential equation*, by Professor Nathan Schwid, University of Wyoming.

The differential equation of heat conduction

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right)$$

is non-linear when the conductivity K is a function of u. Here the diffusivity $K/c\rho$ is taken as $\alpha^2 + \beta^2 u$, where the ratio β^2/α^2 is small. The heat is considered as flowing in the x direction only in a plate of finite width. The solution of the resulting non-linear equation is approximated by a modification of a method given by Kirchoff about 1890 in the *Annalen der Physik* for an analogous problem involving flow of heat in one direction in a semi-infinite solid with the same type of diffusivity as above.

5. *Expansion of functions in combinations of generalized hypergeometric functions*, by Professor Leonard Bristow, University of Wyoming, introduced by C. F. Barr.

The purpose of this paper is to obtain the expansion of a suitable arbitrary function of a real variable in a series of solutions of a self adjoint differential equation of the Cauchy or Euler type containing a parameter. There are one-point boundary conditions (taken to be at $x=1$) together with regularizing conditions at the regular singular point (taken to be at $x=0$) of the differential equation. A Green's function is obtained. Fourier series, Fourier-Bessel, and Dini expansions in Bessel functions are obtained as special cases.

6. *Introduction to sequential analysis*, by Professor Abraham Wald, Columbia University.

7. *Principles of sequential analysis*, by Professor Abraham Wald, Columbia University.

These papers by Professor Wald were invited addresses.

8. *Ivory Towers*, by Miss Ruth I. Hoffman, Denver, Colorado.

There is need for college mathematics teachers to step down and become acquainted with the content of secondary mathematics, the problems and factors that influence the type of courses offered, and the quality of these courses. The university people should know of the valiant struggle that mathematics teachers in secondary education are making to keep up with modern educational trends, and to meet the needs of the present student body while still teaching sound mathematics, and even showing the beauty, as well as the usefulness, of mathematics.

9. *On certain equations involving radicals*, by Harlan Bartram, University of Colorado, introduced by A. J. Kempner.

The following problem was discussed: Given

$$\sqrt{a+bi}+\sqrt{c+di}=f, \qquad i=\sqrt{-1}.$$

If a, b, c, and d are real, and the signs of the radicals are properly chosen, when will f be real? The necessary and sufficient condition for this was found to be that $(b^2-d^2)^2=4(a-c)(ad^2-cb^2)$.

10 *Some teaching devices in undergraduate mathematics*, by Professor S. R. Smith, University of Wyoming.

Experience has shown that the majority of students entering college have difficulty in mathematics courses. At least part of this difficulty is due to lack of organization of their work, particularly in the solution of problems, and to the interpretation of the solutions found. Teaching devices, not necessarily new, are suggested to facilitate the solution of systems of quadratic equations, the discussion and sketching of plane curves in analytic geometry, and the application of the first and second derivatives in calculus.

11. *The book says so*, by Professor A. E. Mallory, Colorado State College of Education.

In this paper attention is called to the importance of teaching the connection between the solution of equations and the concept of a function.

12. *What constitutes good mathematics for undergraduates?* by Professor A. G. Clark, Colorado A. and M. College.

The author used the article, *Can We Teach Good Mathematics to Undergraduates?* by R. G. Helsel and T. Radó, which appeared in the January, 1948 issue of this MONTHLY, as the basis for his discussion. He agreed in part with the opinions of Helsel and Radó, but the extent of the agreement was dependent upon the connotation given the word "elegant," a term which mathematicians seem to have appropriated. The concept of "efficient" mathematics for undergraduates was presented.

13. *Report on entrance requirements*, by Professor A. J. Kempner, University of Colorado.

This paper was a brief report on the discussions which were held and the resolutions which were passed at the meetings of the Mathematical Association at Athens, Georgia, and the National Council of Teachers of Mathematics at Indianapolis, Indiana, in connection with the problems of lowering college entrance requirements and standards in general, and those pertaining to mathematics in particular.

Following a short discussion of the last paper, the following resolution was unanimously adopted: The Rocky Mountain Section of the Mathematical Association of America approves whole-heartedly the recent action of the Mathematical Association and the National Council of Teachers of Mathematics in expressing their desire for the closest cooperation in the critical problems confronting secondary and college mathematics.

J. R. BRITTON, *Secretary*

CALENDAR OF FUTURE MEETINGS

Joint Meeting with American Society for Engineering Education, Troy, New York, June 20–21, 1949.

Thirty-first Summer Meeting, Boulder, Colorado, August 29–30, 1949.

Thirty-third Annual Meeting, New York City, December 30, 1949.

ALLEGHENY MOUNTAIN West Virginia University, Morgantown, May 7, 1949
ILLINOIS, Bradley University, Peoria, May 13–14, 1949
INDIANA, University of Notre Dame, Spring, 1949
IOWA, Drake University, Des Moines, April 15–16, 1949
KANSAS, Manhattan, April 2, 1949
KENTUCKY
LOUISIANA-MISSISSIPPI, University of Mississippi, Oxford, Spring, 1949
MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA
METROPOLITAN NEW YORK Brooklyn College, April 9, 1949
MICHIGAN
MINNESOTA
MISSOURI
NEBRASKA, Lincoln, May, 1949
NORTHERN CALIFORNIA
OHIO, Ohio State University, Columbus, April 2, 1949
OKLAHOMA
PACIFIC NORTHWEST, Oregon State College, Corvallis, March 25–26, 1949
PHILADELPHIA, Haverford College, November 26, 1949
ROCKY MOUNTAIN, Colorado School of Mines, Golden, April, 1949
SOUTHEASTERN, University of Alabama, University, March 18–19, 1949
SOUTHERN CALIFORNIA, John Muir Junior College, Pasadena, March 12, 1949
SOUTHWESTERN
TEXAS, Denton, Spring, 1949
UPPER NEW YORK STATE, University of Buffalo, April 30, 1949
WISCONSIN, Lawrence College, Appleton, May 14, 1949

any one college or university, the team of three must be named on the application. Fewer than three from one college or university may compete as individuals.

The examination may be given at any place where a team, or at least three candidates, can be assembled. Exceptions to this rule may be made by the Director in cases where it would entail unusual inconvenience to a contestant. Sealed copies of the examinations will be sent to the supervisor of the examination in time for the examination day and are not to be opened before the hour set.

The prizes to be awarded to the departments of mathematics of the institutions with the winning teams are $400, $300, $200, and $100, in the order of their rank. In addition, there will be prizes of $40, $30, $20 and $10 awarded to the members of these teams according to the rank of the team; a prize of $50 to each of the five highest contestants and a prize of $20 to each of the succeeding five highest contestants. Each of the winners will receive a suitable medal. Honorable mention will be given to several teams next in order after the four winning teams and to several individuals next in order after the ten individual winners. For further encouragement of the Competition, there will be awarded at Harvard University (or at Radcliffe College in the case of a woman) an annual $1500 William Lowell Putnam Prize Scholarship to one of the first five contestants, this to be available either immediately or on the completion of the student's undergraduate work.

Reports on the nine previous competitions and examination questions will be found in this MONTHLY for May, 1938, 1939, 1940, 1941, 1942, October, 1946. August–September, 1947, December, 1948, and August–September, 1949.

THE MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-second annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado School of Mines, Golden, Colorado, April 22 and 23, 1949. There were three sessions with Professor I. L. Hebel presiding at each.

Among the ninety-eight persons who registered were the following forty-nine members of the Association: R. V. Anderson, W. G. Brady, Leonard Bristow, G. L. Burton, F. M. Carpenter, Nancy V. Cheney, A. G. Clark, G. S. Cook, G. A. Culpepper, David DeVol, Mary C. Doremus, W. R. Eikelberger, O. J. Falkenstern, F. N. Fisch, Katherine C. Garland, R. H. Glass, H. T. Guard,

Leota C. Hayward, I. L. Hebel, H. K. Hilton, Ruth I. Hoffman, LeRoy Holubar, R. J. Howerton, Burrowes Hunt, J. A. Hurry, C. A. Hutchinson, B. W. Jones, A. J. Kempner, Claribel Kendall, A. J. Lewis, M. L. Madison, W. K. Nelson, Greta Neubauer, K. L. Noble, O. H. Rechard, A. W. Recht, L. W. Rutland, Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, K. H. Stahl, J. M. Staley, J. F. Stockman, E. P. Tovani, V. J. Varineau, W. W. Varner, J. F. Wagner, Lillie C. Walters.

At the business meeting, the following officers were elected for the coming year: Chairman, Professor A. J. Lewis, University of Denver; Vice-Chairman, Professor A. E. Mallory, Colorado State College of Education; Secretary-Treasurer, Professor J. R. Britton, University of Colorado.

The program of papers presented was:

1. *The Lill circle*, by Professor (Emeritus) W. J. Hazard, University of Colorado, introduced by A. J. Kempner.

The Lill circle for finding the roots of $ax^2+bx+c=0$, mentioned by Maurice d'Ocagne in his "Calcul Simplifie et Nomographie" as a special case of a general graphic approximation to the roots of the nth degree equation, is here shown to be the XY-plane section of the paraboloid of which the XZ section is the usual parabola plotted from $y=f(x)=x^2+bx/a+c/a$. Both the circle and the parabola show the roots of the equation as the points where the X axis pierces the surface of the paraboloid.

2. *A four-number game*, by Mr. Burrowes Hunt, University of Colorado.

In *Scripta Mathematica* (March 1948) Benedict Freedman showed that if from an ordered set of four positive integers one forms the set of the absolute values of their differences taken in cyclic order, and repeats this process, one arrives at the set 0, 0, 0, 0, in a finite number of steps. Mr. Hunt considered the same game for sets of four positive real numbers. The result is that, in general, any set of real numbers leads to the set 0, 0, 0, 0, but there is a double infinity of exceptional sets all of whose differenced sets consists of positive numbers. If x is the positive root of either of the equations $x^3=1\pm(x^2+x)$, then the set 0, 1, $1+x$, $1+x+x^2$ is exceptional, as is any set derived from it by replacing each element a by $ka+m$.

3. *Is mathematics practical?* by Mr. R. J. Howerton, Regis College.

The author discussed the question of "pseudo-practicality" of problems in textbooks, and the growing tendency for mathematics teachers to be on the defensive with respect to their subject. The problem of teaching the "why" of mathematics rather than the "how" from elementary classes through college work was discussed. Placing mathematics on a plane with history, psychology, literature, and other cultural subjects was proposed. The author maintained that, for the average liberal arts college student, mathematics was highly impractical, and that the only true justification for the subject was on a cultural basis.

4. *The construction of a statistical quality control chart and some interpretations to be made from it*, by Professor J. F. Wagner, University of Colorado.

The bases of a statistical quality control chart are these: (1) Variation in the manufacture or measure of a product quality is to be expected; (2) The data themselves determine the expected spread of the data; (3) The control, or action, limits are then set at such values as to minimize, economically, the waste of looking for trouble when there is none and not looking for trouble when there is some; (4) We shall use the so-called 3σ limits which include all but about one out of every 400 variates in the distribution, unless there is a significant departure from normalcy; (5) The

occurrence of a point outside the control limits shall then be the signal to look for an assignable cause of variation beyond the natural variability of the process.

With the aid of a table of data and a control chart grid which was distributed to each member of the audience, an actual example of this technique taken from industry was discussed. The points lying outside of the control limits were identified with their assignable cause. The meaning of a run or loss of random scattering of the points on a control chart was presented.

5. *A construction for a monoidal quartic*, by Mr. W. G. Brady, University of Wyoming.

In this paper a 4:1 correspondence between the points of two conics is shown to lead to a monoidal quartic, and configurations leading to various types of triple points are discussed.

6. *The central limit concept in an elementary course in statistics*, by Professor H. T. Guard, Colorado A. and M. College.

The author discussed some of the pedagogical problems encountered in the teaching of elementary statistics to students having little mathematical background. Sampling experiments for the verification of the central limit theorem were discussed.

7. *Delta-V, a conical shell*, by Professor F. M. Carpenter, Colorado School of Mines.

For certain types of volumes of revolution the use of a cylindrical element leads to the correct numerical result because of compensating errors. Often an exercise can be analyzed and solved correctly in cartesian coordinates by using a conical shell for the element of volume.

8. *Degenerate conics*, by Professor A. J. Lewis, University of Denver.

The author shows some of the elementary methods of determining when the general equation of the second degree in two variables will give a degenerate conic.

9. *Linkages in relation to certain aspects of college geometry*, by Professor M. L. Madison, Colorado A. and M. College.

The author gave a brief historical sketch of linkages from the time James Watt patented his "parallel motion" in 1874. The use of linkwork models, a number of which were exhibited, as aids in teaching college geometry, analytic geometry, and plane geometry was discussed.

10. *Materials for teaching mathematical meanings in the elementary school*, by Professor Lucy L. Rosenquist, Colorado State College of Education, introduced by A. E. Mallory.

The mathematical meanings that need to be taught in the elementary school are the various relationships between "groups." These groups are the chance groupings met in everyday experience, and the standard groupings of our number system. The processes of addition, subtraction, multiplication, and division are methods of changing chance groupings into standard groupings. Children learn to handle groups with progressively more mature methods as their understanding of groups and group relationships develops. Concrete materials which aid in developing this understanding should have the following characteristics: (1) Compact contours; (2) Patterned arrangement, or capability of being easily arranged in patterns; (3) Freedom from elements that embed the number ideas. These materials are not to be used as demonstration materials by the teacher. Pupils should have opportunities to manipulate materials in discovering solutions to their problems, and in recognizing constant relationships between groups. The explanation of these individual discoveries to the class affords opportunity for clarification of the ideas, and stimulates insight into the meaning of the number system and the computational processes.

11. *Looking backward and forward*, by Professor A. J. Kempner, University of Colorado.

After the program of papers, a joint meeting was held with the Mathematics Section, Eastern Division, Colorado Education Association. There were raised problems relating to the reorganization of mathematics training in the schools of Colorado. Later, a panel, consisting of representatives from elementary, secondary, and college levels attempted to give solutions to these problems.

W. K. NELSON, *Acting Secretary*

THE APRIL MEETING OF THE KANSAS SECTION

The thirty-fourth annual meeting of the Kansas Section of the Mathematical Association of America was held at Kansas State College in Manhattan, on Saturday, April 2, 1949. Sessions were held in the morning and afternoon. Professor R. G. Sanger presided at these sessions. The morning session was a joint meeting with the Kansas Association of Teachers of Mathematics.

The attendance was one hundred fifty-five including the following forty-two members of the Association: Sister M. Nicholas Arnoldy, R. W. Babcock, Wealthy Babcock, Florence L. Black, Frances N. Breneman, Virginia L. Chatelian, W. R. Cowell, L. E. Curfman, Lucy I. Dougherty, Paul Eberhart, Walter Fleming, Albert Furman, W. H. Garrett, Laura Z. Greene, Edison Greer, J. R. Hanna, K. C. Hsu, Emma Hyde, W. C. Janes, L. E. Laird, C. F. Lewis, Anna Marm, Margaret E. Martinson, Thirza A. Mossman, E. P. Northrop, S. T. Parker, P. S. Pretz, G. B. Price, O. M. Rasmussen, C. B. Read, C. A. Reagan, L. M. Reagan, R. G. Sanger, G. W. Smith, R. G. Smith, W. T. Stratton, C. B. Tucker, Gilbert Ulmer, E. B. Wedel, A. E. White, Ferna E. Wrestler, P. M. Young.

At the business meeting the following officers were elected for next year: Chairman, R. G. Smith, Kansas State Teachers College; Vice-Chairman, L. M. Reagan, University of Wichita; Secretary-Treasurer, Anna Marm, Bethany College.

The following papers were presented:

1. *The role of mathematics in general education*, by Professor E. P. Northrop, College of the University of Chicago.

The speaker deplored the fact that teachers of mathematics, in attempting to formulate courses within a program of general education, are in the habit of thinking in terms of subject-matter (content) alone, or at most of deriving ends (aims, objectives) from subject-matter. He pointed to general lack of agreement among teachers concerning what subject-matter is most appropriate, and expressed skepticism of the meaningfulness of courses constructed from the standpoint of subject-matter. He proposed an approach to the problem through initial consideration of ends—of the kinds of abilities and understanding the student ought to acquire from a general education. He listed those ends which he regarded as important and which he felt could best be achieved through the study of mathematics, and pointed out that at least some of them could be served equally well by alternative choices of subject-matter. Briefly put, the speaker argued for an ends-to-means rather than a means-to-ends (or means alone) approach to the formulation of mathematics courses

As the Iowa Section meets jointly with the Iowa Academy of Science, the papers presented at this meeting are eligible to compete for the prize. The committee selected the paper by Professor E. W. Anderson on *Elastic deflection of a split ring* as the entry from Mathematics.

FRED ROBERTSON, *Secretary*

APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-third annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Denver, Denver, Colorado, April 28 and 29, 1950. There were three sessions with Professor A. J. Lewis presiding at each.

The meeting was attended by approximately one hundred persons including the following fifty-three members of the Association: R. V. Anderson, C. F. Barr, D. L. Barrick, W. G. Brady, W. E. Briggs, J. R. Britton, R. L. Calvert, R. C. Campbell, F. M. Carpenter, F. L. Celauro, Nancy V. H. Cheney, A. G. Clark, G. S. Cook, G. A. Culpepper, L. C. Dawson, David DeVol, J. R. Everett, O. J. Falkenstern, A. B. Farnell, F. N. Fisch, Katherine C. Garland, H. T. Guard, R. R. Gutzman, Leota C. Hayward, I. L. Hebel, LeRoy Holubar, Burrowes Hunt, C. A. Hutchinson, B. W. Jones, Claribel Kendall, A. J. Lewis, C. C. MacDuffee, J. C. McKenzie, W. K. Nelson, Greta Neubauer, K. L. Noble, D. O. Patterson, H. C. Peterson, A. W. Recht, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, M. E. Sperline, K. H. Stahl, J. M. Staley, P. O. Steen, J. F. Stockman, E. P. Tovani, V. J. Varineau, W. W. Varner, Lillie C. Walters.

At the business meeting, the following officers were elected for the coming year: Chairman, Professor D. O. Patterson, Colorado State College of Education; Vice-Chairman, Professor F. L. Celauro; Secretary-Treasurer, Professor J. R. Britton, University of Colorado.

The following program of papers was presented:

1. *A note on operators*, by Mr. H. C. Peterson, University of Denver.

2. *A nonlinear differential equation of heat conduction type*, by Professor Nathan Schwid, University of Wyoming.

The solution of the differential equation for the flow of heat in one direction, when the thermal conductivity K and the specific heat C each is of the form $\alpha+\beta u$, where u is the temperature and the ratio β/α is small, was considered for a semi-infinite and for a finite bar. With suitable boundary conditions a solution can be obtained if the ratio K/C depends upon the temperature.

3. *Some properties of Fibonacci sequences*, by Mr. David DeVol, University of Colorado.

Defining Fibonacci sequences by the property $u_{n+1}=u_n+u_{n-1}$, several relations between the terms are easily obtained by the manipulation of two-by-two matrices whose elements are terms of the sequence. The speaker concluded by pointing out a geometric connection between the Fibonacci sequences and the sequences of polygonal numbers.

4. *Determination of a class of solvable biquartic equations*, by Professor L. C. Dawson, Colorado A and M College.

Solvable biquartic equations of the form

$$x^8 + ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = 0$$

may be formed by assigning certain real values to the coefficients. We impose the condition that the general biquartic be expressed as a difference of two squares, thus reducing the given biquartic to two quartics each solvable by known methods. This procedure yields two necessary conditions: $a^2-4c=0$ and $(a+b)^2-4(c+d+e+f+g)=0$, whereby the coefficients may be chosen so that the biquartic decomposes into a pair of quartics. A similar procedure is applicable to the determination of a class of solvable bicubics.

5. *A problem in the theory of runs*, by Professor A. G. Clark, Colorado A and M College.

In a set of independent trials of an event where the probability of a specified outcome is constant, an asymptotic expression was obtained for P_n, the probability that a run of given length will result for the first time with the nth trial. With this definition of n, $E(n)=\sum_{j=1}^{n} P_j=1/2$. Furthermore, $E[n-E(n)]^2$ was determined as a measure of the lack of stability of n. Attention was focussed on the extent to which the solution by elementary methods of this problem in the classical theory of probability makes use of subject matter pertinent to nearly every course offered in the usual undergraduate curriculum in mathematics.

6. *A note on the calculation of residues*, by Professor C. A. Hutchinson, University of Colorado.

The expression for the residue of an analytic function at a pole of order n is obtained as a determinant of order $n-1$. In an illustrative example, the determinant is evaluated by means of a second-order linear difference equation.

7. *Linear equations without determinants*, by Professor C. C. MacDuffee, University of Wisconsin.

8. *Cross-purposes in education*, by Professor C. C. MacDuffee.

This was an evening address at which Professor MacDuffee was the guest speaker.

9. *Progress in mathematics by the U.S.S.R. since World War II*, by Mr. R. J. Howerton, Regis College.

Since January 1948, all Russian scientific journals have been published in the languages of Russia only. A survey was made of the titles and authors of the papers appearing in the six leading mathematical journals of the U.S.S.R. for 1948–49. Four of these were carried back through 1947 and one, *Akademiya Nauk, S.S.S.R., Doklady*, was carried back through 1946 since it carried the greatest number of papers for 1947–48. A classification was then made of the papers into six general categories of mathematics. The following conclusions were drawn: (1) There was a general increase of activity in 1949 over 1948, the greatest increase being shown by topology and group theory; (2) The most profitable journal for an American (Russian reading) would be *Akademiya Nauk, S.S.S.R., Doklady*, unless he were in the field of applied mathematics, in which case *Prikladnaia Matematika i Mechanika* would be the most fruitful; (3) Due to the difficulty caused by transliteration from the Latin alphabet to the Russian and back again, no conclusive evidence was obtained to show an increase in the number of Germanic names among the authors of papers; (4) The work of the Russians seems to be of the highest quality and would do credit to any American Journal. (The same results were obtained by Mr. Paul W. Howerton in the field of organic chemistry. See *Russian literature in the field of organic chemistry*, Journal of Chemical Education, April, 1949); (5) Several writers have turned out a large volume of work, the most prolific being N. G. Chebotarev, with ten papers in two years; (6) There is no evidence of any political slant to any of the papers read.

10. *Problems in the training of teachers of mathematics*, Professor A. W. Recht, University of Denver.

After the program of papers, a joint meeting was held with the Mathematics Section, Eastern Division, Colorado Education Association. The discussion was concerned with the formation of the Colorado Council of Teachers of Mathematics.

J. R. BRITTON, *Secretary*

MAY MEETING OF THE INDIANA SECTION

The twenty-seventh annual meeting of the Indiana Section of the Mathematical Association of America was held at Wabash College, Crawfordsville, Indiana, on Saturday, May 6, 1950. Two sessions were held at which Professor Ralph Hull of Purdue University, Chairman of the Section, presided.

There were sixty-two in attendance including the following thirty-six members of the Association: Juna L. Beal, L. G. Black, Stanley Bolks, C. F. Brumfiel, G. E. Carscallen, W. W. Chambers, T. E. Cheatham, H. E. Crull, M. W. DeJonge, V. E. Dietrich, P. D. Edwards, W. R. Fuller, E. L. Godfrey, Michael Golomb, S. H. Gould, G. H. Graves, J. R. Hadley, N. R. Hughes, Ralph Hull, M. W. Keller, E. L. Klinger, R. A. Lufburrow, R. B. Merrill, P. T. Mielke, P. M. Nastocoff, C. C. Oursler, P. W. Overman, Philip Peak, J. C. Polley, Arthur Rosenthal, M. E. Shanks, Jane A. Uhrhan, R. O. Virts, J. L. Wilson, Florence A. Wirsching, W. D. Wood.

The following officers were elected: Chairman, H. E. Crull, Butler University; Vice-Chairman, M. W. Keller, Purdue University; Secretary, J. C. Polley, Wabash College.

On the matter of awarding Association medals as prizes in high school mathematics contests the chairman was authorized to appoint a committee with power to act. The committee was instructed to investigate the possibility of making such awards in connection with the Indiana State Mathematics Contest and the Indiana Science Talent Search.

The annual meeting of 1951 will be held on Saturday, May 5, the place of meeting to be announced later.

The following papers were presented:

1. *Mathematics for engineers*, by Professor M. E. Shanks, Purdue University.

Of two significant trends in mathematics for freshmen, terminal courses designed solely to fill the cultural gap, and a unified non-compartmentalized course in algebra, trigonometry, and analytic geometry, in part cultural but chiefly motivated by a need for bringing so called advanced ideas down into the undergraduate program, the latter was emphasized. In the author's opinion the need of the modern engineer for the advanced ideas, for pure mathematics, is essential, and once the engineer recognizes that the less traditional course could clearly increase his mathematical "power" he would welcome the change.

2. *A proof of the existence of a real zero for a polynomial of odd degree with real coefficients which is not dependent on continuity*, by Professor J. C. Polley, Wabash College.

Since it is desirable for students in the natural science area to complete the calculus by the end of their sophomore year at the latest, and since it is not possible to assume that a beginning freshman has any knowledge of trigonometry, it is proposed that trigonometry be made an incidental part of a course in analytic geometry. The material of trigonometry can be presented as an application of the analytic method and can replace other illustrations. In particular, the theorem of Pythagoras in its analytic form (the "distance formula") leads immediately to the law of cosines and to the addition formulas. It was the opinion of the speaker that the point of view that this approach would entail would serve the student's needs far more than the excessive emphasis on the solution of triangles which is so often found in standard courses in trigonometry.

12. *A note on the effect of high school preparation in mathematics as measured by the Nebraska mathematics classification examination*, by H. M. Cox, University of Nebraska.

Questions on general mathematics (Part I) and questions on elementary high school algebra (Part II) differentiate sharply between students with two (or less) and three (or more) semesters of high school algebra. However, and for the effective use of the examination, there occur gradations in ascending order of mean score in accordance with the amount and variety of high school courses in mathematics. The Nebraska examination correlates satisfactorily with Section VI (Mathematics) of the Cooperative General Culture Test.

LULU L. RUNGE, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-fourth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado State College of Education, Greeley, Colorado, on Friday and Saturday, April 20 and 21, 1951. Professor Dale O. Patterson, Chairman of the Section, presided at all the sessions.

Of the approximately one hundred thirty persons who registered, the following fifty were members of the Association: C. F. Barr, W. E. Briggs, J. R. Britton, R. G. Buschman, F. M. Carpenter, A. G. Clark, C. H. Cook, G. S. Cook, David Devol, Mary C. Doremus, A. B. Farnell, F. N. Fisch, R. R. Gutzman, Leota C. Hayward, I. L. Hebel, LeRoy Holubar, Burrowes Hunt, C. A. Hutchinson, B. W. Jones, M. W. Jones, A. J. Kempner, Claribel Kendall, J. S. Leech, Garner McCrossen, H. C. McKenzie, M. L. Madison, D. C. B. Marsh, Jr., W. K. Nelson, Greta Neubauer, K. L. Noble, D. O. Patterson, H. C. Peterson, Lily B. Powell, G. B. Rice, O. H. Rechard, A. W. Recht, L. W. Rutland, Jr.. Nathan Schwid, W. N. Smith, L. C. Snively, M. E. Sperline, K. H. Stahl, P. O. Steen, J. F. Stockman, E. P. Tovani, E. L. Vanderburgh, V. J. Varineau, W. W, Varner, J. F. Wagner, Lillie Walters.

At the business meeting, it was voted to hold the next annual meeting at Western State College, Gunnison, Colorado, in May, 1952. The following officers were elected for the ensuing year: Chairman, Professor C. H. Cook, Western State College; Vice-Chairman, Professor B. W. Jones, University of Colorado; Secretary-Treasurer, Professor J. R. Britton, University of Colorado.

The program of papers for the Friday afternoon and Saturday morning sessions was as follows:

1. *Sidelights on certain topics in elementary statistics*, by Professor A. G.

Clark, Colorado A. & M. College.

Various topics of the elementary course in statistics were discussed. These included bounds for the coefficient of correlation, the median as the value of M that minimizes $\sum_i |M-x_i|$, and the probit diagram method for fitting distribution curves to sample data.

2. *Necessary and sufficient conditions on p and r that the equation $x^4+px^2+r=0$ be normal over the rational field*, by Mr. W. E. Briggs, University of Colorado.

Let t, $-t$, t', $-t'$ be the roots of the irreducible equation $x^4+px^2+r=0$, where t is an arbitrary root, and p and r are rational. The equation will be normal if $R(t)=R(-t)=R(t')=R(-t')$, where $R(t)$ is the field of all numbers of the form $a_0+a_1t+a_2t^2+a_3t^3$ with the a_i rational. The necessary and sufficient condition is that t' be an element of $R(t)$, or that $a_0=a_2=0$, with $a_1=p/\sqrt{r}$, $a_3=1/\sqrt{r}$, in which case the Galois group is the four group, or with $a_1=(p^2-2r)/\sqrt{p^2r-4r^2}$, $a_3=p/\sqrt{p^2r-4r^2}$, which gives the cyclic group of order four. This implies that either r or p^2r-4r^2 is a rational square.

3. *Remarks on complex numbers and their functions*, by Professor (Emeritus) A. J. Kempner, University of Colorado.

4. *Generalized functional dependence*, by Professor H. M. Jurney, Colorado School of Mines, introduced by Professor I. L. Hebel.

The functional dependence of n functions $u_i(x_1, \cdots, x_m)$, $i=1, 2, \cdots, n$, of m variables was discussed. The results may be expressed in the form of a theorem:

A relationship of the form $\phi(u_1, \cdots, u_n)=0$, exists for all values of $x_1, \cdots, x_m$ in some given domain of these variables if and only if the rank of the "Jacobian matrix" J_{mn} is less than n, where

$$J_{mn}=\begin{pmatrix} \partial u_1/\partial x_1, \cdots, \partial u_1/\partial x_m \\ \cdots\cdots\cdots\cdots \\ \partial u_n/\partial x_1, \cdots, \partial u_n/\partial x_m \end{pmatrix}.$$

5. *Periodic solutions of nonlinear differential equations*, by Professor A. B. Farnell, University of Colorado.

A discussion was given of the use of fixed point theorems in proving the existence of periodic solutions of nonlinear differential equations, and, by way of illustration, the proof of the existence of such a solution for a particular equation was given.

6. *On automorphs of conic sections*, by Professor B. W. Jones, University of Colorado.

The linear transformations $x=\alpha x'+\beta y'$, $y=\gamma x'+\delta y'$, with $\alpha\delta-\beta\gamma=1$ which leave invariant the quadratic form x^2+sy^2, $s\neq 0$, were shown to satisfy the conditions $\alpha^2+s\gamma^2=1$, $\alpha=\delta$, and, if $\alpha\neq 0$, $\beta=-s\gamma$. Hence β may be defined as the "conic cosine" of an angle θ and γ the "conic sine." If $s=1$ we have the circular functions, if $s=-1$, the hyperbolic functions. It was shown that such transformations may be used to eliminate the xy term in the equation of any conic section.

7. *The IBM card-programmed electronic calculator*, by Mr. W. W. Varner, University of Colorado.

This illustrated presentation described the physical appearance and operation of the recently released semi-portable IBM card-programmed electronic calculator. A specific problem was presented and the details of programming introduced to illustrate the versatility of the machine as well as the technique of programming. A brief discussion of the arrangement of the calculation to mini-

mize storage requirements was included to call attention to the critical problem of storage limitation.

8. *Do you enjoy the problem sections in the Monthly?* by Mr. Hans Stetter and Mr. Donald Tucker, Colorado A. & M. College, introduced by Professor M. L. Madison.

Representative problems selected from the advanced problems section of late issues of this MONTHLY were solved. The problems proposed in the MONTHLY can serve as a challenge to the undergraduate major in mathematics, and many of these problems can be solved by ingenious elementary devices.

9. *A note on income tax calculations*, by Professor W. K. Nelson, University of Colorado.

10. *Occupational outlets in industrial and business fields for majors in mathematics*, by Professor S. R. Smith, University of Wyoming.

In the unavoidable absence of Professor Smith, this paper was read by Professor Greta Neubauer.

11. *Recent efforts and achievements in the revision of the high school mathematics program, and their significance in college*, by Professor C. F. Barr, University of Wyoming.

Professor Barr presented a review of the content and grade placement of high school algebra and geometry. He then developed historically the opinions of well-known mathematicians and the various responses of mathematics teachers to these opinions. Two large resulting movements were described: one, the "two-track" movement in which algebra and geometry were taught to the superior pupils while a course with a utilitarian flavor was presented to those not capable of following the algebra-geometry track; the other movement being not the "two-track" one, but the socializing and popularizing of algebra and geometry, which were urged upon a majority, if not all, of the pupils. The objections to each of these programs were reviewed. The author proposed that a third program, consistent with the accepted purposes of mathematics, be considered, namely, the development of a course compiled from the everyday experiences of all normal citizens. This course he urged should be required of all pupils at some time in their high school program, regardless of their intellectual abilities, and that it be supplemented by algebra and geometry of the classical type if the student intended to pursue mathematics or if he expected to train in any technological field. The age level at which this course should be required was discussed briefly, with the observation that perhaps systematic experimentation alone would furnish any dependable answer.

The after-dinner address Friday evening was given by the guest speaker, Professor G. B. Price, University of Kansas. Professor Price gave an illustrated lecture on the topic, *Experiences of a Mathematician as an Operations Analyst with the Eighth Air Force in England.*

J. R. BRITTON, *Secretary*

THE APRIL MEETING OF THE IOWA SECTION

The Iowa Section of the Mathematical Association of America held its thirty-eighth annual meeting at Wartburg College, Waverly, Iowa, on Friday and Saturday, April 20–21, 1951. The Chairman, Professor D. L. Holl of the Iowa State College, presided at both sessions.

(3) *Phragmén-Lindelöf theorems for generalized subharmonic functions*, by Professor L. K. Jackson, University of Nebraska, introduced by the Secretary.

The notion of subharmonic function is generalized by replacing the dominating family of harmonic functions by a more general family of functions. It is shown that an analogue of the principle of the maximum modulus holds for these generalized subfunctions. This property is used to prove theorems of the Phragmén-Lindelöf type. Solutions of certain types of elliptic differential equations are shown to be examples of such functions.

(4) *Ideals and the prime factorization theorem*, by Miss F. Marion Clarke, University of Nebraska.

The author demonstrated the failure of the fundamental theorem of arithmetic in certain algebraic fields, showed the validity of an analogous theorem with respect to ideals instead of integers, sketched the generalization of this theorem for transcendental extensions and indicated the rule of irreducibility and maximality in characterizing the property of being prime.

(5) *A development of the identities for* $\sin(\alpha+\beta)$ *and* $\cos(\alpha+\beta)$, by Professor A. K. Bettinger, Creighton University.

The new feature of this development is a variation in the geometric construction which greatly simplifies the proof. Application was also made to the rotation formulas in plane analytic geometry.

(6) *Some recent developments of the analysis of the logical foundations and methods of algebra*, by Professor H. B. Ribeiro, University of Nebraska.

Included was a discussion of metamathematical proofs in algebra followed by an introduction to Tarski's mathematical theory of arithmetical classes of algebraic systems and its applications.

The afternoon session was devoted to a discussion of the problems of the teaching of secondary school mathematics. A panel composed of persons representing the university, the small college, the high school, and the administrators, led the discussion. Members of the Association on the panel were Professor W. G. Leavitt, University of Nebraska, and Professor C. B. Gass, Nebraska Wesleyan University.

EDWIN HALFAR, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-fifth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Western State College of Colorado, Gunnison, Colorado, on May 23 and 24, 1952. Professor C. H. Cook, Western State College of Colorado, presided.

Forty-nine persons registered including the following thirty-seven members of the Association:

C. F. Barr, J. R. Britton, R. G. Buschman, F. M. Carpenter, H. W. Charlesworth, C. H. Cook, G. S. Cook, Mary C. Doremus, Albert Edrei, F. N. Fisch, H. T. Guard, R. R. Gutzman, J. R. Hanna, I. L. Hebel, Anna S. Henriques, C. A. Hutchinson, B. W. Jones, M. W. Jones, A. J. Kempner, Claribel Kendall, F. A. Kros, J. S. Leech, M. L. Madison, Greta Neubauer, D. O. Patterson, Lily B. Powell, Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, K. H. Stahl, P. O. Steen, Wilmont Toalson, E. P. Tovani, E. L. Vanderburgh, V. J. Varineau, W. W. Varner.

At the business meeting the following officers were elected: Chairman, Professor B. W. Jones, University of Colorado; Vice-Chairman, Professor C. F. Barr, University of Wyoming; Secretary-Treasurer, Professor J. R. Britton, University of Colorado. It was decided to hold the next annual meeting at the University of Colorado, the date to be announced later. Upon a motion by H. T. Guard it was voted to donate the sum of $25 to the Wald Memorial Fund.

The following papers were presented:

1. *The spirit of discovery in mathematics*, by Professor B. W. Jones, University of Colorado.

Attention was called to some of the similarities between our international situation and our educational difficulties. In both we are apt to blame sinister, all-powerful, and all-wise forces outside our control. Actually what danger there is at present is chiefly from within. It is up to us to adopt an aggressive rather than a defensive attitude in both cases and with due modesty to realize that we have already made some impressive improvement over the past. Not the least of our opportunities in the field of education is to cultivate in our students from kindergarten upward a spirit of discovery. We should not do all the exploration for them but should rather encourage them in their natural curiosity so that our pioneering heritage may be perpetuated in the realm of the mind.

2. *Mathematics applied to the calculation of rocket trajectories*, by Mr. F. A. Kros, University of Colorado.

The following method can be used to determine the position of a rocket at any instant using data from three camera stations. Let the lines of sight from the camera to the rocket be written in the form $x=x_0+\lambda r_1$, $y=y_0+\mu r_1$, $z=z_0+\nu r_1$, where $(x_0, y_0, z_0$ are the coordinates of the camera, λ, μ, ν are the direction cosines of the line of sight and r_1 is the distance from the camera to any point (x, y, z) on this line. Consider a function $D=f(r_1, r_2, r_3)$ such that D is the sum of the squares of the distances between points on each of the three lines. Minimizing this function gives us three linear equations in the r's, the solution of which determines three points on the lines of sight. The average of these three points gives the centroid of the triangle determined by them. This centroid is taken as the best approximation of the rocket's position.

3. *Forms for the solution of spherical triangles*, by Mr. E. L. Vanderburgh, Pueblo College.

A booklet was distributed which the author uses in teaching spherical trigonometry in seven to ten lessons. Three forms, each with the formulas needed and with solution outlines, are used to solve the six cases of oblique spherical triangles. The number of formulas is kept to a minimum and each one is derived in a form students of trigonometry can easily follow. By using the forms, it was pointed out, students can spend their time on the numerical work and easily check results on the form in the place provided. Also, since the complete plan is on the form, one can solve similar problems many years later by use of the forms and a very little review. Six students, over a period of four years, helped prepare the booklet as a part of their trigonometry courses.

4. *A minimum problem in Banach spaces*, by Professor J. S. Leech, Colorado College.

In Monatshefte für Mathematik, XXXII Band, 1922, pp. 204–218, Georg Pick considers the following problem: Let $f(z)$ be a function analytic in the unit circle, $|z|<1$. Among all such functions satisfying the conditions (1) $f(z_k)=\alpha_k$, $k=1, 2, \cdots, n$, where $z_1, z_2, \cdots, z_n$ and $\alpha_1, \alpha_2, \cdots, \alpha_n$ are given complex numbers such that $|z_k|<1$, what function makes the integral $I=\int_0^{2\pi}|f(e^{i\theta})|^2 d\theta$ a minimum? Pick shows that this problem always has a unique solution.

A generalization of this problem to arbitrary Banach spaces may be stated as follows: If $f_1, f_2, \cdots, f_n$ are n given functionals defined on a Banach space X, what element $x \epsilon X$ satisfying (2) $f_k(x) = \alpha_k$, $k = 1, 2, \cdots, n$, has the least norm? We will refer to this as the minimum problem.

The principal result obtained is: In a Banach space X, in order that every minimum problem have a unique solution, it is necessary and sufficient that X be reflexive and strictly convex. In particular, L_2 is both reflexive and strictly convex; hence every minimum problem in L_2 has a unique solution. In this case, it is shown how to construct this minimum solution.

5. *The roots of a certain exponential equation*, by Professor W. N. Smith, University of Wyoming.

The equation considered is of the form

$$A_1(\lambda, \nu) - A_2(\lambda, \nu)e^{\lambda R_1} + A_3(\lambda, \nu)e^{\lambda(R_1+R_2)} - A_4(\lambda, \nu)e^{\lambda R_2} = 0,$$

where R_1 and R_2 are complex constants, and the $A_i(\lambda, \nu)$ are asymptotically representable by power series in $1/\lambda$ with coefficients which are at most quadratic in ν. It is desired to solve for λ as a function of the complex variable ν, with $|\lambda|$ taken to be large and $|\nu|$ small. The equation is studied by means of approximation equations. Two of these are explicitly solvable for λ and are independent of ν. The other two may be regarded, after certain conditions have been imposed upon ν, as defining λ as an infinity of distinct single-valued functions of ν. As ν approaches zero along a suitable path these functions are found to lie in subregions of the λ-plane. Finally it is shown that the roots of the approximation equations actually represent the roots of the given equation asymptotically.

6. *A singular integral equation*, by Professor Albert Edrei, University of Colorado.

J. R. BRITTON, *Secretary*

THE MAY MEETING OF THE UPPER NEW YORK STATE SECTION

The annual meeting of the Upper New York State Section of the Mathematical Association of America was held at Hobart and William Smith Colleges, Geneva, New York, on May 10, 1952. The Chairman of the Section, Professor C. W. Munshower of Colgate University, presided at the morning session; the Vice-Chairman, Professor J. F. Randolph of the University of Rochester, presided at the afternoon session. At the conclusion of the afternoon session a tea was served to members and guests.

Ninety-seven persons attended the meeting, including the following seventy-two members of the Association:

H. T. R. Aude, H. W. Baeumler, Frances E. Baker, M. R. Bates, W. R. Baum, R. A. Beaver, R. L. Beinert, Dorothy L. Bernstein, W. W. Bessell, H. F. Bligh, F. J. H. Burkett, K. A. Bush, E. A. Butler, W. B. Carver, Nancy Cole, Geraldine A. Coon, A. E. Danese, W. A. Dolid, E. J. Downie, Walter H. Durfee, William H. Durfee, G. V. Emerson, H. W. Eves, Jean B. Feidner, A. D. Fleshler, C. W. Foard, A. H. Fox, J. E. Freund, H. M. Gehman, B. H. Gere, J. C. Gibson, Lillian Gough, N. G. Gunderson, H. K. Holt, Anna M. Howe, J. R. F. Kent, D. E. Kibbey, F. W. Lane, R. D. Larsson, Caroline A. Lester, R. C. Luippold, R. W. MacDowell, Dis Maly, E. W. Marchand, Harriet F. Montague, Mabel D. Montgomery, L. J. Montzingo, D. S. Morse, Abigail M. Mosey, C. W. Munshower, W. V. Nevins, III, F. D. Parker, W. B. Pitt, Theresa L. Podmele, L. R. Polan, J. F. Randolph, C. E. Rhodes, M. F. Rosskopf, P. T. Schaefer, Edith R. Schneckenburger, W. A. Small, S. T. Smith, Ruth W. Stokes, Mary C. Suffa, Nura D. Turner, G. W. Walker, R. J. Walker, F. C. Warner, A. E. Whitford, Mary E. Williams, A. G. Wootton, Frances M. Wright.

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-sixth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, Colorado, on April 17 and 18, 1953. Professor B. W. Jones, Chairman of the Section, presided at all the sessions.

Of the eighty-five persons who registered, the following fifty-five were members of the Association:

C. F. Barr, D. L. Barrick, B. C. Bellamy, W. E. Briggs, J. R. Britton, R. K. Butz, F. M Carpenter, Sarvadaman Chowla, G. S. Cook, F. W. Donaldson, W. E. Dorgan, F. N. Fisch, C. A. Grimm, Arnold Grudin, H. T. Guard, R. R. Gutzman, Marian S. Gysland, C. L. Harbison, Leota C. Hayward, I. L. Hebel, Ruth I. Hoffman, LeRoy Holubar, P. F. Hultquist, J. A. Hurry, C. A. Hutchinson, B. W. Jones, A. J. Kemper, Claribel Kendall, J. S. Leech, D. C. B. Marsh, Jr., Garner McCrossen, H. C. McKenzie, E. B. McLeod, Jr., W. E. Mientka, W. K. Nelson, Greta Neubauer, D. K. Parks, Lily B. Powell, O. M. Rasmussen, O. H. Rechard, A. W. Recht, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, K. H. Stahl, P. O. Steen, E. L. Swanson, C. W. Thomson, E. P. Tovani, E. L. Vanderburgh, V. J. Varineau, W. W. Varner, J. F. Wagner.

At the business meeting, the following officers were elected for the coming year: Chairman, Professor M. L. Madison, Colorado Agricultural and Mechanical College; Vice-Chairman, Professor Nathan Schwid, University of Wyoming; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The program of papers for the meetings was as follows:

1. *Some results in number theory using a partial summation method*, by Mr. W. E. Briggs, University of Colorado.

In the classical proofs of theorems concerning the representation of primes by binary quadratic forms, it is necessary to use facts about the continuity, differentiability, and behavior as $s \to 1^+$ of the series $\sum (ax^2+2bxy+cy^2)^{-s}$ and other series similar to it. The summation is extended over all x, y which make the form prime to $2D$, where $D=b^2-ac$, and which satisfy certain other conditions if $D>0$. These facts can all be proved simply by estimating the sum as $\sum_{n=1}^{\infty} n^{-s}[T(n) - T(n-1)]$, where $T(n)$ is the number of lattice points within $ax^2+2bxy+cy^2=n$ which make the form prime to $2D$ and satisfy the other conditions if $D>0$.

2. *Polynomials associated with matrices*, by Professor R. K. Butz, Colorado Agricultural and Mechanical College.

Notation was developed to handle the matric equation $AX=XB$, where A and B are specified matrices of order n and m, respectively, defined over an arbitrary field F, and X is to be determined in terms of parameters using only those operations with respect to which F is closed. The approach to the problem was that given by W. V. Parker (*The matrix equation* $AX=XB$, Duke Mathematical Journal, vol. 17, no. 1, 1950, p. 43).

3. *Some topics in the theory of numbers*, Professor Sarvadaman Chowla, University of Colorado.

4. *An approximation method in certain nonlinear boundary value problems*, by Professor Nathan Schwid, University of Wyoming.

In the differential equation of heat conduction,

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right),$$

the physical quantities c, the specific heat, and K, the thermal conductivity, are usually considered constant. When these quantities are more realistically regarded as linear functions of the temperature u, the equation is nonlinear. A method of approximation to the solution of the equation, subject to suitable boundary conditions, is here discussed. The method is applicable to situations where the solution for constants c and K is in the form of an infinite series of orthogonal functions each term of which has an exponential factor with negative exponent.

5. *Linear diophantine equations and additive number theory*, by Professor Emeritus A. J. Kempner, University of Colorado. (By invitation.)

We know little about solutions of linear diophantine equations with prescribed restrictions, except for such cases as all solutions positive, or all solutions bounded, *etc.* It is interesting that large groups of problems in the additive number theory can be paraphrased into problems in linear diophantine equations with a certain type of restriction on the solutions. Thus,

$$1x_0 + 3x_1 + 5x_2 + \cdots + (2m+1)x_m + \cdots = n$$

has a solution $4 \geqq x_0 \geqq x_1 \geqq \cdots \geqq x_m > 0$ for all positive integral n; $28 = 4\cdot 1 + 4\cdot 3 + 1\cdot 5 + 1\cdot 7$; but has a solution $3 \geqq x_0 \geqq x_1 \geqq \cdots \geqq x_m > 0$ when and only when $n \neq 4^r(8t+7)$. The paraphrase is based on the simple fact: Given a set, finite or infinite, of positive integers $a_0, a_1, \cdots, a_m, \cdots$, with $a_{m+1} > a_m$ (for convenience), and the set $d_0 = a_0$, $d_1 = a_1 - a_0$, $d_2 = a_2 - a_1, \cdots$, then the two statements are equivalent: (a) a given positive integer n is the sum of at most k elements a_m (repetition allowed), and (b) the diophantine equation $d_0x_0 + d_1x_1 + \cdots + d_ma_m + \cdots = n$ has a solution $k \geqq x_0 \geqq x_1 \geqq \cdots \geqq x_m > 0$. Application is made to Pythagorean numbers ($a^2 + b^2 = c^2$), the formulation of Fermat's theorem ($a^n + b^n = c^n$), to polygonal and pyramidal numbers, to such equations as $3^x = 2^y + 1$, or $3^x = y^3 + z^3$, *etc.* Emphasis is placed on the Waring-Hilbert theorem on powers, and the Waring-Kamke theorem on polynomials with rational coefficients and integral function values for integral argument values. The Waring-Kamke theorem contains the Waring-Hilbert theorem as a very special case. The paraphrase of the Waring-Kamke theorem may be stated as follows: Let $a_0 = 1, a_1, \cdots, a_m, \cdots$ (positive integers, increasing) be the elements of an arithmetic progression of order k, and let the (positive) first differences $d_0 = a_0$, $d_1 = a_1 - a_0$, $d_2 = a_2 - a_0, \cdots$ also be increasing (for convenience). Then there exists a positive integer $N = N(k; a_0, \cdots, a_k)$, independent of n and of $a_{k+1}, a_{k+2}, \cdots$, such that $d_0x_0 + d_1x_1 + \cdots + d_mx_m + \cdots = n$ has a solution $N \geqq x_0 \geqq x_1 \geqq \cdots \geqq x_m > 0$ (n any positive integer). If $a_0 > 1$, there exists an N as above, and a positive integer $L = L(k; a_0, \cdots, a_k)$ such that in every interval of length L there is at least one positive integer n for which the equation has a solution $N \geqq x_0 \geqq \cdots \geqq x_m > 0$.

The preceding paper was the invited address for the evening session following the customary banquet.

6. *On sets of quasi-conjugate matrices*, by Professor V. J. Varineau, University of Wyoming.

A set of quasi-conjugate matrices is defined by removing the commutativity restriction from the usual definition of conjugate matrices. Thus, a set, $A_1, A_2, \cdots, A_n$, of $n \times n$ matrices over a field $\mathfrak{F}$ is quasi-conjugate if the matrices A_i are similar and if $I|xI - A_1| = (xI - A_1)(xI - A_2)\cdots(xI - A_n)$. Elementary properties of such sets are presented and conjectures about general existence theorems are made.

7. *A problem from the* MONTHLY: *Number 4479*, by Professor Emeritus A. J. Kempner, University of Colorado.

Making use of elementary properties of the roots of unity, it is shown that $a_1, a_2, \cdots, a_j, \cdots$ (all $\neq 0$) can be determined so that each $\sum_{j=1}^{\infty} a_j^k$, $k = 1, 2, 3, \cdots$, converges (conditionally) to zero. Each a_j is of the form $\gamma_j \cdot \epsilon_j$, γ_j real, ϵ_j some root of unity.

8. *The content and method for a general mathematics course for adults*, by Miss Ruth I. Hoffman, Byers Junior High School, Denver, Colorado.

9. *The rapid growth of numerical analysis since 1943 and the challenge it offers to the university teacher*, by Mr. W. W. Varner, University of Colorado.

The mushrooming of numerical analysis and its inherent problem of error consideration has greatly increased the need of all teachers of mathematics, engineering, and the sciences to be meticulous in demanding that problem solutions be written in such a manner that the error or uncertainty in every result be clearly and unmistakably given. Certain aspects of this problem were discussed.

10. *On the improvement of service courses in freshman mathematics*, by Professor I. L. Hebel, Colorado School of Mines.

An outline is presented of a revised approach to the teaching of freshman mathematics adopted at Colorado School of Mines. The traditional sequence of topics is replaced by a unification that stresses the analytic geometry viewpoint throughout and that maintains the necessary rigor of a pre-engineering mathematics course. Principal results of the initial trial of the plan include increased student interest, improved faculty instruction, and a better prepared student for sophomore courses. The author and his staff feel that such a curriculum is a forward step in the solution of the perplexing freshman teaching problem.

11. *Mathematics used in university departments other than mathematics or engineering*, by Professor O. M. Rasmussen, University of Denver.

A report was presented on a part of a study using the methods of textbook analysis, interviews, and questionnaires to determine those mathematical skills and concepts that are desirable as preparation for non-mathematics course work for university students not majoring in mathematics or the physical sciences. The mathematics needed for a vast majority of these students is quite elementary and many of the students do not possess sufficient arithmetical maturity to enable them to gain maximum benefit from a large number of university courses. An understanding of elementary statistics is needed in many courses throughout the university.

J. R. BRITTON, *Secretary*

THE APRIL MEETING OF THE TEXAS SECTION

The annual meeting of the Texas Section of the Mathematical Association of America was held at Texas Christian University, Fort Worth, Texas, on April 24-25, 1953. Professor C. B. Wright, Chairman of the Section, presided at the sessions. Professor L. R. Ford, who was an invited guest, contributed much to the success of the meeting.

There were one hundred eight persons in attendance, including the following sixty-two members of the Association:

T. A. Abouhalkah, R. C. Ailara, A. W. Ashburn, A. V. Banes, Ina M. Bramblett, H. E. Bray, Myrtle C. Brown, M. L. Coffman, L. A. Colquitt, J. V. Cooke, Don Cude, F. W. Donaldson, G. H. Dubay, L. K. Durst, Terrell Ellis, L. R. Ford, Gordon Fuller, R. L. Glass, Blanche B. Grover, W. T. Guy, Jr., E. H. Hanson, E. A. Hazelwood, E. R. Heineman, Fay H. Johnson, Ruth Kissel, E. C. Klipple, H. A. Luther, Hazel L. Mason, Lida B. May, Dorothy McCoy, W. K. McNabb, V. A. Miculka, Harlan C. Miller, B. C. Moore, E. D. Mouzon, Jr., C. A. Murray, Albert Newhouse, Bob Parker, H. C. Parrish, C. J. Pipes, C. B. Rader, Sr., L. W. Ramsey, Dorothy L. Rees, C. L. Riggs, Virginia E. Roberts, C. A. Rogers, R. Q. Seale, C. R. Sherer, D. P. Shore, Sister Mary of Perpetual Help, D. W. Starr, W. G. Stokes, W. W. Taylor, Earl Thomas, F. E. Ulrich, R. S.

where b and d are the orthonormalized bisectors of the given angle and its supplement. The equation was transmuted to a primordial prototype

$$A \cdot r = 0,$$

whose solution was

$$r = 0/0.$$

A second view started from a single prototype which led to a factorable 4th degree equation with a free choice parameter with which to control its irreducibility. A mutation curve was drawn for this which was shown to trisect the angle.

FOSTER BROOKS, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-seventh annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado Agricultural and Mechanical College, Fort Collins, Colorado, on Friday afternoon and evening and Saturday forenoon, April 30 and May 1, 1954. Professor M. L. Madison, Chairman of the Section, presided at all three sessions.

Seventy-five registered for the meeting, including the following forty-nine members of the Association:

C. F. Barr, D. L. Barrick, J. R. Britton, R. G. Buschman, R. K. Butz, F. M. Carpenter, A. G. Clark, Sarvadaman Chowla, G. S. Cook, Rev. F. T. Daly, W. E. Dorgan, H. T. Guard, R. R. Gutzman, C. L. Harbison, Leota C. Hayward, I. L. Hebel, LeRoy Holubar, J. E. Householder, P. F. Hultquist, C. A. Hutchinson, B. W. Jones, A. J. Kempner, Claribel Kendall, R. B. Kriegh, J. S. Leech, M. L. Madison, W. E. Mientka, M. W. Milligan, W. K. Nelson, Greta Neubauer, D. O. Patterson, O. M. Rasmussen, O. H. Rechard, A. W. Recht, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, K. H. Stahl, J. McD. Staley, P. O. Steen, E. P. Tovani, E. L. Vanderburgh, W. W. Varner, J. F. Wagner, F. J. Wall, C. R. Wylie, Jr., A. Zirakzadeh.

Officers elected at the meeting for 1954–1955 were: Chairman, Professor Nathan Schwid, University of Wyoming; Vice-Chairman, Professor C. R. Wylie, Jr., University of Utah; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *On expressing the matrix A^t as a polynomial in t*, by Professor R. K. Butz, Colorado Agricultural and Mechanical College.

This paper discusses the notion of $E_0F(X)$ matrices as introduced by G. B. Huff (*Matrices such that A^t is a polynomial in t and principal idempotent elements*, Bull. Amer. Math. Soc., vol. 59, 1953, p. 54). Emphasis is placed on the fact that the proofs of the main theorems require only the more elementary concepts of matrix theory and on methods of finding $F(X)$ given a matrix A with elements in the field of complex numbers. The clarity with which some classical results follow by the use of this notion is pointed out.

2. *An optimum solution of N equations in M unknowns with N greater than M*, by Mr. Leon Rutland, University of Colorado.

A problem in engineering design led to a consideration of the system of equations

$$\sum_{j=1}^{m} a_{ij}x_j = t_i \qquad (i = 1, 2, \cdots, n), (n > m),$$

where the desired optimum solution of the system is that set of x's for which the largest absolute value of any of the deltas in the set of equations

$$\Delta_i + \sum_{j=1}^{m} a_{ij}x_j = t_i, \qquad (i = 1, 2, \cdots, n), (n > m),$$

is as small as possible. Several theorems giving solutions to the problem under various conditions were either proved or stated with the proofs being omitted. A numerical example, carried through on a digital computer, was cited to indicate that even though there are as many as forty equations the method is feasible and the answer can be readily attained.

3. *The method of Frobenius*, by Professor R. H. Cook, South Dakota School of Mines and Technology, introduced by the secretary.

The usual textbook presentation of the method of Frobenius effectively camouflages two important points: (1) that the method involves a Taylor's expansion, and quite often leads to just a Taylor's expansion; (2) the conditions under which the method is applicable. This paper suggests a modified approach which has neither of the above disadvantages, is easily taught, and is sufficiently flexible to be applicable to many non-linear problems.

4. *Approximate solutions to a certain functional equation*, by Professor C. A. Rogers, Colorado A and M. College.

The following is investigated: Given a non-negative $g(x)$, defined for all $x>0$, and which is strictly monotonic increasing and everywhere differentiable, to find a closed-form $f(x)$, reasonably computable, such that $f[f(x)]$ is at least approximately equal to $g(x)$. It was indicated how this approximation could be accomplished for certain g-functions, with examples.

5. *Some infinite series*, by Dr. W. E. Briggs, Professor S. Chowla, Professor (Emeritus) A. J. Kempner, and Research Assistant W. E. Mientka, University of Colorado, presented by Mr. Mientka.

It is proved that

$$\sum_{1}^{\infty} \frac{\sigma_n}{n^2} = 2\zeta(3),$$

where

$$\sigma_n = \sum_{t=1}^{n} \frac{1}{t}.$$

6. *Effect of rotation on the normal mode frequencies of transverse vibration of a cantilever beam*, by Professor R. H. Cook and Mr. L. J. Eatherton, South Dakota School of Mines and Technology, presented by Mr. Eatherton.

The differential equation,

$$EI\frac{\partial^4 y}{\partial x^4} - \frac{\partial^2 y}{\partial x^2}\int_x^L \rho AS\Omega^2 dS + \rho Ax\Omega^2\frac{\partial y}{\partial x} + \rho A\frac{\partial^2 y}{\partial t^2} = 0$$

describes the vibrations, in a vertical plane, of a cantilever beam which rotates about a vertical axis through its clamped end. This equation and appropriate boundary conditions are considered

by the use of Taylor's expansion. The normal mode frequencies are calculated in terms of Ω, the rotational speed. Results show the dependence upon both Ω^2 and Ω^4 for the first and second modes and upon Ω^2 for the third mode. They are in excellent agreement with existing experimental data.

7. *Block designs*, by Professor Burton W. Jones, University of Colorado.

The definition and significance of balanced incomplete block designs are briefly described and methods of exclusion sketched.

8. *The mapping of the circles of S_2 into the points of S_3*, by Professor C. R. Wylie, Jr., University of Utah. (By invitation).

If the coefficients a, b, c, d in the general equation of a circle

$$d(x^2 + y^2) - 2ax - 2by + c = 0$$

are interpreted as homogeneous point coordinates in S_3, point circles are mapped into points on the paraboloid

$$V \equiv a^2 + b^2 - cd = 0,$$

proper real circles are mapped into finite points outside V, improper real circles (lines) are mapped into real points at infinity, and imaginary circles are mapped into finite points within V. Pencils and bundles of circles are represented in S_3 by lines and planes, respectively, and may be classified according to the intersection of their images with V. Two circles which are orthogonal are represented by points each of which lies in the polar of the other with respect to V. Conjugate pencils of circles are represented by lines conjugate with respect to V. Various theorems from college geometry were interpreted in S_3, and the classical constructions for circles satisfying three conditions were considered as problems in descriptive geometry in S_3.

9. *The University of Colorado Engineering Experiment Station analog computer—The UCEESAC*, by Mr. Walter W. Varner, University of Colorado.

A description of the Boeing analog computer recently installed at the University of Colorado was given. Types of problems that can be solved as well as restrictions on their solution were given. Finally a simple pair of simultaneous differential equations were considered and the simplicity of forming the wiring diagram shown.

10. *Fitting empirical equations to fluid meter data*, by Professor S. R. Smith, University of Wyoming.

Empirical equations of the form

$$C = \frac{R}{a + bR} + de^{fR}*$$

were fitted to both flow nozzle and orifice meter data and residuals determined. R, C curves were fitted to data for the fluids stream, oil and water for $0 < R \leqq 3{,}040{,}000$. C is the coefficient of discharge of the meter and R its corresponding Reynolds number, both dimensionless.

11. *On $ff(x) = F(x)$, $F(x)$ given (real), $f(x)$ unknown*, by Professor (Emeritus) A. J. Kempner, Dr. W. E. Briggs, Professor S. Chowla and Mr. W. E. Mientka, University of Colorado, presented by Professor Kempner.

For the real case the geometrical interpretation employed in studying $f(x) - x = 0$ can be extended so as to lead immediately to results such as: there exist totally discontinuous functions $f(x)$

* Equation first used by I. D. Ruggles, former graduate student in mathematics, University of Wyoming.

for which $ff(x)$ is single-valued and continuous; or, the function $y=g(x)$ given by $x-y+\pi/2 = \mu \cos(x+y)$, $|\mu| \leq 1/2$, is a single-valued inverse iterate of $f(x)=x+\pi$. It also makes plain the plausibility of introducing iterations of $f^{(n)}(x)$ of any rational (or even any real) index n.

12. *Figure it out for yourself*, by Professor A. W. Recht, University of Denver.

Trained mathematicians are at a premium; everyone realizes the great part mathematics plays in a technical civilization. Yet election of mathematics in high schools is waning, despite efforts of government and mathematical groups stressing urgent need of training of the talented. The problem is to reach the 85% who will use mathematics in normal pursuits and also train the talented 15%. The paper suggested revisions of policy, including training of really professional mathematics teachers, emphasis on students doing own work, insistence on 100% accuracy, promotion of "first the problem, then the mathematics," and strengthening of fundamental concepts to avoid mathematical accidents in home and industry.

13. *Textbooks on elementary mathematics*, by Professor J. S. Leech, Colorado College.

Attention is called to the fact that in a large majority of textbooks many terms are defined erroneously or ambiguously. Many theorems are stated and "proved" without any or with incomplete hypotheses. In all of these cases, the author believed that the correct definitions and statements of theorems not only do not increase the difficulty of the subjects, but serve to clarify concepts and contribute greatly to understanding.

14. *Freshman mathematics separation*, by Professor I. L. Hebel, Colorado School of Mines.

This paper dealt with the mathematics department's experience over the last seven years in assigning and classifying freshman mathematics students.

F. M. CARPENTER, *Secretary*

THE APRIL MEETING OF THE SOUTHWESTERN SECTION

The fourteenth annual meeting of the Southwestern Section of the Mathematical Association of America was held at Arizona State College, Tempe, Arizona, on April 16, 1954. Professor M. S. Hendrickson, Chairman of the Section, presided.

Forty persons attended the meeting including the following twenty-seven members of the Association:

O. B. Ader, C. E. Aull, C. E. Buell, J. H. Butchart, D. G. Duncan, J. F. Foster, Jr., R. S. Fouch, F. C. Gentry, R. F. Graesser, R. E. Graves, W. P. Heinzman, M. S. Hendrickson, Carol Karp, Max Kramer, Lincoln LaPaz, R. B. Lyon, W. W. Mitchell, Jr., E. D. Nering, J. L. Olpin, E. J. Purcell, L. C. Snively, A. H. Steinbrenner, Deonisie Trifan, Earl Walden, D. L. Webb, Charles Wexler, Oswald Wyler.

The following were elected officers for the year 1954: Chairman, Professor D. L. Webb, University of Arizona; Vice-Chairman, Professor R. L. Westhafer, New Mexico College of Agriculture and Mechanic Arts; Secretary-Treasurer, Professor W. W. Mitchell, Jr., Phoenix College (four year term); Lecturers, Professor Max Kramer, New Mexico College of Agriculture and Mechanic Arts (one year), Professor D. G. Duncan, University of Arizona, (two years).

9. *A note on Fourier coefficients*, by Professor R. P. Gosselin, Youngstown College.

Let $c(g)$ be the nth Fourier (exponential) coefficient of $g(x)$. Let $f(x)$ belong to L^q, q an integer $\geqq 2$. Let $c_n(f)$ be positive and decrease with $1/|n|$. By use of Parseval's formula, it is shown that $(c_n(f))^q \leqq A_q c_n(f^q)/(|n|+1)^{q-1}$, where A_q is a constant depending only on q. As an application of this inequality, a proof of the following result, due to Hardy and Littlewood (*Journal of the London Math. Soc.*, vol. 6, 1931, pp. 3–9), is obtained: If $f(x)$ belongs to L^r, $r \geqq 2$, and $c_n(f)$ decreases, then

$$\sum_{n=-\infty}^{+\infty} |n|^{r-2}(c_n(f))^r \leqq B_r \int^{2\pi} |f(x)|^r dx.$$

10. *The tensor form of the equations of hydrodynamics*, by Mr. W. H. Lane, Wright Air Development Center, introduced by the Secretary.

The tensor forms of the general energy equation and of the Navier-Stokes equations of motion for a viscous incompressible fluid are considered. The special case for spherical coordinates in which the velocity and temperature fields are assumed to be inversely proportional to the power of a radial vector is developed, and the resulting class of exact solutions is discussed.

11. *On ideals in the ring of linear multidifferential polynomials*, by Mr. Frank Levin, University of Cincinnati, introduced by Professor H. D. Lipsich.

The ring of linear multidifferential polynomials is a noncommutative ring of polynomials in several indeterminates. This ring is not a principal ideal ring, and, therefore, the results are stated ideal-theoretically. A basis of an ideal of multidifferential polynomials which corresponds to the basis of an ideal in the principal ideal ring of ordinary differential polynomials is the *canonical basis* of the ideal. With this basis one is able to provide an upper bound for the length of a basis of the ideal and to give a necessary and sufficient condition for solvability of multidifferential equations.

FOSTER BROOKS, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-eighth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, Wyoming, on Friday afternoon and evening and Saturday forenoon, April 22 and 23, 1955. Professor Nathan Schwid, Chairman of the Section, presided at all three sessions.

There were 62 persons registered for the meeting, including the following 46 members of the Association:

J. W. Ault, G. E. Bardwell, C. F. Barr, B. C. Bellamy, W. E. Briggs, J. R. Britton, R. K. Butz, F. M. Carpenter, Sarvadaman Chowla, E. L. Crow, W. E. Dorgan, F. N. Fisch, H. T. Guard, Leota C. Hayward, Anna S. Henriques, Archie Higdon, J. E. Householder, Sr., P. F. Hultquist, C. A. Hutchinson, A. J. Kempner, Claribel Kendall, R. B. Kriegh, L. J. Lange, E. B. McLeod, Jr., M. L. Madison, W. D. Marsland, Jr., W. E. Mientka, W. K. Nelson, Greta Neubauer, D. O. Patterson, J. W. Querry, O. M. Rasmussen, O. H. Rechard, A. W. Recht, Calvin A. Rogers, L. W. Rutland, Jr., Nathan Schwid, S. R. Smith, W. N. Smith, L. C. Snively, P. O. Steen, W. J. Thron, E. P. Tovani, V. J. Varineau, W. W. Varner, C. R. Wylie, Jr.

Officers elected at the meeting for 1955–1956 were: Chairman, Professor

C. R. Wylie, Jr., University of Utah; Vice-Chairman, Professor R. R. Gutzman, Colorado School of Mines; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *Remarks on the functional equation* $f[f(z)]=e^z-1$, by Professor W. J. Thron, University of Colorado.

If the functional equation has a solution $f(z)$ which is holomorphic in a sufficiently large neighborhood of the point $z=0$ then it can be shown that $f(0)=0$. Using this one can determine a unique formal power series solution $f(z)=\sum c_n z^n$. Let r be the radius of convergence of this series. By means of Picard's theorem, Hadamard's factorization theorem, and a result of Pólya, it is established that $r<\infty$. The statement that $r=0$ but that there exists a solution of the functional equation which is holomorphic for all z, not on the negative real axis, concludes the paper.

2. *Note on hemispheric numerical integration of the barotropic model*, by Major J. F. Blackburn, USAF Academy, and W. L. Gates, presented by Major Blackburn, who was introduced by the Secretary.

Under the assumption of frictionless, adiabatic flow in hydrostatic and quasi-geostrophic equilibrium, the barotropic equation was solved numerically for hemispheric flow. The method of solution was similar to the scheme outlined by Charney, Fjørtoft and von Neumann (*Tellus*, November, 1950) applied to a smaller area. The procedure consists of the cyclical calculation of the absolute vorticity advection at time t for each point of a finite difference grid, the solution of the resulting finite difference equations for $\partial z/\partial t$ at each point by a method of relaxation, and the calculation of the heights z at time $t+\Delta t$ using centered differences over a short time interval.

3. *Some simple geometrical properties of the space* L^2 by Mr. A. E. Labarre, Jr., University of Wyoming, introduced by the Secretary.

Geometrical interpretations of the Parseval and Riesz-Fischer theorems are given. The space L^2 is the direct product of the even functions of L^2 and the odd functions of L^2. How the operation of differentiation in L^2 can be interpreted as an orthogonal transformation is explained.

4. *The number of lattice points in an n-dimensional tetrahedron*, by Professors Sarvadaman Chowla and W. E. Mientka, University of Colorado, presented by Professor Mientka.

Let the $a_i (1 \leqq i \leqq n)$ be positive integers relatively prime in pairs, and $A=\Pi_{i=1}^n a_i$. In this paper we find exact expressions (which are polynomials in η/A and the a_i) for (i) the number of solutions in non-negative integers x_i of $\sum_{i=1}^n a_i x_i=\eta$ whenever $\eta \equiv 0 \pmod{A}$, (ii) the number of lattice points in the tetrahedron bounded by the planes $\sum_{i=1}^n a_i x_i=\eta$ $(x_i \geqq 0)$ again provided $\eta \equiv 0 \pmod{A}$.

5. *A possible measure of asymmetry in a line*, by Professor Calvin A. Rogers, Colorado Agricultural and Mechanical College.

Eight requirements were set up, formally expressing intuitive convictions about the asymmetry in the x-axis of two points P_1 and P_2 with same abscissa and ordinates, y_1 and y_2. From these, it was deduced that one of the simplest functions satisfying all requirements was the fraction $(y_1+y_2)^2/(y_1^2+y_2^2)$.

6. *Remarks on the distribution of primes*, by Professors A. J. Kempner and Sarvadaman Chowla, University of Colorado, presented by Professor Kempner.

From the extended Prime Number Theorem two formulae are derived:

$$\frac{\pi(\delta x)-\pi(\gamma x)}{\pi(\beta x)-\pi(\alpha x)}=\frac{\delta-\gamma}{\beta-\alpha}\cdot\left\{1+\mathcal{C}(\alpha,\beta,\gamma,\delta)\cdot\frac{1}{\log x}\right\}+o\left(\frac{1}{\log x}\right)$$

and

$$[\pi(\delta x)-\pi(\gamma x)]-[\pi(\beta x)-\pi(\alpha x)]=[(\delta-\gamma)-(\beta-\alpha)]\cdot\frac{x}{\log x}+\mathcal{C}'(\alpha,\beta,\gamma,\delta)\cdot\frac{x}{\log^2 x}+o\left(\frac{x}{\log^2 x}\right).$$

Specialization of $\alpha, \beta, \gamma, \delta$ leads to results concerning the distribution of primes.

7. *Knots and quadratic forms*, by Professor K. A. Hirsch, University of London and University of Colorado. (By invitation).

The speaker discussed certain topological properties of knots by considering the invariants of related quadratic forms.

8. *The power series coefficients of L-series*, by Dr. W. E. Briggs, University of Colorado.

Consider $L_k(s)=\sum_1^\infty \chi(n)n^{-s}$ where χ is a real non-principal character mod k. This series can be presented by the power series $\sum_1^\infty L^{(r)}(1)(s-1)^r/r!$. These coefficients can be determined by evaluating the r-th derivative of the defining series to obtain

$$L^{(r)}(1)=(-1)^r\sum_1^k \chi(t)\gamma_{r,k,t},\quad\text{where}\quad \gamma_{r,k,t}=\lim_{x\to\infty}\left[\sum_{n\leqq x,\,n\equiv t(k)}\frac{\log^r n}{n}-\frac{\log^{r+1}x}{k(r+1)}\right].$$

Similarly the power series coefficients of the zeta function can be determined by considering

$$h(s)=\zeta(s)-\frac{s}{s-1}=s\int_1^\infty\frac{[x]-x}{x^{s+1}}dx=\sum_0^\infty\frac{h^{(r)}(1)}{r!}(s-1)^r$$

and evaluating the expression obtained by differentiating the integral r times. This gives $h^{(r)}(1)=(-1)^r\gamma_r$ for $r>0$ and $h(1)=\gamma_0-1$ where $\gamma_r=\gamma_{r,1,0}$.

9. *A problem in interpolation*, by M. L. J. Lange, University of Colorado.

Given a polynomial of degree n with coefficients in a field K, $f(x)=(x-\alpha_1)^{m_1}(x-\alpha_2)^{m_2}\cdots(x-\alpha_k)^{m_k}$, and with the α_i in the root field K' of $f(x)$, and given a polynomial $g(z)$ of arbitrary degree with coefficients in K', the problem is to find a polynomial $h(x)$ of degree $\leqq n-1$ with coefficients in an extension field K'' such that $F(x)=g(h(x))-x$ is divisible by $f(x)$. The author showed that an $h(x)$ with the required properties exists if and only if for all α_i with $m_i>1$ the equation $g(z)-\alpha_i=0$ has at least one simple root. He also gave a method for actually constructing such a polynomial $h(x)$.

10. *An outline of the mathematics curriculum and schedule at the USAF Academy*, by Colonel Archie Higdon, USAF Academy.

The United States Air Force Academy mathematics curriculum consists of courses in college algebra, plane and spherical trigonometry, analytical geometry, differential and integral calculus, applied calculus, and elementary differential equations for a total of 21 quarter hours.

Students will be sectioned according to demonstrated ability in mathematics with an average of 12.5 students per section. They will be graded almost every day and daily preparation is mandatory. The top sections will cover some advanced topics in each course not required for those with less aptitude for mathematics. These top sections will contain many students who would be admitted with advanced standing in civilian schools. All students are required to complete all four years at the U. S. Air Force Academy regardless of previous college training.

11. *A new Poisson equation analog computer*, by Mr. W. W. Varner, University of Colorado.

A new computer has been completed at the University for the very rapid solution of the general second order partial differential equation

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + ku + F(x, y, u) = 0$$

with appropriate boundaries. It also solves the Poisson equation in three dimensions with a grid of 960 nodes or mesh points which can be arranged very easily and quickly into Cartesian, cylindrical, spherical, and other coordinate systems. It can handle very complicated boundaries, source and sink conditions, and transients.

12. *A review of the* 1954 *Oregon Summer Conference*, by Professor F. M. Carpenter, Colorado School of Mines.

F. M. CARPENTER, *Secretary*

CALENDAR OF FUTURE MEETINGS

Thirty-ninth Annual Meeting, Rice Institute, Houston, Texas, December 30, 1955.

Thirty-seventh Summer Meeting, University of Washington, Seattle, Washington, August 20–21, 1956.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

ALLEGHENY MOUNTAIN

ILLINOIS, Eastern Illinois State College, Charleston, May 11–12, 1956.

INDIANA, Wabash College, Crawfordsville, May 5, 1956.

IOWA, Grinnell College, Grinnell, April 20–21, 1956.

KANSAS

KENTUCKY

LOUISIANA-MISSISSIPPI, McNeese State College, Lake Charles, Louisiana, February 17–18, 1956.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Catholic University, Washington, D. C., December 3, 1955.

METROPOLITAN NEW YORK

MICHIGAN, University of Michigan, Ann Arbor, March, 1956.

MINNESOTA, South Dakota State College, Brookings, October 15, 1955.

MISSOURI, Fontbonne College, St. Louis, Spring, 1956.

NEBRASKA

NEW ENGLAND, Organizational Meeting, University of New Hampshire, Durham, November 26, 1955.

NORTHERN CALIFORNIA

OHIO, April, 1956.

OKLAHOMA, Oklahoma City University, October 28, 1955.

PACIFIC NORTHWEST, Oregon State College, Corvallis, June, 1957.

PHILADELPHIA, University of Pennsylvania, Philadelphia, November 26, 1955.

ROCKY MOUNTAIN

SOUTHEASTERN, UNIVERSITY OF GEORGIA, Athens, March 16–17, 1956.

SOUTHERN CALIFORNIA, Pomona College, Claremont, March 17, 1956.

SOUTHWESTERN, New Mexico College of Agriculture and Mechanical Arts, Las Cruces, April, 1956.

TEXAS, Southwest Texas State Teachers College, San Marcos, April, 1956.

UPPER NEW YORK STATE, Alfred University, Alfred, April 28, 1956.

WISCONSIN, Marquette University, Milwaukee, May, 1956.

7. *Commutativity of finite matrices*, by Dr. Olga Taussky-Todd, National Bureau of Standards, introduced by the Secretary.

Various generalizations of the concept of commutativity of a pair A, B of finite matrices are being discussed. One such generalization consists in assuming that all polynomials $p(A, B)$ have as eigenvalues $p(\alpha_i, \beta_i)$ where α_i, β_i are the eigenvalues of A and B taken in a special order. A more recent generalization assumes that only all linear combinations $\lambda A+\mu B$ have as eigenvalues $\lambda\alpha_i+\mu\beta_i$. A further generalization is obtained by studying the matrices in the pencil $\lambda A+\mu B$ which have multiple characteristic roots. The vanishing of the higher commutators of A and B also plays a big role. If in particular $B=A^*$ (the complex conjugate and transpose of A) then the vanishing of the higher commutators sheds light on the nature of A itself.

R. P. BAILEY, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The thirty-ninth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Utah, Salt Lake City, Utah, on May 4 and 5, 1956. Professor C. R. Wylie, Jr., Chairman of the Section, presided at all three sessions.

There were 77 persons registered for the meeting, including 48 members of the Association.

Officers elected at the meeting for 1956–1957 were: Chairman, Professor R. R. Gutzman, Colorado School of Mines; Vice-Chairman, Professor J. S. Leech, Colorado College; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *Connectedness in partially ordered sets*, by Professor L. E. Ward, Jr., University of Utah.

A partially ordered space is a triple $(X, T, <)$ where (X, T) is a topological space, $(X, <)$ is a partially ordered set, and there obtains some harmonious relation between T and $<$. Toward the end of characterizing various partially ordered spaces theoretically, results of the following type are of interest: order hypothesis→topological conclusion. Sample theorem: if the set of $a \leqq x$ is compact for each $x \in X$, if the partial order is continuous and dense, and if each pair of points of X have a common predecessor, then X is connected.

2. *Convergence of sequences of linear fractional transformations*, by Professor W. J. Thron, University of Colorado.

Let $\{T_n(z)\}$ be a sequence of nonsingular linear fractional transformations. Let Z be the set in the complex plane, such that $\{T_n(z)\}$ converges for all $z \in Z$. Finally, let $T(z)$ be the function to which the sequence converges on Z. Possible domains of convergence Z and corresponding limit function $T(z)$ are determined. Among other results it is proved that $T(z)$ must either be a constant for all $z \in Z$, or a constant for all but one value of Z, or a linear fractional transformation (in this case Z is the whole plane).

3. *Orthonormal functions used for the approximate solution of integro-differential equations*, by Professor F. M. Stein, Colorado Agricultural and Mechanical College.

Under certain conditions the integro-differential equation $U(u) \equiv L(u) - \int_a^b h(x, t)u(t)dt = f(x)$

with boundary conditions $U_i(u)=0 (i=1, 2, \cdots, m)$ has a unique solution of the form $u(x)=\int_a^b H(x, t)f(t)dt$, where $H(x, t)$ is the Green's function of the problem. Let $S_n(x)$ be a sum of functions of the complete set $\phi_i(x)$ which are orthonormal with respect to the weight function $\rho(x)$ and which satisfy the boundary conditions. Under the criterion that $\int_a^b |f(x)-U[S_n(x)]|^r dx$ shall be least for $r>0$, this paper examines the sufficient condition that $S_n(x)$ as well as its first m derivatives be uniformly convergent to the corresponding derivative of the solution $u(x)$ as n becomes infinite.

4. *The Dirichlet series transformation*, by Professor J. R. Britton, University of Colorado.

Let $F(t)$ be a complex valued function defined for nonnegative integral values of t. If a is a positive constant, the Dirichlet series $\sum_{t=0}^{\infty} a^{-st}F(t)$ is the Dirichlet series transform, $D\{F(t)\}$. The series converges, for example, if $F(t)=\theta(k^t)$, $k>0$, $t>t_0$. Simple properties of the transform were developed, in particular, the convolution theorem

$$[D\{F(t)\}][D\{G(t)\}] = D\left\{\sum_{t=0}^{\tau} F(t-\tau)G(\tau)\right\}.$$

Application was made to "summation" equations of the convulution type $\sum_{t=0}^{\tau} F(t-\tau)G(\tau)=H(t)$, where $G(t)$ and $H(t)$ are given functions, and $F(t)$ is to be found.

5. *Series solution of linear differential equations*, by Mr. C. A. Grimm, South Dakota School of Mines and Technology.

By the use of the exponential shift formula,

$$P(\Delta)e^{mt}f(t) = e^{mt}P(\Delta+m)f(t)$$

and the substitution $x-a=e^t$, one easily arrives at the formula $P(\Delta)(x-a)^m=P(m)(x-a)^m$. Then by writing (if possible) a linear differential equation in the form $f(\Delta)y+(x-a)^k g(\Delta)y=0$ with the assumed solution $y=\sum_{n=0}^{\infty} c_n(x-a)^{\lambda+Un}$, $c_0\neq 0$, the recurrence relation, standardly found by the method of Frobenius, is readily obtained.

6. *Homogeneous continua*, by Professor C. E. Burgess, University of Utah.

A brief history of results on homogeneous continua was given, and some related unsolved problems were mentioned.

7. *Finite geometries and difference sets*, by Professor B. W. Jones, University of Colorado.

Cyclic finite geometries were defined and it was shown that any such geometry leads to a perfect difference set. Conversely, any perfect difference set leads to a finite geometry. One or two generalizations were indicated.

8. *Decimal expansions*, by Dr. Bodo Volkmann, University of Mainz, Visiting Professor, University of Utah. (By invitation.)

Borel proved in 1909 that almost all real numbers are normal, *i.e.*, their decimal expansion involves each of the possible digits with the same asymptotic frequency. A number of generalizations of Borel's theorem to the digit distribution of non-normal numbers were obtained by Besicovitch, Knichal, Eggleston, and the speaker. These theorems describe certain sets of real numbers in terms of their Hausdorff (or fractional) dimension. Among the unsolved problems on decimal expansions are questions such as whether numbers like e, π, or $\sqrt{2}$ are normal in any scale, and what conclusions can be drawn from the normality of a real number in a given scale about its digits distribution in any other scale.

9. *Some constants associated with the Riemann zeta-function*, by Dr. W. E. Briggs, University of Colorado.

The Riemann zeta-function has the representation

$$\zeta(s) = \frac{1}{s-1} + \sum_{0}^{\infty} \frac{(-1)^n \gamma_n}{n!} (s-1)^n$$

where

$$\gamma_n = \lim_{n \to \infty} \left[\sum_{i=1}^{N} \frac{\log^n i}{n} - \int_1^N \frac{\log^n x}{x} dx \right]$$

and satisfies the functional equation $\zeta(s) = 2^s \pi^{s-1} \sin s\pi \Gamma(1-s)\zeta(1-s)/2$. By letting $s = 1-2m$, where m is a positive integer, it can be shown that infinitely many γ_n are positive, and infinitely many are negative. From a representation of γ_n as an infinite series, it can be shown that $\gamma_n = O\{(n/2e)^n\}$. Using the fact that $\zeta(s) - 1/(s-1)$ is an entire function of order one, it can be shown that if $\epsilon > 0$, then $n^{-\epsilon n} < |\gamma_n| < n^{\epsilon n}$ is true for infinitely many n.

10. *Production delay period*, by Professor E. A. Davis, University of Utah.

In the production of economic goods a "production delay period" elapses between the time factors are joined in a firm's productive mechanism and the time resulting output materializes. The delay period, "time", acts somewhat as a productive factor whose cost is expressed through the interest rate. The author considered adjustments of a firm attempting to maximize "profit" integrals, strict competition being assumed, under the assumption that the production delay period for the firm was: a) of zero length; b) of fixed length; c) of length subject to entrepreneur control. The latter two cases lead naturally to a theory involving producer expectations.

11. *Summing of series with missing terms*, by Dr. H. J. Fletcher, Brigham Young University.

A method is presented to sum many series of the form $\sum a_n b_n$ where the series $\sum a_n$ is known and b_n is a periodic function of n. The method consists of expressing b_n as a finite sum of sines and cosines, and summing the corresponding trigonometric series. In particular, if the series $\sum a_n \sin nx$ can be summed to a known function, then the corresponding sum which has terms missing periodically can be summed to a known function.

12. *Fitting empirical equations to data from small orifice fluid meters*, by Professor S. R. Smith, University of Wyoming.

Two empirical equations, $C = R/(a+bR)$ and $C = d + k/\sqrt{R}$ were fitted to the orifice meter data (C, R data was determined by Professor Eric Lindahl, University of Wyoming). Water was the fluid used in each case. C is the coefficient of discharge of the meter and R its corresponding Reynolds number, both dimensionless.

13. *Errors in linear systems*, by Professor C. A. Hutchinson, University of Colorado. (Presented by title.)

14. *Coordination of college and high school mathematics topics*, by Professor O. M. Rasmussen, University of Denver.

The author expressed the need for coordination of mathematics programs in the high schools and colleges, especially with respect to topics of modern mathematics. This need was shown by a brief review of some of the present activity concerning modern topics in mathematics. It was emphasized that coordination should take place on national, regional, and local levels with each individual aiding in any manner possible.

F. M. CARPENTER, *Secretary*

4. *Solution of the integrodifferential equation of transfer by successive approximations*, by Dr. P. M. Anselone, Radiation Laboratory, Johns Hopkins University.

The equations

$$\mu \frac{\partial I(\tau, \mu)}{\partial \tau} = I(\tau, \mu) - 1/2 \int_{-1}^{+1} I(\tau, \mu')d\mu', \quad \int_{-1}^{+1} I(\tau, \mu')\mu' d\mu' = F \text{ (a constant)},$$

where $0 \leqq \tau < \infty$ and $-1 \leqq \mu \leqq 1$, plus certain auxiliary conditions, define a classical problem in transfer theory. The existence and uniqueness of the solution was established by E. Hopf, who obtained the Neumann series solution of a related integral equation. The Wick-Chandrasekhar technique for approximating I involves replacing the two integrals above by sums corresponding to the $2n$ point Gauss quadrature formula. The resulting problem is solved to yield an approximation, I_n, to I. The "double-Gauss" formula, in which the n point Gauss formula is applied separately to each of the intervals $-1 \leqq \mu \leqq 0$ and $0 \leqq \mu \leqq 1$, also yields an approximation. The principal result obtained is that the sequence $\{I_n : n \geqq 1\}$ corresponding to the double-Gauss formula converges to I uniformly on each compact subset of the domain of I.

5. *Matrix analysis for production scheduling and inventory control*, by Professor D. N. Chorafas, Catholic University of America.

With the use of high speed electronic data processing systems, mathematical techniques which seem to be very complicated or unduly involved became of importance and interest for the solution of engineering production problems.

Matrix analysis can be used to advantage for the solution of problems in Engineering Production. Commodity requirements for the initial, the intermediate and the final steps can be set in the form of a rectangular matrix. Then with a simple matrix multiplication engineering management is able to study the input-output requirements of any production system.

The speaker discussed the mechanics of the method from the conception of the model to the data processing through an electronic digital computer and evaluated the method with respect to its potentialities for future application in industry.

6. *Intermittent rotors*, by Mr. Michael Goldberg, Bureau of Ordnance, Navy Department, Washington, D. C.

The shape of the least area which, when placed at random on a square lattice of points, always includes at least one of the points was shown by J. J. Shaffer and D. B. Sawyer to be a square to which has been added the areas included by two parabolic arcs, one on each of two opposite edges of the square. The speaker showed that this shape is one of a family of convex curves which may be rotated through all orientations in the plane while passing through at least three of the four vertices of a square. Extensions to a series of similar problems were indicated.

In addition, the following hour lectures were presented by invitation of the joint program committee:

1. *Geometry in the mathematics curriculum*, by Professor W. L. Chow, The Johns Hopkins University (auspices of MAA).

2. *Quaternions and Clifford numbers*, by Professor Marcel Riesz, Institute of Fluid Dynamics, University of Maryland (auspices of SIAM).

R. P. BAILEY, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The fortieth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the Colorado School of Mines, Golden, Colorado, on Friday afternoon and evening and Saturday forenoon, May 3 and 4, 1957. Professor R. R. Gutzman, Chairman of the Section, presided at all three sessions. There were 116 persons registered for the meeting, including 68 members of the Association.

Officers elected at the meeting for 1957–1958 were: Chairman, Professor D. O. Patterson, Colorado State College; Vice-Chairman, Professor N. C. Hunsaker, Utah State Agricultural College; Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *On generalized Legendre polynomials*, by Professor Arne Magnus, University of Colorado, introduced by the Secretary.

A recurrence formula is developed for the polynomials $P_k=P_k(\phi_1, \cdots, \phi_n)$ defined by $[1+\phi_1 t+\cdots+\phi_n t^n](m/n)=1+P_1 t+\cdots+P_1 t^k+\cdots$ and application made to the polynomial solutions of the partial differential equation $u_x \cdot v_y - u_y \cdot v_x = 1$ where $u=u(x, y)$ and $v=v(x, y)$.

2. *The Laplace transform in discontinuous solutions of a partial differential equation*, by Professor V. W. Bauman, Colorado School of Mines, introduced by the Secretary.

Using the Laplace transform to solve problems involving the equation (1) $Y_{tt}(x, t)=a^2 Y_{xx}(x, t)$, equation is assumed to be (2) $s^2 L\{y\}=a^2(\partial^2/\partial x^2)[L\{Y\}]$. In some problems the solution, $Y(x, t)$, or its partial derivatives of first order are only sectionally continuous. In this case the members of (2) are not the transforms of members of (1), both members of (2) having additional terms which involve the salti in the discontinuous functions. It was shown in this paper that the true transformed equation always reduces to equation (2) if the solution or its partial derivatives of first order have only finite discontinuities on the lines in the xt-plane, $t=C\pm x/a$.

3. *The economic index numbers of Divisia*, by Professor E. A. Davis, University of Utah.

An expository account of the "historical" index numbers, for prices and quantities of goods traded, due to Divisia (F. Divisia, *Economique Rationelle*, Paris, 1928) was presented. Relationships between these quantities and the index formulae of Laspeyres were noted and, in particular, a device for approximating the former by means of products of Laspeyres indexes was indicated.

4. *A mathematical analysis of fuel burnout in nuclear reactors*, by Captain A. W. Banister, United States Air Force Academy, introduced by the Secretary.

Mathematical analysis of nuclear reactors is necessary in order to provide quantitative information regarding certain design and operational features. One such item is the effect of fuel burnout in modern reactors operating at high power levels. An approach to this problem can be made by writing a differential equation descriptive of a generalized reactor volume element, and introducing perturbations in the reactor constants to simulate burnout. By making certain approximations justifiable on physical grounds, the equation can be transformed into a linear, non-homogeneous type, easily solvable by the operator method. The solution then provides quantitative information on changes in the power distribution function, and adjustments necessary to maintain level operation.

5. *A characterization of n-groups*, by Professor D. W. Robinson, Brigham Young University.

The generalized groups defined by W. Dörnte (*Untersuchungen über einen verallgemeinerten Gruppenbegriff*, Math. Z., vol. 29, 1928, pp. 1–19) are systems of elements with a polyadic operation satisfying an extension of the associativity and solvability axioms for ordinary groups. This note points out that these systems can be characterized as well by replacing the solvability axiom with a generalization of the identity-inverse axiom for groups.

6. *Infinite exponentials*, by Professor W. J. Thron, University of Colorado.

For every $n>1$ let $t_n(z)=e^{a_n z}$, where the a_n are complex numbers. Define $T_n(z)$ to be: $T_1(z)=t_1(z)$, $T_n(z)=T_{n-1}(t_n(z))$. Then $\{T_n(1)\}$ is called an infinite exponential. It is proved that an infinite exponential converges if $|a_n| \leq e^{-1}$ for all n.

7. *Speed-up college mathematics*, by Professor I. L. Hebel, Colorado School of Mines.

A review of the progress of a highly selected group of twenty entering freshmen who have been allowed to progress at an accelerated pace with a view to completing 21 semester hours of college mathematics (through Differential Equations) in three semesters. From the results of this first attempt to do something for the better-than-average student, the conclusion reached is that many additional students taking the "standard" courses could profitably be placed in the accelerated program. It is anticipated that about 50 of the next group of 300 entering freshmen will be assigned to a similar group.

8. *Antenna theory*, by Dr. James Wait, Boulder Laboratories, National Bureau of Standards. (Invited Address.)

The calculation of the radiation field of a flush mounted antenna in the tangent plane (the classical light-shadow boundary) is not readily treated by either geometrical optics or the residue-series. In the former case the field is indeterminate and in the latter case the convergence is extremely poor and would actually diverge in the illuminated region. Despite the fact that the harmonic series is cumbersome, it is valid in this transition zone between the illuminated and shadow regions of space. Therefore, it is desirable to attempt to adapt the harmonic-series representation to surfaces of large radius of curvature. This is the purpose of the present paper.

9. *Orthogonal functions whose k-th derivatives are also orthogonal*, by Professor F. M. Stein, Colorado State University.

Several sets of orthogonal functions possess the property that the k-th derivatives of these functions form sets of orthogonal functions, perhaps with a different weight function, $w_k(x)$, but over the same interval. This paper shows that the set $\{1, \cos nx, \sin nx\}$ possesses this property. Also, if the orthogonal functions are polynomials they must be those of Hermite, Jacobi, or Laguerre; and these are the only polynomials possessing this property under the definition of orthogonality that for $\{\phi_n(x)\}$,

$$\int_a^b w(x)\phi_n(x)\phi_m(x)dx = c_n\delta_{mn}.$$

10. *Introduction to SOMAC*, by Professor R. R. Gutzman, Colorado School of Mines.

The speaker explained the basic operating exponents of the analog computer. He discussed the methods of programming a differential equation of the type

$$a_0\frac{d^2y}{dx^2} + a_1(dy/dx)^2 + a_2y = F(x),$$

and simultaneous linear algebraic equations. Different ways of generating $F(x)$ were considered.

11. *The relation of regular semigroups to groups*, by Mr. H. G. Moore, University of Utah.

In this paper theorems are given which relate regular semigroups and inverse semigroups with groups. A regular semigroup with cancellation is a group as is every regular semigroup generated by a single element. Cancellation may be relaxed slightly if certain other conditions are imposed on the regular semigroup. Every regular semigroup possesses subsets which are groups, and if not all the elements of the semigroup are idempotent it possesses nontrivial subsets which are groups.

12. *Likes and dislikes—when and why?*, by Professor O. M. Rasmussen, University of Denver.

A preliminary report of a survey in progress as an attempt to find when and why students like or dislike mathematics. Thus far it appears that grades 7, 9, and 10 are the critical ones with the teachers receiving credit or blame in most cases. Parental encouragement appears to be a factor

among those who now like mathematics. Information reported is from a survey of students at all levels in a university. Therefore, the results of changing conditions of the past five years are not included.

13. *The new Bachelor of Science Degree in Applied Mathematics at the University of Colorado*, by Professor L. W. Rutland and Professor J. R. Britton of the University, presented by Professor Rutland.

An announcement was made of the new degree being offered in the Department of Applied Mathematics at the University of Colorado. The curriculum includes study in the basic engineering sciences and applied mathematics subjects. The applied mathematics work includes intermediate differential equations, advanced calculus (*e.g.*, Taylor, *Advanced Calculus*), introduction to applied mathematics (*e.g.*, Wylie, *Advanced Engineering Mathematics*), statistics, algebraic methods, computing machines, and a senior seminar.

F. M. CARPENTER, *Secretary*

CALENDAR OF FUTURE MEETINGS

Forty-first Annual Meeting, University of Cincinnati and Hotel Sheraton-Gibson, Cincinnati, Ohio, January 31, 1958.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

ALLEGHENY MOUNTAIN, Washington and Jefferson College, Washington, Pennsylvania, May, 1958.

ILLINOIS, Illinois College, Jacksonville, May 9–10, 1958.

INDIANA, DePauw University, Greencastle, October 18, 1957.

IOWA

KANSAS

KENTUCKY, University of Kentucky, Lexington, April, 1958.

LOUISIANA-MISSISSIPPI, Loyola University, New Orleans, February 21–22, 1958.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Georgetown University, Washington, D. C., December 7, 1957.

METROPOLITAN NEW YORK

MICHIGAN, University of Michigan, Ann Arbor, March, 1958.

MINNESOTA, State Teachers College, Mankato, October 5, 1957.

MISSOURI, University of Missouri, Columbia, Spring, 1958.

NEBRASKA, University of Nebraska, Lincoln, April 19, 1958.

NEW JERSEY, Fairleigh Dickinson University, Rutherford, November 2, 1957.

NORTHEASTERN, Dartmouth College, Hanover, New Hampshire, November 30, 1957.

NORTHERN CALIFORNIA, San Francisco State College, January 18, 1958.

OHIO, Denison University, Granville, April, 1958.

OKLAHOMA, Oklahoma City University, October 25, 1957.

PACIFIC NORTHWEST

PHILADELPHIA, November 30, 1957.

ROCKY MOUNTAIN, Colorado State College, Greeley, Spring, 1958.

SOUTHEASTERN, University of Florida, Gainesville, March 14–15, 1958.

SOUTHERN CALIFORNIA, Pasadena City College, March 8, 1958.

SOUTHWESTERN, University of New Mexico, Albuquerque, April 11–12, 1958.

TEXAS, Baylor University, Waco, April, 1958.

UPPER NEW YORK STATE, Ecole Polytechnique and University of Montreal, Montreal, Quebec, Canada, May, 1958.

WISCONSIN, Carroll College, Waukesha, May, 1958.

5. *A representation symbol applied to Waring's theorem, modulo p*, by Professor J. D. Elder, St. Louis University.

The author gave an expository account of the representation symbol, $[a, b, c]$, introduced by Sr. M. F. Torline, C.S.J. in her doctoral dissertation. Implicative properties were given, and their uses in problems connected with Waring's theorem were discussed.

6. *Electronic computers, information and education*, by Professor P. C. Hammer, University of Wisconsin. (By invitation.)

If it is agreed that a proof is in a chain of symbols, machines can and do prove theorems. More generally, they answer questions. The principal role of computing machines is to prove propositions and answer questions. It is an obstacle to the use of machines that mathematicians consider they are dealing with infinite processes, which they say cannot be done by machine. While it may be debated whether there are infinite processes, there is no doubt that no one deals with any process with infinite means. Hence, there are no stated proofs which could not be duplicated on a computer.

MARY L. CUMMINGS, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-first annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado State College, Greeley, Colorado, on Friday afternoon and evening and Saturday forenoon, May 9 and 10, 1958. Professor D. O. Patterson, Chairman of the Section, presided at all three sessions. On Saturday morning the Section held a joint meeting and luncheon with the Colorado Council of Teachers of Mathematics.

There were 107 persons registered for the meeting, including 67 members of the Association. Officers elected at the meeting for 1958–1959 were: Chairman, Professor N. C. Hunsaker, Utah State Agricultural College; Vice-Chairman, Professor J. W. Ault, United States Air Force Academy; and Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *Solving boundary value problems by use of Green's function in conjunction with the Laplace transform and separation of variables*, by Professor L. C. Barrett, South Dakota School of Mines.

The primary purpose of this paper is to point out how an influence function, *i.e.* Green's function, may be utilized together with the Laplace transform and separation of variables to facilitate a solution of boundary value problems of engineering and physics. Among the notable features of the method are: (a) Its capacity to yield the inverse of certain Laplace transforms without requiring recourse to complex variable theory. (b) The method enables one to escape the tedium of the step-by-step procedure, and subsequent use of superposition, usually followed in solving such problems by separation of variables. (c) Time-dependent boundary conditions present no special difficulty to the method.

2. *The radiation of waves from a point source*, by Professor R. W. McKelvey, University of Colorado, introduced by the Secretary.

The object of the paper is to obtain by a new method, a known expression for a *radiation solution* of the generalized wave equation,

$$\frac{\partial^2 u}{\partial t^2} = \sum_{i,j=1}^{3} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{3} a_i \frac{\partial u}{\partial x_i} + au.$$

The coefficients a_{ij}, a_i, a are variable functions of space, but are constant in time. The matrix (a_{ij})

is positive-definite. [For an exact definition of a radiation solution, see Courant-Hilbert, *Mathematischen Physik II*, Berlin, 1937, p. 453]. The formula in question has been obtained by the methods of Hadamard. [*loc. cit.*, p. 154]. The procedure given here avoids many of the complications of those methods, while preserving the spirit. It consists of a construction process, resembling Hadamard's construction of the fundamental solution. [J. Hadamard, *Lectures on Cauchy's Problem*, New York.]

3. *Coset spaces in topological groups and their relation to the group*, by Mr. D. A. Ford, Graduate Assistant, University of Utah.

A topological group is an abstract group defined on the elements of a topological space where x^{-1} is continuous in x, and the product xy is continuous in x and y simultaneously. If H is a subgroup of a topological group G, then the space of left cosets G/H is a topological space and if H is invariant, G/H is a topological group. The paper deals with the relation of topological properties among the group, subgroup and coset space. For example, if H and G/H are compact, then G is compact.

4. *Geometries based on the undefined terms "sets" and "inclusion,"* by Professor Aboulghassem Zirakzadeh, University of Colorado.

E. V. Huntington, in 1913, introduced a set of axioms for Euclidean Geometry which was based on the undefined terms "set" and "inclusion." Using these undefined terms and changing the given axioms, it is possible to find other geometries, finite and infinite. A minimum set of axioms is given to insure that the resulting geometries are sufficiently regular. The consistency and independence of these axioms is proven.

5. *Variation of parameters in solving systems of difference equations*, by Professor L. C. Barrett and Professor F. E. Dristy, South Dakota School of Mines, presented by Professor Dristy.

The primary purpose of this paper is to illustrate how the method of variation of parameters, so familiar in finding particular integrals of nonhomogeneous linear differential equations, may be extended to determine a particular solution of a nonhomogeneous system of linear difference equations. At the same time a technique is developed for solving such a system which, in contrast to the usual procedure, may be applied directly to the system. Thus, the usual reduction of the system to a single difference equation before solving becomes unnecessary.

6. *Heat conduction with variable thermal conductivity in a sphere*, by Professor Nathan Schwid, University of Wyoming.

When the dependence of the diffusivity coefficient k/cp of the heat conduction equation upon the temperature is sufficiently significant to warrant its consideration, the equation becomes nonlinear. The quantity k, the thermal conductivity, is then a function of temperature with $c\rho$, the product of specific heat and density, constant. If we take $k/c\rho$ as $\alpha+\beta u$, where β/α is small, and u is the temperature, an approximation to the temperature in a sphere under simple boundary conditions is obtained which approximates the solution for substantial values of t.

7. *The geometry of* $f(n, \alpha) = \sum e^{ik\alpha}$, $k=0, \cdots, n$, by Professor Emeritus A. J. Kempner, University of Colorado.

An obvious vector construction is combined with the geometrical multiplication in the plane of complex numbers of all points on one curve by all points of a second curve to obtain results of which the following is representative: The "Wertevorrat" (Set of values assumed) of $\sum\sum e^{i(k_1\alpha_1+k_2\alpha_2)} = f(n_1, n_2)$, α_1/π, α_2/π irrational, k_1, k_2 independently over 0, 1, $\cdots$, n_1, and 0, 1, $\cdots$, n_2, respectively, n_1, $n_2 < \infty$ is given by the cardioid $a\rho \sin(\alpha_1/2) \sin(\alpha_2/2) = 1 - \cos(\theta+\alpha_1/2+\alpha_2/2)$. In this cardioid the functional values are distributed everywhere densely.

8. *Mathematics program for outstanding cadets at USAFA*, by Professor W. Milliken, United States Air Force Academy.

Three levels of mathematics have been established at the Air Force Academy. The regular course is the usual two-year engineering mathematics course with spherical trigonometry, some statistics and differential equations added. A cadet may advance from this course to the accelerated course at the beginning of the second semester. The accelerated course covers the same material plus a course in elementary statistics in a year and a half. The super-accelerated course covers this material in one year. Thus, there is extra time available for cadets in the faster programs to take additional advanced mathematics courses or other electives.

9. *Mathematical education in Europe, Britain, and the United States*, by Professor W. W. Rogosinski, King's College, Durham University, England; Visiting Professor, University of Colorado.

An attempt is made to point out and to explain the striking differences in mathematical education, both at high school and university level, as seen in Continental Europe, Britain, and the United States of America. The explanation is sought in a different philosophy of education in general which, in turn, is conditioned by different history and tradition: the scholastic idealism of Europe, its realistic variant in Britain, and the social (and materialistic) trend in American education.

10. *A sequel to Euclid*, by Professor H. S. M. Coxeter, University of Toronto. (Invited Address.)

11. *Coaxal circles and inversion*, by Professor H. S. M. Coxeter, University of Toronto.

Any two given circles belong to a pencil of coaxal circles consisting of all the circles orthogonal to any two circles, α and β, orthogonal to the two given circles. The arbitrariness of α and β is established by inverting the two given circles into straight lines or concentric circles. When inverted with respect to a sphere whose center is outside the plane, two orthogonal pencils of coaxal circles yield sections of a sphere (the inverse of the plane) by pencils of planes through two polar lines. Such a pencil of circles is hyperbolic (*i.e.*, intersecting), parabolic (touching), or elliptic (disjoint) according as the common line of the planes is a secant, a tangent, or an exterior line.

12. *Oscillation and non-oscillation of second order complex differential equations*, by Mr. R. W. Hunt, Graduate Assistant, University of Utah.

The primary object of this paper was to investigate the zeros on $a \leqq x < \infty$ of solutions of the differential equation $(py')' + fy = 0$, with p and f complex-valued continuous functions of the real variable x. By the use of an associated system of two real, second-order equations obtained by writing p, f, and y in polar form, two sufficient conditions for disconjugacy (at most one zero on $a \leqq x < \infty$) of all nontrivial solutions were obtained. Then a special form of this equation, $(y'/q)' + \bar{q}y = 0$, q complex-valued, was changed to the first order system $y' = q\bar{z}$, $z' = -q\bar{y}$, with solutions $s(x) = s[a, x; q]$ and $c(x) = c[a, x; q]$ corresponding to the boundary conditions $y(a) = 0$, $z(a) = 1$. Finally $s[a, x; q]$ and $c[a, x,; q]$ were shown to have two properties analogous to well-known properties of the real sine and cosine functions; namely, $|s|^2 + |c|^2 \equiv 1$ and, for $k = 1$, $s[a,x; kq] = ks[a, x; q]$, $c[a, x; kq] = c[a, x; q]$.

13. *A multiple integral approach to Taylor's theorem*, by Professor L. C. Barrett and Mr. D. W. Willett, Student, South Dakota School of Mines, presented by Mr. Willett.

This note presents several elementary geometrical considerations, involving lengths, areas, and volumes, which lead quite naturally to a multiple integral approach to Taylor's theorem.

14. *Evaluation of a limit from the theory of heat flow*, by Dr. H. R. Bailey, Mathematician, Ohio Oil Company Research Center, Littleton, Colorado, introduced by the Secretary.

The problem of heat conduction in an infinite homogeneous medium from the surface of a cylinder whose radius is increasing with time is solved by the Green's function method. The solution is obtained as an integral of the form $I=\int_0^t f(t,\tau)d\tau$. A method is given to obtain an explicit evaluation of this integral for $t\to\infty$ for the case of the cylinder radius increasing at a constant velocity. It is shown that the integral can be divided into two parts, $I=\int_0^{t/N} f(t,\tau)d\tau+\int_{t/N}^t f(t,\tau)d\tau$, where the last integral goes to zero as $t\to\infty$ and the integrand in the range $[0, t/N]$ can be replaced by an asymptotic expression which can be integrated explicitly as a function of N. Finally the desired limit is obtained by passing to the limit as $N\to\infty$.

15. *An application of the decomposition of a matrix into principal idempotents*, by Professor D. W. Robinson, Brigham Young University.

As a simple application of the decomposition of a (diagonable) matrix into principal idempotent elements, this note provides a proof of the following well-known result: if the nth derivative of a function f exists at α, then it can be computed as the limit of $h^{-n}\sum_{m=0}^{n}\binom{n}{m}(-1)^m f[\alpha+(n-m)h]$ as h approaches zero.

16. *Families of Sturm-Liouville systems*, by Mr. E. L. Dunn, Colorado State University, introduced by Professor F. M. Stein, Colorado State University.

A family of Sturm-Liouville systems is defined as the collection of all Sturm-Liouville systems whose equations can be obtained by repeatedly differentiating and integrating a Sturm-Liouville equation. For each system, similar boundary conditions apply to the same interval. In this paper the conditions are developed such that a Sturm-Liouville system may generate a family. It is shown that (1) if a Sturm-Liouville system generates a family, the kth derivatives and antiderivatives of its eigenfunctions form orthogonal sets, and (2) if the eigenfunctions of the generating system are not polynomials the sets of eigenvalues for all members of a family are identical. The case when the eigenfunctions are polynomials must be considered separately.

17. *Let's not go off the deep end!* by Professor A. W. Recht, University of Denver.

The general theme is in opposition to the idea of introducing Boolean algebra, sets, and similar types of theoretical mathematics into high school and elementary college courses. We are already teaching too much "gifted" mathematics to the general student, and not teaching successfully the kind of mathematics the 85 per cent or perhaps the 100 per cent, ought to have before they go into the so-called superior pure mathematics. Maybe we have an inferiority complex, and are running away from our real job, which is to bring up all people in our democracy to their full potentialities with a more democratic kind of mathematics.

18. *The work of the Commission on Mathematics*, by Professor Henry Van Engen, University of Wisconsin, Madison.

See this MONTHLY, Report of the May Meeting of the Wisconsin Section.

F. M. CARPENTER, *Secretary*

MAY MEETING OF THE WISCONSIN SECTION

The twenty-sixth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at Carroll College, Waukesha, Wisconsin, on May 3, 1958, Professor R. D. Wagner, Chairman, presiding. Sixty-nine attended the meeting, including forty members of the Association.

At the business meeting of the Section the following officers were elected for the coming year: Chairman, Prof. J. V. Finch, Beloit College, Beloit, Wisconsin; Vice-Chairman, Prof. C. B. Hanneken, Marquette University, Milwaukee, Wisconsin; Secretary-Treasurer, Sister Mary Felice, Mount Mary College, Milwaukee, Wisconsin.

Mr. J. W. Kennedy gave the following report of the 1958 high school mathematics contest: A preliminary contest was held on Feb. 27, in 237 schools in the state, with

puter program, introduced by Ramon E. Moore in "range" arithmetic, is a very hopeful step in the right direction. The need for seeking processes that are numerically stable must be emphasized. Finally, some implications of function theoretical concepts were touched upon.

6. *Minimum variance of estimates under stratified sampling*, by Professor W. R. Van Voorhis, Fenn College.

When a population is stratified for purposes of sampling, the variance of the estimated sample mean, x, depends not only upon the allocation of the sample of size n to the several k strata, but also upon the location of x_i, the points of stratification. Necessary and sufficient conditions to yield minimum variance have been established by Dalenius but these conditions do not yield the explicit values of x_i that must be known before an optimum stratification can be made. It is shown that no general proof of the existence of uniqueness is possible. For the case of "proportional" allocation, it is shown that there exists at least one optimal solution for k strata. A method of successive approximations beginning with a first feasible solution is discussed, and examples are given for several well-known distributions.

7. *Mutation view of conics (shadow transformations and primal states)*, by Dr. Beckham Martin, Owens-Illinois Glass Company, Toledo, Ohio.

In the presentation the following salient remarks were made: (A) There had to be a clean break with conventional geometry, which has reached a state of stagnation, before one could ever hope to reach new pinnacles of achievement. (B) Mutation Geometry is the science of intangible change (shadow-transformations). The discussion began with the general conic equation: (1) $Ax^2+Bxy+Cy^2=Dx+Ey+F$. A primal state number P was calculated: (2) $P=2/(\sqrt{B^2+(A-C)^2}+A+C)$ by which (1) was transformed shadow-wise to its primal state: (3) $ax^2+bxy+cy^2=dx+ey+f$ from which the properties of the representative conic may be read off at sight. Example: The eccentricity is given by (4) $e^2=2-(a+c)$

8. *The 1959 mathematical program*, by Professor R. L. Wilson, Ohio Wesleyan University.

A contrast is made between the type of mathematics currently being used in the physical and nonphysical sciences and the type of mathematics so applied a decade or more ago. Implications are drawn for the mathematical education of students in the various fields of specialization. Alternative suggestions for meeting this situation are made.

9. *Composite pattern of primitive Pythagorean triangles formulated by arithmetical progression*, by Mr. R. J. Irwin, Eddie Painton Associates, Inc., Cleveland, Ohio.

The method presented is believed to be easier and quicker to compile than the methods more commonly used. The non-Pythagorean Triangles are eliminated very readily as they follow a rhythmic appearance in these tables. Periodic checks throughout the tables automatically correct preceding calculations. These tables discovered two (probably typographical) errors in existing published tables. The interesting relation of Pythagorean Triangles to prime numbers is also shown.

FOSTER BROOKS, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-second annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Utah State University, Logan, Utah, on Friday afternoon and evening and Saturday forenoon, May 8 and 9, 1959. The meeting was divided into several sessions with Professors N. C. Hunsaker, Joe Elich, J. H. Barrett, and Harvey Fletcher presiding. There were 94 persons registered for the meeting, including 60 members of the Association.

Officers elected at the meeting for 1959–1960 were: Chairman, Colonel J. W. Ault,

United States Air Force Academy; Vice-Chairman, Professor L. W. Rutland, University of Colorado; and Secretary-Treasurer, Professor F. M. Carpenter, Colorado School of Mines.

The following papers were presented:

1. *A method of approximating the roots of an equation by quadratic formulae*, by Professor Stephen Kulik, Utah State University.

Two zeros of an analytic function $f(z)$ are approximated with a prescribed accuracy by one application of a quadratic formula. The coefficients of the quadratic equation depend on $D_n(z)$ which is calculated recursively,

$$D_n = f'D_{n-1} - f''fD_{n-2}/2! + \cdots + f^{(n-1)}(-f)^{n-2}D_1/(n-1)! + f^{(n)}(-f)^{n-1}D_0/(n-1)!,$$
$$D_0 = 1, D_1 = f', D_2 = f' - f''f; \quad \text{where} \quad D_n = D_n(z), f^{(n)} = f^{(n)}(z), \ n = 0, 1, \cdots.$$

The roots of the quadratic equation converge to the two zeros of $f(z)$ which are nearer to the number z than the remaining zeros.

2. *A note on a simple matrix isomorphism*, by Professor D. W. Robinson, Brigham Young University.

Let C be the field of complex numbers. Let $\phi:\alpha\phi=A$ be the well-known ring isomorphism of C onto a real subsystem of 2-by-2 matrices over C. Let $f(x)$ be a polynomial over C. It is shown that $\phi(f(\alpha))=f(\phi(\alpha))$ if and only if $f(\bar{\alpha})=\overline{f(\alpha)}$, where $\bar{\alpha}$ is the complex conjugate of α. This result is then generalized by considering (1) a ring isomorphism of the n-by-n matrices over C onto a real subsystem of the $2n$-by-$2n$ matrices over C, and (2) functions of matrices.

3. *Definition of "plus" and "times" for the natural numbers*, by Mrs. Jean J. Pederson, Olympus Senior High School of Salt Lake City and University of Utah.

A function may be defined as a set of ordered pairs such that no two distinct pairs of the set have the same first element. Introducing the natural numbers according to the technique of Peano, explicit definitions, as sets of ordered pairs, may then be exhibited for the addition and multiplication functions as applied to the natural numbers.

4. *An extreme value problem for honor students*, by Captain R. C. Rounding and Captain R. L. Eisenman, United States Air Force Academy.

This paper explores the relationship between a problem in which the volume of a cylindrical solid is given and the relative dimensions are to be found to minimize surface area, and the corresponding problem of a rectangular region wherein the area is constant and the perimeter is to be minimized. It is presented as a problem for Honor Students as an example of a technique in mathematical research.

5. *The multiplicity and positiveness of the characteristic numbers of second order Sturm-Liouville systems involving generalized boundary conditions*, by Professor L. C. Barrett, South Dakota School of Mines and Technology.

This paper is concerned with Sturm-Liouville systems that possess boundary conditions involving left and right hand limits of the dependent variable and its first derivative at one or more interior points of a given fundamental interval. Two theorems are cited which provide introductory information concerning the nature of the characteristic functions of such systems. A third theorem desoribes the orthogonality of the characteristic functions. It is then shown that the characteristic numbers are simple roots of the characteristic equation. Sufficient conditions that these characteristic values be non-negative are also given.

6. *Distribution of zeros of solutions of complex differential equations*, by Mr. N. H. Mines, University of Utah.

A zero-free region of a nontrivial solution of the complex differential equation $[K(z)W']'$

$+G(z)W=0$ for $K(z)\equiv 1$ was obtained by E. Hille (*Transactions American Mathematical Society*, vol. 23, 1922). The same method can be used to obtain a zero free region of a nontrivial solution of the general equation requiring only analyticity of the coefficients and $K(z)\neq 0$.

7. *A vector solution of simultaneous linear equations*, by Professor C. A. Grimm, South Dakota School of Mines and Technology.

Three points in the plane, $Ax+By+Cz=D$, $D\neq 0$, are sufficient to determine, to a scalar multiple, A, B, C. The vector $[A, B, C]$ is orthogonal to the plane, and is a scalar multiple of the cross product of two vectors in the plane determined by the three points in the plane. By generalizing this idea a method is developed by which any set of n nonhomogeneous linear equations in n variables may be solved by evaluating one n by n determinant.

8. *Homogeneous production function*, by Professor E. A. Davis, University of Utah.

9. *Lattice points*, by Professor T. M. Apostol, California Institute of Technology (Lecture sponsored by Mathematical Association of America).

10. *The Caltech experiment in calculus*, by Professor T. M. Apostol, California Institute of Technology. (Invited Address).

11. *An undergraduate course on the topology of a line*, by Professor C. E. Burgess, University of Utah. (Invited Address).

The author described an introductory undergraduate topology course which is based upon axioms that describe a linearly ordered, separable, connected space with no first point and no last point. Such a course has been offered at the University of Utah each year for the last several years. A similar address was given before the Wisconsin Section at Whitewater, Wisconsin, May 11, 1957 (this MONTHLY, vol. 64, 1957, p. 627).

12. *Particular solutions for nonhomogeneous, linear, ordinary difference equations*, by Professors Forrest Dristy and L. C. Barrett, South Dakota School of Mines and Technology, presented by Professor Dristy.

In this paper an identity is derived which relates adjoint difference expressions in much the same way that Lagrange's identity of differential equation theory relates adjoint differential expressions. It is then shown how this identity may be used to determine a particular solution of a nonhomogeneous linear ordinary difference equation once the complementary function is known. Thus, the method provides an alternative to the familiar method of variation of parameters.

13. *Application of Fourier series to difference equations*, by Lieutenant J. N. Christiansen, United States Air Force Academy.

A method for obtaining general solutions to linear difference—differential equations is presented. The method is applied to a simple example and the solution is plotted for a special case. The method presented is valuable in that it requires no knowledge of mathematics beyond that usually gained from a course in advanced calculus. It is easy to apply and reduce the solution to quadratures in a very few steps. Also, the solution contains the initial conditions explicitly. The method is analogous to the use of the Fourier transform for finding solutions to partial differential equations.

14. *Geometrical motivations for determinant type proofs of mean value theorems*, by Mr. R. A. Jacobson and Professor L. C. Barrett, South Dakota School of Mines and Technology, presented by Mr. Jacobson.

The usual proofs of the mean value theorems involve the process of applying Rolle's Theorem to functions or determinants happily designed to yield the desired conclusions. The determinants thus employed are usually introduced without comment. In this paper it is shown that these determinants may be motivated by an analysis originating in a geometrical setting.

15. *An approach to the foundations of intuitionism*, by Mr. David Drake, University of Colorado.

The concept of a formal proof was adapted to intuitionist philosophy by replacing axioms and primitive rules of inference with other assertability criteria, such as metamathematical observation. The primitive symbols of logic were defined by means of words having reference to finite and perceptually concrete situations. An axiom of formalized intuitionist logic was then derived—for the given interpretation of symbols—by the modified proof method.

16. *The program of advanced placement in mathematics at Colorado State University*, by Professor F. M. Stein, Colorado State University.

Many students enter Colorado State University with more than the minimum background to enter the regular sequence of mathematics courses. This is first an outline of the method used to select those in this group who could start the sequence at an advanced level, and second a report on the success of the program thus far.

17. *A method of solving Diophantine quadratic equations*, by Professor B. W. Jones, University of Colorado.

Here a method, originating in some ideas of Edgar Emerson, is given for finding all the integral solutions of certain equations of the form $ax^2-by^2=c$ where a, b, and c are positive integers, given a finite number of such solutions. Geometrically this is equivalent to use of a zigzag pattern of lines. This applies also to some quadratic indefinite forms with cross products. In some respects this method is an improvement over the traditional use of the Pell equation.

F. M. CARPENTER, *Secretary*

THE MAY MEETING OF THE WISCONSIN SECTION

The twenty-seventh annual meeting of the Wisconsin Section of the Mathematical Association of America was held on May 2, 1959, at Wisconsin State College, Platteville, Wisconsin, Professor J. V. Finch, Chairman, presiding. There were 69 present, including 33 members of the Association and 30 members of the Wisconsin Mathematics Council, 15 of whom are also members of the Association.

At the business meeting, the following officers were elected for the coming year: Chairman, Professor C. B. Hanneken, Marquette University; Vice-Chairman, Professor Henry Van Engen, University of Wisconsin; Secretary-Treasurer, Sister Mary Felice, S.S.N.D., Mount Mary College.

The following report of the Section's fourth annual mathematics contest for high school students was given by the Contest Committee Chairman, Professor Earl Swokowski, Marquette University:

For the second consecutive year a preliminary contest examination consisting of multiple-choice questions was given for the purpose of enabling the teachers to better select the participants in the final contest. This examination was given to 12,600 students in 283 high schools on February 26, 1959. The top contestant in each school was awarded a certificate.

The final contest was held on April 11, in 27 centers distributed throughout the State, with 900 participating from 187 schools. Cash prizes of $50, $25, $10, and $1 were awarded to each student in the top 15 percent of the contestants, divided into four groups respectively. In addition initialed M.A.A. award pins were given to the twenty in the top two groups and a certificate to the rest of these groups.

After some discussion during which various suggestions were offered for subsequent contests, the Section Chairman was instructed to continue to carry on the contest as in the past two years.

After an address of welcome by Professor Bjarne Ullsvik, President of Wisconsin

novelty to see computation applied to rules of numerical analysis. From tables of Gauss, Laguerre, Hermite, and Lobatto quadrature formulas which have recently been computed with great accuracy, and using the "method of inspection," P. Rabinowitz and the lecturer have formulated and proved a number of asymptotic results relating the abscissas to the weights. Other conjectures, including one by G. Szegö, seem plausible numerically, but proofs are still to be supplied.

7. *Nets and calculus*, by Professor B. J. Pettis, University of North Carolina. (By invitation)

D. B. LLOYD, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-third annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the United States Air Force Academy, Colorado Springs, Colorado, May 6 and 7, 1960. On Saturday the Section enjoyed a joint meeting and lecture with the Rocky Mountain Section of the Society for Industrial and Applied Mathematics. The meeting was divided into several sessions with Professors J. W. Ault, R. M. Elrick, J. W. Querry, J. S. Leech, and A. W. Recht presiding. There were 132 persons registered for the meeting, including 102 members of the Association.

Officers elected at the meeting for 1960–1961 were: Chairman, Professor L. W. Rutland, University of Colorado; Vice-Chairman, Professor L. C. Barrett, South Dakota School of Mines and Technology; Secretary-Treasurer, Professor Leota Hayward, Colorado State University; and Director of High School Mathematics Contest, Professor D. C. B. Marsh, Colorado School of Mines.

The following papers were presented:

1. *Cauchy's theory of characteristics*, by Professor R. W. McKelvey, University of Colorado.

This was a discussion of the recent studies of A. Plis and T. Wazewski on the domain of existence of solutions of the first order nonlinear partial differential equation $F(x, u, u_x)=0$, $x=(x_0, \cdots, x_n)$. Wazewski's derivation [Bull. Acad. Polon. Sci. Cl. III No. 4, 1956, pp. 131–135] of Plis' results [same source, pp. 125–129] is in the context of the classical Cauchy theory of characteristics. The method can be applied, by analogy, to the quasi-linear equation

$$\sum a_i(x, u)u_{x_i}, -b(x, u) = 0,$$

and is simple enough to find a place in elementary textbooks.

2. *Variation of parameters by vector methods*, by Professors L. C. Barrett and C. A. Grimm, South Dakota School of Mines and Technology, presented by Professor Grimm.

For the equation, $a_0y'''+a_1y''+a_2y'+a_3y=f(x)$, where the a's and $f(x)$ are functions of x continuous on a closed interval over which $a_0\neq 0$, whose reduced equation has the independent solutions y_1, y_2, y_3, we assume the particular solution $y=u_1y_1+u_2y_2+u_3y_3=\vec{U}\cdot\vec{Y}$, $\vec{U}$ to be determined by substitution. From this dot product representation it follows immediately that $y=\int_{x_0}^{x}[\vec{Y}(x)\cdot\vec{w}(t)f(t)][a_0(t)W(t)^{-1}]dt$, $\vec{w}=\vec{Y}\times\vec{Y}'$, $W=\vec{w}\cdot\vec{Y}''$, the Wronskian. It was shown how to generalize the results to other orders of equations.

3. *Existence and stability of periodic solutions of weakly nonlinear differential equations*, by Dr. H. R. Bailey, The Ohio Oil Company, Denver Research Center.

Existence and stability theorems for periodic solutions of weakly nonlinear differential systems have been given recently in a number of papers using a convergent method of successive approximations. This method was originally considered by Lamberto Cesari in 1940 for linear systems.

In the present paper the existence and stability theorems are specialized to the case of a weakly nonlinear differential equation and then applied to a nonlinear Mathieu equation. A summary of the corresponding results in more general situations is given.

4. *On solutions of second order differential equations by changing variables*, by Professor F. M. Stein and Mr. R. D. Finley, Colorado State University, presented by Mr. Finley.

In this paper the authors consider the various methods of changing the dependent and independent variables in the equation $y''+P(x)y'+Q(x)y=0$, where it is assumed that the solutions are known to exist, to reduce it to some classical form or to an equation which may be solved by standard means.

5. *Particular solutions for systems of:* (1) *nonhomogeneous, linear, ordinary differential equations*, (2) *nonhomogeneous, linear, ordinary difference equations*, by Professors L. C. Barrett and R. A. Jacobson, South Dakota School of Mines and Technology, presented by Professor Barrett.

In this paper Lagrange's identity and the bilinear concomitant, so familiar in ordinary differential equation theory, are generalized and then applied, as an alternative to variations of parameters, in solving systems of nonhomogeneous linear ordinary differential equations. A parallel treatment of difference equations is also given.

6. *An integral transform*, by Professor F. M. Hudson, Western State College.

If it is possible to write a differential equation in the form (1) $g(gF')'+gF'+F=0$, where g represents any function of x, then the solution of the equation can usually be found by use of the integral transform $T[F(x)]=\int_0^\infty K(m, x)F(x)dx$. The kernel $K(m, x)$ will depend on g and will be given by the equation $K(m, x)=g^{-1}\exp[\int -(m/g)dx]$. This transform will have as its basic differentiation property $T[gF']=mT[F]$. If an equation is written in the form (1), then the most convenient transform will be suggested by the function g. The Laplace and Mellin Transforms are special cases of the transform $T[F(x)]$.

7. *School Mathematics Study Group experimentation in Colorado*, by Professors W. E. Briggs and L. W. Rutland, Jr., University of Colorado, presented by Professor Rutland.

The objectives and accomplishments of the School Mathematics Study Group and of the Colorado Junior High Center and Colorado Geometry Center were reviewed. An outline was given of the SMSG texts for grades seven through twelve.

8. *The power series coefficients of certain L-functions*, by Professor W. E. Briggs, University of Colorado, and Professor R. G. Bushman, Oregon State College, presented by Professor Briggs.

An example is given to indicate a general method for determining the power series coefficients of functions defined by Dirichlet series. If χ is a principal character modulo k and $h=\phi(k)/k$, then $L(s)=\sum_{n=1}^\infty \chi(n)n^{-s}$ can be written as $h/(s-1)+\sum_{r=0}^\infty (-1)^r V_r(s-1)^r/r!$. Let $L_1(s)=L(s)-hs/(s-1)$. Using an integral representation of $L(s)$, the authors derive the Theorem: *If* $u<0$, *then* $\sum_{n\leqq x} n^u\chi(n)\log^r n=h\int_1^x t^u \log^r t dt+(-1)^r L_1^{(r)}(-u)+0(1)$. By setting $u=-1$ and letting $x\to\infty$, it follows that $V_0=\lim_{x\to\infty}[\sum_{n\leqq x} n^{-1}\chi(n)-h\log x]+h$ and

$$V_r=\lim_{x\to\infty}\left[\sum_{n\leqq x} n^{-1}\chi(n)\log^r n-\{h/(r+1)\}\log^{r+1}x\right], \qquad r=1, 2, \cdots.$$

9. *New teacher education program in mathematics at Colorado State College*, by Professor D. O. Patterson, Colorado State College.

The program in mathematics at Colorado State College is to become, beginning in September 1960, more extensive in its offering and follow fairly closely the recommendations of the National Commission on Mathematics. New courses for secondary school teachers include such titles as "set theory," "modern algebra," "probability theory," "analysis," and "statistics." For the elementary school teachers new courses in mathematics are "arithmetic for elementary teachers" and "foundations of arithmetic."

10. *Astronautics program at USAF Academy*, by Colonel R. C. Gibson, USAF Academy.

One of the principal values of the astronautics program at the Air Force Academy is pedagogic, in that mathematics, physics and chemistry are brought together in a challenging and timely way during the two senior semesters.

The astronautics courses are using most, if not all of the mathematics taught in the regular cadet mathematics sequence. Both the mathematics department and the astronautics department are finding it extremely challenging to design the courses to maximize the cadet's ability to use and appreciate the magnificence of mathematics.

11. *Are mathematicians playing fair with Uncle Sam?* by Professor A. W. Recht, University of Denver.

Recent Russian advances in science forced a hysterical Uncle Sam into a crash program to support so-called "modern mathematics." NSF millions have been spent, and perhaps squandered, to support mathematical study groups and institutes to indoctrinate high school teachers. College professors have gone on a binge of riding their mathematical hobbies at government expense. Payola in mathematics is not on the grand scale of TV payola and rigged programs, but has had a profound effect on the morale of mathematics education. Have the mathematical touts had Uncle Sam gamble on the wrong horses? Have the mathematicians a real solution or a racket?

12. *Applications of mathematics in technology*, by Professor N. W. McLachlan, Visiting Professor of Applied Mathematics, University of Colorado (Invited address)

13. *Anti-associative systems*, by Professor C. H. Cunkle and Mr. D. R. Rogers, Utah State University, presented by Mr. Rogers.

A binary operation defined on a set S is anti-associative provided that $a(bc) \neq (ab)c$ for each a, b, c in S. Examples of anti-associative systems of n elements ($n>1$) with one binary operation were constructed, and these were characterized for sets of 2 and 3 elements. A generalization of associativity was defined for binary operations mapping a finite set onto itself. These operations fell into classes which formed the elements of a permutation group whose identity element was the class of associative operations.

14. *A new nomographic treatment of quartic equations*, by Professor C. R. Wylie, Jr. and Mr. Elbert Johnson, University of Utah, presented by Professor Wylie.

The equation $\phi_1(t)+a\phi_2(t)+b\phi_3(t)+c=0$ can be written as the pair of equations $\phi_1(t)+a\phi_2(t)-p=0$, $b\phi_3(t)+c+p=0$, for each of which a nomogram can be constructed, provided that in the second equation $c+p$ be treated as one of the variables. The contributions of this paper are (1) a description of a mechanical device to facilitate the simultaneous use of these nomograms in the trial and error case when a, b, and c are given and t is required, and (2) the application of this procedure to the solution of the quartic equations $Ut^4+t^3+Vt^2\pm t+W=0$ which are obtained from the general quartic equation $x^4+a_1x^3+a_2x^2+a_3x+a_4=0$ via the substitutions $x=(\pm a_3/a_1)^{1/2}t$, the plus or minus sign being chosen according as $a_1a_3>0$ or $a_1a_3<0$.

15. *Probability in differential equations*, by Captain R. L. Eisenman, USAF Academy.

The solution $y=a-b\cos(Ct+d)$, where a, b, d are constants and C is a random variable, is eventually distributed as $F(k)=\cos(a-k)/b$ regardless of the distribution of C. The problem was motivated by perturbation of a satellite in circular orbit.

16. *An elementary method of determining initial conditions for missile trajectories*, by Professor C. H. Cunkle, Utah State University.

A missile trajectory can be predicted by solving a set of simultaneous differential equations. Such solutions can be very costly, and, in the process of designing a missile, methods are sought for reducing the number of solutions required. By assuming that the range is a quadratic function and using three-point interpolation, the proper initial conditions for a desired range are approximated in a minimum number of trials. The process involves only elementary analytic geometry.

17. *A matrix application of Newton's identities*, by Professor D. W. Robinson, Brigham Young University.

A novel proof, which is based upon Newton's identities, is given of the following theorem. *Let A be an n-by-n matrix over a field of characteristic zero or prime $p>n$. Then A is nilpotent if and only if trace $A^k=0$, $k=1, \cdots, n$.* Other applications are also suggested.

18. *An application of Markov processes in inventory theory*, by Professor P. W. Zehna, Colorado State College.

In recent investigations in the area of inventory depletion, it was possible to define a sequential model for issuing items from a stockpile. By considering the ages of the items to be random variables, a stochastic process was determined for each of two issuing schemes that were of interest. Further examination revealed imbedded Markov processes in each scheme. It was then possible to find unique stationary absolute probability distributions for each scheme and compare them on the basis of the statistical equilibrium afforded by the stationary distributions.

19. *Reflections of a mathematician*, by Professor L. J. Mordell, Visiting Professor of Mathematics, University of Colorado. (Invited address—SIAM)

The topics discussed were selected from the following: What is mathematics, and what are the difficulties in its study? How are mathematicians made, and how do they work? How do problems arise, and how are they solved? What help is given by the electronic computers? What part is played by memory and luck, and what kind of mistakes and errors do mathematicians make? Finally there are the aesthetic and international aspects of mathematics.

The following papers were presented by title:

20. *Limits of iterated discontinuous functions*, by Professor Emeritus A. J. Kempner, University of Colorado.

21. *On Whitworth's Exercise 667*, by Lt. D. R. Barr, USAF Academy.

At least four solutions of problems equivalent to Exercise 667 in Whitworth's *DCC Exercises in Choice and Chance* have appeared in the literature since the publication of the latter in 1897 (see references in J. O. Irwin, J. Roy Statist. Soc., vol. 118, pp. 393–396). A new solution, using only basic definitions and the formula for the simultaneous occurrence of exactly m among N events, is presented.

22. *Green functions for systems of differential equations*, by Professors L. C. Barrett and R. A. Jacobson, South Dakota School of Mines and Technology.

The present note is concerned with extending the familiar concept and usage of Green's function to nth order systems of ordinary linear differential equations with two-point boundary conditions.

23. *Methods for calculating principal idempotents*, by Mr. J. C. Higgins, Brigham Young University.

24. *On a generalization of Pythagorean theorem*, by Professor Aboulghassem Zirankzade, University of Colorado.

The generalized Pythagorean theorem, in Euclidean n-space, is well known and a simple proof, using methods of vector analysis, exists. A more elementary proof, using only plane and solid Euclidean geometry, is given for the case $n=3$ or $n=4$. It is also shown how this method could be modified to prove the generalized theorem for the case $n>4$. Because of the existence of this simple proof, the generalized theorem becomes a suitable topic to be offered in secondary school.

F. M. CARPENTER, *Secretary*

THE MAY MEETING OF THE WISCONSIN SECTION

The twenty-eighth annual meeting of the Wisconsin Section of the Mathematical Association of America was held at Mount Mary College, Milwaukee, Wisconsin, on May 7, 1960. Professor C. B. Hanneken, Chairman of the Section, presided. This meeting was held jointly with the May meeting of the Wisconsin Mathematics Council and there were 129 present, including 52 members of the Association and 62 members of the Wisconsin Mathematics Council.

A defending missile battery is regarded as a server who attempts to perform some operation on each element before that element reaches O. If an element in the queue reaches O without having been served, the server is subject to a risk of disability. The dependence of the process on various parameters (distance between queue elements, probability of disability of server, initial distance of first queue element from objective, etc.) is studied. Various problems concerning the probability distribution of the number of elements served are described. The role of simulation (and its relation to mathematical analysis) in studying such processes is discussed.

2. *Some second thoughts on artificial intelligences*, by Dr. Bradford Dunham, International Business Machines Corporation.

3. *The place of programed instruction in mathematics education*, by Mr. Lewis Eigen, Vice-President, Center for Programed Instruction.

Dr. Bradford F. Hadnot of International Business Machines Corporation announced the formation of the Division of Mathematics of the New York Academy of Sciences and invited all members of the Association to participate in the activities of the Division.

MARY P. DOLCIANI, *Secretary*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The 44th annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Colorado, Boulder, on April 28–29, 1961. The following officers were elected: Chairman, Professor L. C. Barrett, South Dakota School of Mines and Technology; Vice-Chairman, Professor D. W. Robinson, Brigham Young University; Secretary-Treasurer, Professor Leota C. Hayward, Colorado State University.

The following papers were presented:

1. *Approximating the kth derivatives of a function by sums of Sturm-Liouville eigenfunctions*, by Professor F. M. Stein, Colorado State University.

The author uses eigenfunctions of a family of Sturm-Liouville systems as defined by Dunn and Stein, SIAM Review, January, 1961, to prove the existence of a sum, $S_n(x)$, of such eigenfunctions which uniformly approximates an arbitrary differentiable function, $f(x)$, and whose kth derivative at the same time uniformly approximates the corresponding derivative of $f(x)$. That is, it is proved that there exists a sum, $S_n(x)$, such that $|f^{(k)}(x)-S_n^{(k)}(x)|<\epsilon$, $k=0, 1, \cdots, m$, for $\epsilon>0$ and for all x on $[a, b]$, the closed interval over which $f(x)$ and its derivatives are defined.

2. *Separation axioms between T_0 and T_1*, by Mr. C. E. Aull and Professor W. J. Thron, University of Colorado.

3. *A continuation of the zeta series and its implications*, by Professor W. E. Briggs, University of Colorado.

A standard method of continuing the zeta series $\zeta(s)=\sum_{n=1}^{\infty} n^{-s}$ to the left of $\mathrm{Re}\, s=1$ can be generalized for any integer a greater than 1 by writing $(1-a^{1-s})\zeta(s)=\sum_{n=1}^{\infty}\beta_n n^{-s}$, where $\beta_n=1$ if $a\nmid n$ and $1-a$ if $a\mid n$. To evaluate the right hand number and its derivatives at $s=1$, first write $\sum_{n\leq x}(\log^k n)/n=(\log^{k+1}x)/(k+1)+\gamma_k+o(1)$. It is now possible to derive the THEOREM. *For integral a and k, $a\geqq 2$, $k\geqq 0$, $\sum_{n=1}^{\infty}(\beta_n\log^k n)/n=(\log^{k+1}a)/(k+1)-\sum_{t=0}^{k-1}\binom{k}{t}\gamma_t\log^{k-t}a$, where the summation on the right is zero for $k=0$.* By solving these equations for γ_t, one immediately obtains the principal result of a paper by Kluyver (Quar. J. Math., vol. 50, 1927, 185–192). In particular this gives $\gamma=\frac{1}{2}\log a-\sum_{n=1}^{\infty}(\beta_n/n)\log_a n$.

4. *Methods of proving mean value theorems*, by Professor L. C. Barrett, South Dakota School of Mines.

The primary purpose of this paper is to emphasize the equivalence of various proofs of the extended law of the mean, including analytic, geometric, vector, and determinant types of proof. A yet more general method of generating mean value theorems is also given.

5. *Utilizing Green's functions to solve nonhomogeneous differential systems*, by Professor L. C. Barrett and Mr. R. A. Jacobson, South Dakota School of Mines.

In this paper we give an example showing how the solution of a system consisting of n ordinary first-order linear differential equations and n linearly independent two-point boundary conditions can be obtained by utilizing a Green function. The equations, as well as the boundary conditions, may be nonhomogeneous.

6. *Symmetric Boolean functions*, by Professor C. H. Cunkle, Utah State University.

7. *Homomorphisms on certain multiplicative semigroups*, by Professor R. S. DeZur, San Diego State College.

8. *Circular and spherical probability problems*, by Professor W. C. Guenther, The Martin Company and the University of Wyoming.

9. *On the permanence of formal laws*, by Professor Edgar Karst, Brigham Young University.

The author tried to generalize proofs in establishing as a main rule: The results of all mathematical operations which follow a certain arithmetical, geometric, or logical iteration pattern are proved to be correct by the permanence of formal laws. He treated four versions of multiplication in the base 8, partly in the decimal mode, with and without conversion to binary, the last one failing because of uncertainty in the logical structure. The third version, based on a modified method of Bhaskara, works for all bases from 2 to 10, and, built in the hardware of an electronic computer, would yield 8 times more versatility, with only a small loss in machine time.

10. *The Committee on the Undergraduate Program in Mathematics*, by Professor R. C. Buck, University of Wisconsin, Chairman of the Committee.

11. *Student versus teacher*, by Professor Edward Anlian, U. S. Air Force Academy.

12. *Guesses on prime numbers*, by Professor Emeritus A. J. Kempner, University of Colorado.

13. *The behavior of solutions of ordinary, self-adjoint differential equations of arbitrary even order*, by Mr. Robert Hunt, University of Utah.

The differential equation $(r(x)y^{(n)})^{(n)}+p(x)y=0$, $r(x)>0$, $p(x)\neq 0$ on $[a, \infty)$ is studied with regards to the existence of various types of zeros of its solutions. Of chief interest in the first part of the paper are solutions with two nth order zeros and solutions $y(x)$ with an nth-order zero followed by an nth-order zero of $r(x)y^{(n)}(x)$. In the latter part of the paper, zeros of types which do not seem to lend themselves to variational methods are considered, and separation and oscillation properties are studied for the case $p(x)>0$.

14. *Thoughtful algebra carries its own insurance*, by Professor A. W. Recht, University of Denver.

Mathematics teachers waste much time marking mistakes in algebra that should never have been made. Errors results from mechanical manipulation, makeshift tricks used and forgotten almost as soon as devised. Better to go back to fundamentals for every step, to provide a strand of good common sense that holds everything logical together. Even then we are not back to fundamentals often enough. To avoid bickering, students are furnished lists of exercises that illustrate mandatory methods with fundamental steps. Final warning: if student doesn't follow directions when they don't seem to matter, how can he follow them when they do matter?

15. *Mathematical curriculum for engineers and scientists*, by Professor I. I. Kolodner, University of New Mexico.

LEOTA C. HAYWARD, *Secretary*

2. *A geometric definition of an analytic function*, by Professor A. W. Goodman, University of Kentucky.

3. *The* (F_n, a_{nk}) *topological space*, by Mr. T. R. Westbrook, University of Louisville.

4. *Eigenvalue problems of ordinary differential equations*, by Mr. James Rolf, University of Kentucky.

V. F. COWLING, *Secretary*

THE MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-fifth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at The South Dakota School of Mines and Technology, Rapid City, South Dakota, May 4 and 5, 1962. The meeting was divided into several sessions with Professors F. M. Carpenter, P. O. Steen, and Lawrence Fearnley, presiding. There were 68 persons registered for the meeting.

Officers elected at the meeting for 1962–1963 were: Chairman, Professor H. J. Fletcher, Brigham Young University; Vice-Chairman, Col. J. W. Ault, United States Air Force Academy; Secretary-Treasurer, Professor Leota C. Hayward, Colorado State University; and Director of High School Mathematics Contest, Professor D. C. B. Marsh, Colorado School of Mines.

The following papers were presented:

1. *Quasi-resolutions of the identity*, by Professor E. R. Deal, Colorado State University.

For certain nonspectral operators, if the conditions that $\{E(\delta)\}$ be a resolution of the identity be weakened to the condition that $\{E(\delta)\}$ be an operator measure, it is still true that T may be represented in the form $T=\int_{\sigma(T)} \lambda E(d\lambda)+N$ where N is a generalized nilpotent operator. Such an operator T is called a quasi-spectral operator. Examples of quasi-spectral operators are given, and sufficient conditions for an operator to be quasi-spectral are given.

2. *Convergence and stability in the numerical integration of ordinary differential equations*, by Professor R. A. Hansen, Brigham Young University.

The use of the class of difference equations $y_{n+k}+a_{k-1}y_{n+k-1}+\cdots+a_0y_n=h(b_ky'_{n+k}+\cdots+b_0y_n')$ for the numerical solution of the initial value problem for an ordinary differential equation $y'=f(x, y)$, $y(a)=y_0$, is considered. Sufficient conditions are indicated which guarantee the convergence of the solution of the difference equation to the solution of the differential equation as the tabular interval h approaches zero. Stability of solution is defined and sufficient conditions are given which insure stability.

3. *Some n-dimensional coverage problems*, by Professor W. C. Guenther, University of Wyoming.

The center of a sphere is aimed at a point target in an n-dimensional coordinate system, with aiming errors being governed by a p.d.f. $f(X)$. Before the sphere arrives the point selects a new position according to a probability law whose p.d.f. is $g(X')$. The probability that the sphere covers the point target when the sphere comes to rest is computed for several choices of $f(X)$ and $g(X')$.

4. *A linear congruence with side conditions*, by Professor David Rearick, University of Colorado.

For positive integral r and n, and integral m, denote by $\phi_r(n, m)$ the number of distinct solutions of the congruence $x_1+x_2+\cdots+x_r=m \pmod n$ with x_i relatively prime to n for all i. It is shown that $\phi_r(n, m)$ and the rth power of Ramanujan's exponential sum $C_n(m)$ form a Fourier transform pair. From this is deduced a formula for $\phi_r(n, m)$ in terms of the Euler ϕ-function.

5. *An integral transform*, by Professor R. H. Niemann, Colorado State University.

The Riemann Stieltjes integral from zero to infinity of $g(z+t)/g(z)$ with respect to $c(t)$ has several interesting special cases. Here $g(z)$ is assumed to be analytic and $e^{pz^q}g(z)$ can be represented in an asymptotic series in a sector of the complex plane that includes the positive real axis. The exponents p and q are polynomials in z and $c(t)$ is a function of bounded variation. If $g=\exp(-z^2/2)$ the integral reduces to the Laplace transform. If g is the reciprocal of the gamma function the integral reduces to the factorial transform and the factorial series if $c(t)$ is chosen properly.

6. *Concave functions and points of inflection*, by Professor L. C. Barrett, South Dakota School of Mines and Technology.

In this note we give an analytic definition for concavity of a function at a point and then extend the definition to concavity over an interval. Generalizations of the concept are noted and point of inflection is also defined. These ideas are enlarged upon by means of theorems and illustrative examples. It is pointed out that the definition of concavity may be formulated in terms of a determinant, or in terms of second order central differences.

7. *Matrices of basis vectors*, by Captains R. L. Eisenman and D. R. Barr, United States Air Force Academy, presented by Captain Barr.

A linear combination of basis vectors can be written as a formal product RC_R, where R is a row matrix of basis vectors and C_R is a column matrix of coefficients. This notation has been used at the Air Force Academy, and has advantages in unifications of concepts and notations of vector analysis and linear algebra, and in formulation and testing of conjectures. These advantages are illustrated by the finding of the relations between the matrices of a linear transformation of a vector space into itself in two different bases and by a generalization of the Coriolis theorems, respectively.

8. *The Fibonacci matrix modulo m*, by Professor D. W. Robinson, Brigham Young University.

The periodic properties of the Fibonacci sequence modulo m (see D. D. Wall, this MONTHLY 67 (1960) 525–532) are studied by considering integral powers of the 2-by-2 matrix with first row (0, 1) and second row (1, 1).

9. *Some convergence results for continued fractions*, by Professor K. L. Hilliam, University of Colorado.

10. *On the convergence criterion of Du Bois Reymond and the theory which has evolved from it*, by Professor Alexander Peyerimhoff, University of Utah.

The theory of convergence—and summability factors is discussed as a generalization of the convergence criterion of Du Bois Reymond. This generalization is obtained by weakening one of the assumptions of the criterion through the idea of summability. Complete results are obtained if the method of summability is connected with a certain mean value theorem—as was observed first by L. S. Bosanquet in the case of the Cesáro method.

11. *Some observations regarding binomial coefficients*, by Dr. T. C. Fry, Consultant to the Director, National Center for Atmospheric Research.

12. *A multiphase diffusion problem*, by Lt. C. F. Lutz, U. S. Air Force Academy.

The diffusion equation is $D_i\partial^2 C_i/\partial X^2 = \partial C_i/\partial t$ where the diffusivity, D, is considered a constant. The solution of the diffusion equation is derived for the experimental situation of an infinite bar of constant cross section, where the bar may be regarded as extending along the x-axis. After time $t=0$ it is found that the diffusion causes a separation of the bar into n-segments corresponding to n pure phases. Thus, there are phase boundaries at points, X_i, where discontinuous changes in C_i are observed. The solution is generalized for the case where D is variable.

13. *A mathematical treatment of the eutectoid in the W-C system*, by Professors G. W. Orton and Rudolph Speiser, United States Air Force Academy and The Ohio State University, presented by Lt. Col. George Orton.

An analysis is made of the three univariant curves about the W-C eutectoid to relate temperature, enthalpy and the activity of carbon in the reactions. The activity of carbon determined experimentally is related to the free energy change in the reaction and the free energy is expressed in terms of enthalpy and entropy. Values are presented for ΔF^0, ΔH^0 and ΔS^0 of each reaction.

14. *The college training of high school teachers*, by Professor W. R. Orton, University of Arkansas.

LEOTA C. HAYWARD, *Secretary*

DISTRIBUTION OF MAA FILMS

Effective September 1, 1962, the Association will discontinue the free distribution of the films by Henkin, McShane, and Hewitt, which were produced by the MAA Committee on Production of Films. These films may be rented from Modern Learning Aids, 3 East 54th Street, New York 22, N. Y.

Schools and individuals wishing to purchase the films should also write to Modern Learning Aids.

CALENDAR OF FUTURE MEETINGS

Forty-sixth Annual Meeting, University of California, Berkeley, January 26–28, 1963.

Forty-fourth Summer Meeting, University of Colorado, Boulder, August 26–28, 1963.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Associate Secretary.

ALLEGHENY MOUNTAIN, Pennsylvania State University, University Park, May 4, 1963.

ILLINOIS, Northern Illinois University, De Kalb, May 10–11, 1963.

INDIANA, Evansville College, October 5, 1962.

IOWA, Iowa State University, Ames, April 19–20, 1963.

KANSAS, Kansas State University, Manhattan, April 20, 1963.

KENTUCKY

LOUISIANA-MISSISSIPPI, Buena Vista Hotel, Biloxi, Mississippi, February 15–16, 1963.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Howard University, Washington, D.C., December 1, 1962.

METROPOLITAN NEW YORK

MICHIGAN, Michigan State University, East Lansing, March 23, 1963.

MINNESOTA, Bemidji State College, November 3, 1962.

MISSOURI

NEBRASKA, University of Nebraska, Lincoln, May 3–4, 1963.

NEW JERSEY, Rutgers, The State University, New Brunswick, November 3, 1962.

NORTHEASTERN, Connecticut General Life Insurance Company, Bloomfield, Connecticut, November 24, 1962.

NORTHERN CALIFORNIA, University of California, Berkeley, January 1963.

OHIO, Ohio State University, Columbus, May 4 1963.

OKLAHOMA, Oklahoma City University, November 10, 1962.

PACIFIC NORTHWEST, Western Washington College, Bellingham, June 14, 1963.

PHILADELPHIA, Franklin and Marshall College, Lancaster, Pennsylvania, November 24, 1962.

ROCKY MOUNTAIN, Brigham Young University, Provo, Utah, Spring, 1963.

SOUTHEASTERN, University of Chattanooga, Chattanooga, Tennessee, March 29–30, 1963.

SOUTHERN CALIFORNIA, University of California, Riverside, March 9, 1963.

SOUTHWESTERN, Arizona State College, Flagstaff, April, 1963.

TEXAS, North Texas State University, Denton, April 19–20, 1963.

UPPER NEW YORK STATE, University of Buffalo April 27, 1963.

WISCONSIN, Carroll College, Waukesha, May 4, 1963.

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-sixth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Brigham Young University, Provo, Utah on Friday and Saturday, May 3 and 4, 1963. Professor M. L. Madison, Governor, and Professor H. J. Fletcher, Chairman, presided. There were 109 persons registered.

The following officers were elected for 1963–64: Chairman, Professor J. S. Leech, Colorado College; Vice-Chairman, Professor I. L. Hebel, Colorado School of Mines; and Secretary-Treasurer, Professor Leota C. Hayward, Colorado State University, Professor E. R. Deal, Colorado State University, was appointed coordinator of High School Mathematics Contests for a three year term.

By-Laws for the section as prepared by Professors F. M. Carpenter and I. L. Hebel were unanimously approved.

The 1964 spring meeting will be held at Colorado College, Colorado Springs, Colorado.

The Friday evening guest speaker was Professor B. W. Volkmann, visiting professor at the University of Utah, who spoke on *Transcendental numbers and their approximation properties.*

The following papers were presented:

1. *Singular families of Sturm-Liouville systems*, by Professor F. M. Stein, Colorado State University.

The following theorem is proved: The only singular Sturm-Liouville systems that generate families of Sturm-Liouville systems are those of Hermite, Jacobi, and Laguerre.

2. *The digital computer in secondary school education*, by R. L. Albrecht, Control Data Corporation.

A program of computer education for secondary school students has been established in Denver and Jefferson Counties, Colorado. More than 200 students are enrolled. In this program, emphasis is placed on the development of precise mathematical problem-solving procedures, the computer is regarded as a tool for the solution of mathematical problems, and auto-instructional teaching methods are used. Textbooks are being developed for teaching computer methods in secondary schools. These books are related to the texts of the School Mathematics Study Group and the University of Illinois Committee on School Mathematics.

3. *Some stochastic arithmetic series*, by Professor E. A. Power, visiting professor, University of Colorado.

Some arithmetical series, whose nth terms are sums of inverse products of n integers, are summed. They arise from a model problem involving scalar neutral π-mesons interacting with fixed sources.

4. *Semi-multiplicative functions and correlation functions*, by Professor D. F. Rearick, University of Colorado.

5. *A characterization of separable metric spaces*, by William Eaton, University of Utah.

A chain in a metric space is a collection of open sets which is simply ordered by set inclusion. The following statements are equivalent in a metric space S, (1) S is separable, (2) every chain C in S has a countable subchain C' such that $\bigcup_{A\in C}A=\bigcup_{B\in C'}B$, (3) every chain C in S has a countable subchain C' such that $\bigcap_{A\in C}A=\bigcap_{B\in C'}B$.

6. *Damped motion of a fixed-free uniform beam subjected to an acceleration pulse*, by E. M. Grenning, Thiokol Chemical Corporation.

An analytical solution is presented for the damped motion of a fixed-free uniform beam subjected to a short acceleration pulse at its fixed end. Deflection due to shear is neglected, thus allow-

ing the use of the Euler-Bernoulli beam equations. To obtain the motion during the acceleration pulse, a transformation of the dependent variable is used to transfer the nonhomogeneity from the boundary conditions to the differential equation. The resulting nonhomogeneous partial differential equation is then solved. Motion subsequent to the removal of the acceleration is obtained using initial conditions as derived from the solution for motion during acceleration.

7. *Modified Lommel functions*, by 1st/Lt. C. N. Rollinger, Instructor in Mathematics, U. S. Air Force Academy.

The modified Lommel function is defined as a special case of the Lommel function when the argument of the latter is imaginary. It is shown that the modified function, which is a particular solution of a nonhomogeneous Bessel equation, can be used to evaluate certain integrals.

8. *Testing for integer cases in binary computers*, by Major H. K. Leland, Assistant Professor of Mathematics, U. S. Air Force Academy.

9. *Interlacing properties of characteristic values of Sturm-Liouville systems involving interface boundary conditions*, by Professor L. C. Barrett and G. E. Bendixen, South Dakota School of Mines and Technology.

The primary purpose of this paper is to describe how the characteristic values of a general second order Sturm-Liouville system with interface boundary conditions interlace those of associated Sturm-Liouville systems of a more elementary type. To illustrate the results, a detailed discussion is presented of the problem of a torsionally vibrating shaft, one part of which is tapered with circular cross-sections, the other part being cylindrical. The shaft parameters, such as the modulus of elasticity in shear and the density, need not be the same for both segments.

10. *On the topology of Boolean rings*, by J. C. Higgins, Brigham Young University.

This paper compares topologies for Boolean rings as found in papers by M. H. Stone and P. R. Halmos. Results from these papers are used to characterize rings which have an operator topology homeomorphic to a prime ideal topology. In such rings every ideal is a simple ideal.

11. *Essential fixed points*, by Professor D. L. Schmidt, Colorado State College.

Let X be a compact metric space with the fixed point property. Let X^X be the set of all continuous mappings on X into X, metrized with the supremum norm. P is an essential fixed point of f if corresponding to each neighborhood U of P there is a neighborhood N of f such that if g is in N then g has a fixed point in U. This definition is due to M. K. Fort, Jr. It is shown that results on essential fixed points can be obtained in a compact Hausdorff space X by making use of the fact that the topology on X is uniform.

12. *The range of a Boolean function*, by N. H. Eggert, Utah State University.

13. *Boolean-like algebra*, by E. D. Goodrich, Utah State University.

14. *A modified Maclaurin integral test*, by Professor L. C. Barrett, South Dakota School of Mines and Technology.

In its most elementary form, Maclaurin's integral test is used to examine series such as $\sum_{j=1}^{\infty} f(j)$ for convergence or divergence when the continuous function $f(x)$ ultimately becomes and remains positive and monotone decreasing. This paper provides a modification of the test which may be applied to series of the type $\sum_{j=1}^{\infty} f(\lambda_j)$ when $f(x)$ has the aforesaid properties and λ_j is the jth element of a strictly monotone increasing sequence. Extensions of this test to other kinds of sequences are also considered, together with its use in connection with boundary value problems.

15. *A differential-difference equation describing mixing in certain biological systems*, by Professor H. R. Bailey, Colorado State University.

16. *Approximate solutions of a system of linear differential equations*, by Professor F. M. Stein and Mr. K. F. Klopfenstein, Colorado State University; presented by Mr. Klopfenstein.

Approximation in the sense of least rth powers, $r \geqq 1$, of the solution vector of the system of n first order linear differential equations $\overline{L}\bar{y}(t) = \overline{D} + \overline{F}(t)\bar{y}(t) = \bar{f}(t)$, subject to the nonhomogeneous two-point boundary conditions $\overline{A}\bar{y}(a) + \overline{B}\bar{y}(b) = \bar{h}$ by a vector of polynomials satisfying the end-point conditions is considered. Existence and uniqueness of approximating vectors of polynomials of degree m, $\bar{p}_m(t)$, are discussed, and it is shown that the sequence $\overline{L}\bar{p}(t)$ converges in the mean of order r to the vector function $\bar{f}(t) = \overline{L}\bar{y}(t)$.

17. *Transform methods for difference equations*, by Professor C. A. Grimm and Mr. Wayne Walther, South Dakota School of Mines and Technology; presented by Mr. Walther.

LEOTA C. HAYWARD, *Secretary*

MAY MEETING OF THE WISCONSIN SECTION

The thirty-first annual meeting of the Wisconsin Section of the Mathematical Association of America was held at Carroll College, Waukesha, Wisconsin, on May 4, 1963. Professor G. L. Bullis, Chairman of the Section, presided. This meeting was held jointly with the May meeting of the Wisconsin Mathematics Council and there were 159 present, including 69 members of the Association and 81 members of the Wisconsin Mathematics Council.

At the business meeting the following officers were elected for the coming year: Chairman, Professor C. E. Flanagan, Wisconsin State College, Whitewater; Vice-Chairman, Professor J. M. Osborn, Jr., University of Wisconsin; Secretary-Treasurer, Professor E. F. Wilde, Beloit College.

The following papers were presented:

1. *The concept of precompactness*, by Professor M. B. Smith, Jr., University of Wisconsin.

A subset M of a topological space S is said to be precompact if and only if every infinite subset of M has a limit point in S. This paper discussed the relation between precompactness and some of the definitions of compactness. Examples of topological spaces were discussed in which there exist precompact sets whose closures contain infinite sets having no limit point. It was proved, however, that in a normal T_1 topological space the closure of a precompact set is compact.

2. *Some problems in the geometry of numbers*, by Professor M. N. Bleicher, University of Wisconsin.

This work attempts to indicate some problems and methods of the geometry of numbers. Determining the average number A_k of representations of the first k integers as the sum of two squares of integers is equivalent to determining the number N_k of lattice points (points with integral coordinates) in the circle with radius $\sqrt{k}$ and center (0, 0), since $kA_k = N_k - 1$. Also, N_k is asymptotic to πk. Difficult unsolved problems remain in estimating $\pi k - N_k$. From Minkowski's convex body theorem, proved by Blichfeldt's method, it follows that (1) $|ax+by| \cdot |cx+dy| < |ad-bc|$, (2) $|y\alpha - x| < y^{-1}$, (3) $(ax^2+bxy+cy^2)^2 < b^2-4ac$, for $b^2-4ac>0$, have infinitely many integral solutions. Analogs of (3) are known for definite forms.

3. *Report of the Pittsburgh meeting of the National Council of Teachers of Mathematics*, by L. C. Dalton, Waukesha Public Schools, Waukesha.

4. *Report on the mathematics scene in Wisconsin*, by A. M. Chandler, State Department of Public Instruction, Madison.

5. *The map color problem on surfaces of higher genus*, by Professor William Gustin, University of Wisconsin.

In 1890 Heawood showed that for any map on a closed surface S_h of positive genus h (with h holes or handles), there is a way of coloring its regions, so that no two adjacent regions be colored

4. The number of divisions required to find the g.c.d. of two numbers is never greater than five times the number of digits in the smaller number, (this depends on the denary scale).
5. There are just five complex fields $F(\sqrt{m})$ for which there is a euclidean algorithm, viz., $m=-1, -2, -3, -7, -11$.
6. There are just five known Fermat primes $2^{2^t}+1$, $(t=0, 1, 2, 3, 4)$.
7. Every map on the sphere can be properly colored if no two regions having a whole segment of their boundaries in common, receive the same color.
8. Kuratowski's Theorem on nonplanar graphs.
9. Heawood's Theorem that every planar graph is 5-chromatic.

7. *Computation of elliptic integrals using Gauss' transformation*, by H. E. Fettis, Applied Mathematics Research Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base.

By means of Gauss' transformation, the computation of the three kinds of elliptic integral may be reduced to routine operations involving only elementary functions, without any further restrictions on the modulus and parameter. The resulting formulae are easily programmed to provide subroutines for a digital computer.

8. *Projective invariants of a curvilinear element*, by Rodney Angotti, University of Akron.

The projective invariants of certain configurations associated with a regular third order differential element, i.e., expansions including the third degree terms in a projective three space are discussed; in particular, a construction of one such invariant is exhibited.

9. *The construction of the real numbers*, by L. D. Rodabaugh, Ohio Northern University.

An extension and refinement of the author's earlier work on this subject as reported to the Illinois Section in May 1951, (see this MONTHLY, 59 (1952) 286).

10. *Some theorems about simple semigroups*, by C. E. Aull, Kent State University.

The following are proved: A semigroup S is a simple semigroup iff for $a, b \in S$, the equation $xay=b$ has at least one solution $x, y \in S$. A simple semigroup S, with identity is a group if any of the following conditions is satisfied: (a) S is commutative, (b) S is left (right) cancellative, (c) S is finite.

11. *Statistical hypothesis modification —a new point of view for statistical inference*, by Thaddeus Dillon, Youngstown University.

Instead of accepting or rejecting the hypothesis $\theta=\theta'$, it is suggested that the hypothesis be modified to $\theta=(Q+\lambda\theta')/(1+\lambda)$, where Q is a statistic and λ is a nonnegative real function of three variables: (1) sample size, (2) population size, and (3) the probability used for comparison to decide whether to accept or reject the hypothesis. Under rather general conditions on the power function such procedures are self-correcting in the sense that the worse of two theories is likely to receive less weight and a really bad theory θ' is likely to receive negligible weight.

FOSTER BROOKS, *Secretary*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-seventh annual meeting of the Rocky Mountain Section of the MAA was held at Colorado College, Colorado Springs, Colorado, on Friday and Saturday, May 1 and 2, 1964.

The following officers were elected for 1964–65: Chairman, F. M. Carpenter, Colorado School of Mines; Vice-Chairman, F. M. Stein, Colorado State University; and Secretary-Treasurer, W. N. Smith, University of Wyoming. E. R. Deal, continues, in his second year of a three-year term, as coordinator of High School Mathematics Contests.

The 1965 spring meeting will be held at the Colorado School of Mines, Golden, Colorado.

Changes in the By-Laws for the section were considered and discussed and a new draft approved for presentation to the Board of Governors for approval.

The Friday evening guest speaker was Professor W. J. LeVeque, visiting professor at the University of Colorado; his topic was *Probability and Number Theory*.

The following papers were presented:

1. *Idempotent matrices* (mod p^a), by J. H. Hodges, University of Colorado.

For positive integers m, a and prime p, the number $N(m, p, a)$ of idempotent matrices (mod p^a) of order m is determined. First, the number for $a=1$ is determined by using canonical forms, involving elementary divisors, for matrices under similarity. Then it is shown that $N(m, p, a+1) = N(m, p, 1)$ for all $a \geqq 1$. The method employed in the second step is the standard recursive one in number theory of using solutions mod p^a to generate solutions mod p^{a+1}. $N(m, p, 1)$ can be expressed as a simple sum involving the number g_r of nonsingular matrices of order r (mod p).

2. *An application of symmetric functions to statistics*, by P. W. Mielke, Colorado State University.

It is well known that symmetric functions have desirable statistical estimation properties. Methodology for treating the two-way classification finite model with disproportionate population subcell sizes is discussed. In particular some symmetric function variance component estimators are introduced which can be applied to this present model even if the sample subcell sizes are disproportionate. An immediate consequence of the use of symmetric functions is the unbiased estimation of the sampling variance for these variance component estimators.

3. *Estimation with some prior information*, by M. M. Siddiqui, Colorado State University.

4. *Chebyshev lines*, by B. L. Foster, Denver Research Center, Marathon Oil Company.

The best fitting line for a set of data points depends on what is meant by best. According to Chebyshev, that line is best which minimizes the worst data deviation. The x-Chebyshev line is the one minimizing the worst x-deviation; the y-Chebyshev line minimizes the worst y-deviation. With uninteresting exceptions, these lines are the same. Using the over-under-over theorem discussed by Scheid (this MONTHLY, 68 (1961) 862), this can be proved by a simple geometrical argument that extends to oblique coordinate systems. A different proof was announced at this meeting of the Association by Professor M. M. Siddiqui.

5. *Some results on T-fractions*, by B. W. Jones and W. J. Thron, University of Colorado.

A T-fraction is a continued fraction of the form $(1+d_0z)+z/(1+d_1z)+z/(1+d_2z)+\cdots$, where z is a complex variable and the d_n are complex numbers. Among convergence criteria for T-fractions given by W. J. Thron (Bull. Amer. Math. Soc., 54 (1948) 206–218) is the following: if $d_n>0$ for $n \geqq 0$, the T-fraction converges for all z, not on the negative real axis, to a function $f(z)$ which is holomorphic in the interior of this region. In the present work the authors show that if $d_n > 0$ for $n \geqq 0$, there exists a bounded, nondecreasing function $\psi(t)$ such that $f(z) = 1 + d_0z + z\int_{-\infty}^{\infty} d\psi(t)/(z-t)$. If t_0 is the largest point of increase of $\psi(t)$ then $z=t_0$ is a singular point of $f(z)$ or if $\{d_n\}$ is unbounded then $f(z)$ has a singularity at $z=0$.

6. *Hankel transforms and entire functions II*, by K. R. Unni, Utah State University.

7. *The value of a coalition in applied games*, by W. C. White, Cadet, USAFA.

8. *The University of Colorado Computer Center for secondary schools*, by R. L. Albrecht, Control Data Corporation.

A center for exploring methods of secondary school computer education has been established by the University of Colorado, College of Engineering, Denver Center, with the cooperation of the Control Data Corporation and the Denver Chamber of Commerce. During the 1963–64 school year, 144 high school students and 26 high school teachers were enrolled in an experimental program. The main objective is the development of methods for using a computer to reinforce classroom training in secondary school mathematics and science. The computer is regarded as a "mathematics laboratory" with which students solve textbook problems, perform mathematical experiments, and process scientific data.

9. *Polypack: a set of polynomial algorithms*, by R. A. Kahn, student, George Washington High School, Denver, Colorado.

Polypack is a set of algorithms which enables a computer to add, subtract, multiply, divide, or evaluate two polynomials. For example, the addition algorithm tells the computer how to add the coefficients of two given polynomials—$A(x)$ and $B(x)$—to obtain $(A+B)(x)$. These algorithms were derived by first taking specific polynomials and performing different operations on them. Usually a general pattern could be observed for obtaining the result in the specific cases. This pattern was then enlarged and a general algorithm for the operation was formed.

10. *BOOTRAN: A logical interpreter for a Control Data* 160-A *computer*, by Randy Levine, student, George Washington High School, Denver, Colorado.

BOOTRAN is an interpretive system for a Control Data 160 or 160-A computer. In an algebraic manner, it does logic or simple Boolean algebra. Hence the name, BOOTRAN. The talk concerns the most recent version, BOOTRAN III, which is the third in a series of programs to develop a system to do advanced Boolean algebra by computer in a method which can be easily understood by the programmer, even though he may not be acquainted with computers in detail. The main distinguishing feature of BOOTRAN III is that it does its logic using Polish Notation, instead of parentheses. Future BOOTRANs will eliminate this and make writing source programs considerably easier.

11. *Number theory*, by Larry Davis, student, Golden High School, Golden, Colorado.

12. *Generalized polynomials*, by Captain D. E. Helton, USAFA.

In 1959 Jan Mikusinski developed generalized functions through convolution quotients. By restricting the class of continuous functions to the Heaviside function $h(t)=1$ for $t\geqq 0$; $h(t)=0$ for $t<0$, one may obtain good partial results. Considering convolution powers of $h(t)$, as we do the integers in developing the rational numbers, then applying pointwise addition, scalar multiplication and last, forming ordered pairs, we arrive at the field of convolution quotients which we call "Generalized Polynomials." This field contains a class of impulse functions, including the Dirac δ function.

The simplicity of proving no zero divisions under convolution is a prime advantage of "Generalized Polynomials."

13. *An Undergraduate Research Program in Mathematics*, by F. M. Stein, Colorado State University.

An Undergraduate Research Program in Mathematics to be conducted at Colorado State University during the summer of 1964 similar to three previous programs is described. These eight week programs, supported by the National Science Foundation, attempt to show the nine well qualified undergraduate mathematics majors who participate how a mathematician works and what he does by directing their work on various non-coursework topics.

14. *General repeated exponentiation*, by R. A. Bruce, student, Colorado State University.

The convergence or divergence of the sequence $(x_n)_1^\infty$, where $x_1=c$, $x_2=x^{x_1}$, $\cdots$, $x_n=x^{x_{n-1}}$, $\cdots$, can be completely determined for $c\geqq 0$ and $x>0$. In particular, when $c=x$, this sequence converges for x, such that $1/e^e\leqq x\leqq e^{1/e}$, and diverges for all other positive x.

15. *Nondecreasing solutions of* $y''=f(x, y)$, by J. W. Bebernes, University of Colorado.

Consider the problem of finding a unique solution of class C^2 on $[a, \infty)$ of the infinite interval boundary value problem (*): $y''=f(x, y)$, $y(a)=-\alpha$ $(\alpha>0)$, $y(x)\leqq 0$, $y'(x)\geqq 0$, for $x\geqq a$. Assume as needed: (1) $f(x, y)$ is continuous on $\{(x, y)\,|\,x\geqq a,\ |y|<\infty\}$, (2) $f(x, y)$ is nondecreasing in y for each fixed x, (3) $f(x, 0)\equiv 0$, $x\geqq a$, (4) there exists a $\delta>0$ such that $y\leqq f(x, y)$ for all $x\geqq a$, $-\delta\leqq y\leqq 0$. By the use of the technique of subfunctions, the following theorems can be proved. THEOREM A: *If* f *satisfies* (1), (2), *and* (3), *then there exists a unique solution of* (*). THEOREM B: *If* f *satisfies* (1), (2), (3), *and* (4) *then there exists a unique negative solution of* (*).

LEOTA C. HAYWARD, *Secretary*

NEWS ITEM

The Stanford Competitive Examination in Mathematics is being discontinued as of the spring of 1965. Over the past twenty-one years students in most of the high schools in the far Western states participated in this annual competition.

This competitive examination, which was initiated and organized by Professors G. Polya and G. Szego, was one of the pioneering efforts to use such competition in high schools as a means of stimulating interest in mathematics and in problem-solving in particular, and to discover and encourage students of unusual ability.

The competition was administered by the Department of Mathematics of Stanford University which records its appreciation of the generous cooperation of the many high school teachers and administrators who gave freely of their time to conduct examination sessions.

THE AFRICAN-AMERICAN INSTITUTE

The African-American Institute, 345 East 46th Street, New York, N. Y. 10017, has immediate openings for high school teachers of mathematics, with preference for those able to teach at least one science as well. Teachers will fill two-year AAI contracts either in Nkumbi International College, north central Zambia, or Kurasini International Education Center, Dar es Salaam, Tanzania. Maintained by AAI, both schools are primarily for refugee students from southern Africa. Certification and at least three years' teaching experience required. Master's degree preferred. Appointees receive round-trip transportation for themselves and dependents, overseas allowances, free housing, various fringe benefits in addition to salary, which ranges from $6,500 to $9,400, depending upon candidates' qualifications and prior earnings. Application forms should be requested from AAI Personnel Assistant at the address above.

CORNELL UNIVERSITY

On July 1, 1965, Cornell University established an intercollege Department of Computer Science in the Colleges of Engineering and Arts and Sciences. To aid the creation of this department and further its growth, the Alfred P. Sloan Foundation has awarded Cornell University a grant of one million dollars. The fields of study and research now represented include programming languages and systems, numerical analysis, data processing and information retrieval, and automata theory and theory of computation. The Department of Computer Science is authorized to grant the Ph.D. and M.S. degrees in Computer Science.

Further information can be obtained by writing to Professor J. Hartmanis, Department of Computer Science, Upson Hall, Cornell University, Ithaca, New York.

MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-eighth annual meeting of the Rocky Mountain Section of the Mathematical Association of America was held at Colorado School of Mines, Golden, Colorado, on Friday and Saturday, May 7 and 8, 1965. The Rocky Mountain Section of SIAM participated.

There were 122 people registered for the meeting including Dean William E. Briggs, Sectional Governor, and Professor Fred M. Carpenter, Section Chairman.

An invited address was given on Friday afternoon by Professor Marvin Marcus of the University of California, Santa Barbara. Professor Marcus spoke on "Some Techniques for Proving Inequalities." On Saturday morning Dr. George W. Morgenthaler, Visiting Professor, University of Colorado and Department Manager, Martin Marietta Corporation, delivered an invited address on "Some Problems in Non-Linear Vibrations in N-degrees of Freedom Systems."

At the banquet Friday night Professor J. R. Lee of Colorado School of Mines presided. The Section was welcomed by Dr. Anton G. Pegis, Assistant to the President, Colorado School of Mines, and Sectional Governor W. E. Briggs gave a brief talk. Following the banquet the Section was entertained by the Adolph Coors Company.

The business meeting was held on Saturday morning, May 8, 1965, with Professor Carpenter presiding.

The secretary distributed copies of the By-Laws adopted May 2, 1964, revised to conform to the suggestions made by Dr. Paul Johnson (UCLA) ex-Chairman of the Committee on Sections. The motion was made, seconded and carried that the By-Laws be adopted as revised.

The chairman appointed Professor Robert W. Ellingwood, Colorado University, to a three year term on the Meeting Committee and Professor William Dorgan, Western State College, to a three-year term on the Nominating Committee. Those standing committees are now:

Meeting Committee: Donald Robinson, Brigham Young University; R. E. Doutt, South Dakota School of Mines; Robert W. Ellingwood, Colorado University.

Nominating Committee: Kenneth Noble, Denver University; Chairman: F. N. Fisch, Colorado State College; William Dorgan, Western State College.

The following officers were elected for 1965–66: Chairman, F. Max Stein, Colorado State University; Vice-Chairman, W. E. Dorgan, Western State College; Secretary-Treasurer, W. Norman Smith, University of Wyoming.

Professor E. R. Deal, Contest Chairman of the Annual High School Mathematics Contest, reported that 143 schools had requested 7618 tests; 135 schools returned results —15 from Wyoming, 30 from Utah and 90 from Colorado.

The following papers were presented at the meetings:

1. *Separation and interlacing theorems*, by L. C. Barrett, South Dakota School of Mines and Technology.

In this paper, separation and interlacing theorems pertaining to zeros of the functions $f_i(\lambda)$ and $F(\lambda)$, which may occur in a quite general equation of the type $F(\lambda)\equiv f_1(\lambda)f_4(\lambda)-f_2(\lambda)f_3(\lambda)=0$ are given. It is pointed out how these theorems may be utilized to isolate the characteristic numbers of a general Sturm-Liouville system involving a single pair of interface boundary conditons.

2. *Descending chain condition rings with cyclic quasi-regular group*, by K. E. Eldridge, University of Colorado.

It is known for associative rings that the set of all quasi-regular elements forms a group with respect to the circle operation. This group is called the quasi-regular group of the ring. Using the fact that the Jacobson radical is a subgroup of the quasi-regular group and the well-known Wedderburn-Artin structure theorem, it is shown that all descending chain condition rings with a cyclic quasi-regular group are finite.

3. *The characteristic functional of a nonhomogeneous Poisson process*, by Meckinley Scott, Colorado School of Mines.

The process considered is a birth process where the probability of a birth in $(t,\ t+\delta t)$ is

$\lambda_n\delta t+o(\delta t)$, when n is the number of individuals existing in the system at time t. The characteristic functional for this process is obtained and the result given in the form of a sum of n-fold integrals. The integrals can easily be evaluated for the special cases where (i) $\lambda_0=\lambda_1=\cdots=a$ (constant) and (ii) $\lambda_n=a+nb$, $b\neq 0$.

4. *An exact perimeter inequality for the pedal triangle*, by A. Zirakzadeh, University of Colorado.

5. *A note on the trigonometric, quasi-trigonometric, Jacobian elliptic, and hyperbolic functions*, by F. M. Stein, Colorado State University.

The Jacobian elliptic functions arise as solutions of certain nonlinear differential equations. It is shown that the trigonometric and hyperbolic functions can be obtained from these differential equations by a proper choice of the parameter involved. New functions are then defined by allowing the parameter to become imaginary.

In a similar manner it is shown that the quasi-trigonometric functions defined in the paper, *Quasi-Trigonometry*, by Strand and Stein, this MONTHLY, 69 (1962) 143–147, also satisfy certain nonlinear differential equations and reduce to known functions for proper choices of the parameter.

6. *Class of solvable sum equations*, by S. W. Reyner and L. C. Barrett, South Dakota School of Mines and Technology.

Given functions $f(z, n)$ and $K(n, k)$ determine $g(z, n)$ so that (1) $f(z, n)=\sum_{k=0}^{n}K(n, k)g(z, k)$. For what $K(n, k)$ can (1) be solved for g by interchanging g and f? Given $\{b_m\}$, define $a_{n,k}=\prod_{m=k+1}^{n}b_m (k<n)$, $a_{n,n}=1$. If $K(n, k)$ is such that (1) is solved for $g(z, n)$ by interchanging f and g, then $K^*(n, k)=a_{n,k}K(n, k)$ also has this property.

7. *Generalized orthogonality and null series*, by Richard Nau and L. C. Barrett, South Dakota School of Mines and Technology.

This paper is primarily concerned with two related problems, namely: (1) that of deriving generalized orthogonality conditions for the characteristic functions of physical systems involving lumped parameters, and (2) an investigation of the behavior of a particular null series within and outside the fundamental interval of convergence. In deriving generalized orthogonality conditions direct use is made of the definition of Stieltjes integral as the limit of a sum.

8. *Pairs of bilinear and quadratic equations in a finite field*, by A. D. Porter, University of Wyoming.

Let $F=GF(q)$ be the finite field of $q=p^r$ elements, p odd, and consider the pair of equations $a_1x_1^2+\cdots+a_nx_n^2=a$; $b_1x_1y_1+\cdots+b_nx_ny_n=b$ with all coefficients from F. Explicit formulas are obtained for the number of simultaneous solutions, $x_1, y_1, \cdots, x_n, y_n$, in F of this system. It is then noted that solutions to the system always exist for $n\geqq 3$.

9. *Geometrical line-fitting*, by B. L. Foster, Denver Research Center-Marathon Oil Company.

For the over-under-over theorem discussed by Scheid (this MONTHLY, 68 (1961) 864), the order of deviations is important, as pointed out by Biesterfeldt. Thus Foster's result (this MONTHLY, 71 (1964) 960) is wrong. A correct version reads: The y-Chebyshev line equals the x-Chebyshev line, for a monotone set of data points. The over-under-over theorem also implies that the Chebyshev line for a data triangle is a midpoint line and that each midpoint line is the Chebyshev line for all directions it subtends (proved by Deal). The exceptional case of a direction parallel to a triangle edge explains the apparent discontinuity in passing from one midpoint line to another.

10. *Early mathematicians' works in meteorology—theories on thunder and lightning*, by H. H. Frisinger, Colorado State University.

A presentation of early theories on thunder and lightning by mathematicians from the ancient Greek period up to the end of the seventeenth century.

11. *There are no generalized functions*, by Greg Canavan, U. S. Air Force Academy.

Laurent Schwartz' approach to generalized functions through functionals defined on a space of test functions gives a rigorous basis for their use. Using the test space $\{e^{-st}; t \geqq 0\}$, the functionals of Schwartz' definition are reduced to the familiar Laplace transform. Generalized functions are taken to be all functions of s which are not the Laplace transforms of ordinary functions. Hence, generalized functions are simply ordinary functions of the complex variable s. An isomorphism exists between the rational functions of s arising in this manner and the rational functions of s (the differential operator) which are the generalized functions in Mikusinski's Operational Calculus.

12. *Impossibilities of a generalized interpolation by transcendental analytic functions and polynomials*, by Daihachiro Sato, University of Saskatchewan.

Since an analytic function is determined by its Taylor series at any point, it is not possible to prescribe arbitrarily all derivatives of an analytic function even at one point. If, instead of determining the values, we merely restrict them to a certain set, it may (or may not) become possible to prescribe all derivatives to be in the set. Attention is given to the cases at which this type of generalized interpolation is not possible. The following elementary examples, among others, which are special cases of more general ones to be given are typical for the impossibilities of this type of generalized interpolations. 1. There is no nonconstant polynomial all of whose higher derivatives assume real values at 1 and i. 2. There is no entire function all of whose higher derivatives assume real values at 1, i and $w=(-1+3i)/2$. 3. Let S be the set of the first k positive integers, i.e., $S=\{1, 2, 3, \cdots, k\}$. If $k>1$, then there is no analytic function all of whose higher derivatives map S into itself.

W. N. SMITH, *Secretary-Treasurer*.

THIRD COOPERATIVE SUMMER SEMINAR

The Mathematical Association of America will sponsor a third Cooperative Summer Seminar for college teachers of mathematics during the period of June 20- August 12, 1966 at Bowdoin College in Brunswick, Maine. Grants from the Alfred P. Sloan Foundation and from the National Science Foundation are furnishing financial support for the Seminar.

Activities will include daily lectures on topics in applied mathematics by Professor G. F. Carrier of Harvard University and on topics in analysis by Professor E. J. McShane of the University of Virginia.

Participants will be selected from applicants who teach in colleges or universities which offer an undergraduate major in mathematics but which do not offer a Ph.D. degree in mathematics.

Brochures describing the Seminar have been sent to all MAA members as well as to department chairmen in all colleges and universities. Additional copies and application forms may be obtained from V. O. McBrien, Director, MAA Cooperative Summer Seminar, Department of Mathematics, College of the Holy Cross, Worcester, Mass. 01610.

ACKNOWLEDGMENT

The Editorial Board acknowledges with thanks the services of the following mathematicians, not members of the Board, who have kindly assisted by evaluating papers submitted for publication in the MONTHLY.

H. L. Alder, C. B. Allendoerfer, A. R. Amir-Moéz, K. W. Anderson, L. H. Andree, G. E. Andrews, P. M. Anselone, T. M. Apostol, B. H. Arnold, M. Auslander, R. G. Ayoub, R. W. Bagley, K. Baker, R. W. Ball, J. H. Barrett, R. G. Bartle, P. T. Bateman, A. F. Beardon, R. A. Beaumont, E. F. Beckenbach, S. K. Berberian, P. W. Berg, S. D.

MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The forty-ninth annual meeting of the Rocky Mountain Section of the MAA was held at Colorado State University, Fort Collins, Colorado, on Friday and Saturday, May 13 and 14, 1966.

There were 122 people registered for the meeting, including Dean W. E. Briggs, Sectional Governor and Professor F. M. Stein, Section Chairman.

An invited address, "Algebraic Topology for Undergraduates(?)" was given on Friday afternoon by Professor A. B. Willcox, Amherst College, Second Vice-President of the Mathematical Association of America. On Saturday morning Professor George Seifert of Iowa State University delivered the SIAM invited address. His title was "Almost Periodic Solutions for Nonautonomous Systems of Ordinary Differential Equations."

At the banquet Friday night Professor M. L. Madison of Colorado State University presided. The Section was welcomed by President W. E. Morgan of Colorado State University. Following the banquet mathematical films were shown and card tables were available.

The business meeting was held on Saturday morning, at 8:45 A.M., with Professor Stein presiding.

Professor R. E. Doutt reported for the Meeting Committee that the 1967 meetings would be held at Western State College in Gunnison, Colorado. He announced that the University of Denver had extended an invitation to meet there in 1968. It was moved, seconded and carried that this invitation be accepted.

It was announced that a letter had been received from Southern Colorado State College at Pueblo requesting that that institution be listed on the rotation list for future meetings. The motion was made, seconded and carried that Southern Colorado State be added to the list of those institutions in District D.

The following officers were elected for 1966–67: Chairman, W. E. Dorgan, Western State College; Vice-Chairman, Kenneth Noble, University of Denver; and Secretary-Treasurer, W. N. Smith, University of Wyoming.

The chairman announced the appointment of Professor R. L. Eisenman of the Air Force Academy, and Professor Neville Hunsaker of Utah State University, to serve on the Meeting and Nomination Committees, respectively. These standing committees are now:

Meeting Committee: R. E. Doutt, South Dakota School of Mines; Chairman: R. W. Ellingwood, University of Colorado; R. L. Eisenman, Air Force Academy.

Nomination Committee: F. N. Fisch, Colorado State College; Chairman: W. E. Dorgan, Western State College; Neville Hunsaker, Utah State University.

Professor E. R. Deal of Colorado State University, Contest Chairman for the Annual High School Mathematics Contest, reported that 140 schools had participated—18 from Wyoming, 24 from Utah and 98 from Colorado.

The following 17 papers were presented at the meeting:

1. *Hilbert space with an indefinite inner product*, by R. W. McKelvey, University of Colorado.

An indefinite inner product $[x, y]$ on a vector space V is a symmetric bilinear functional such that $[x, x]$ may be positive, negative or zero. A Nevanlinna space is a Hilbert space with positive definite inner product (x, y) and a second indefinite inner product given by $[x, y]=(Jx, y)$ where J and J^{-1} are bounded self-adjoint operators. The talk is a brief exposition of the theory of Nevanlinna space: subspaces, orthogonal projectors, Cartesian sum decompositions, and the spectral resolution for a self-adjoint operator.

2. *The generalized Jordan canonical form*, by D. W. Robinson, Brigham Young University.

One of the topics usually considered in a course in linear algebra is the study of various matrix representations for a linear transformation on a vector space over a field. If the field is algebraically closed, then the most useful and well-known representation is the Jordan canonical form. However, it is not as well known that, by means of a slight extension, this form may essentially be used over a much larger class of fields. The purpose of this paper is to suggest a way to bring this "generalized Jordan canonical form" into the classroom.

3. *Approximate continuity*, by J. E. Kimber, Jr., Utah State University.

4. *Trilinear equations in a finite field*, by A. Duane Porter, University of Wyoming.

Let $F=GF(q)$ be the finite field of $q=p^r$ elements, p arbitrary, and let $N(n,a)$ denote the number of solutions in F of $a_1x_1y_1z_1+\cdots+a_nx_ny_nz_n=a$. Also, let $N(n, a_i, b_i, a, b)$ denote the number of solutions in F of the system $a_1x_1y_1z_1+\cdots+a_nx_ny_nz_n=a$, $b_1x_1y_1z_1+\cdots+b_nx_ny_nz_n=b$, where all coefficients are from F. Explicit formulas for both $N(n, a)$ and $N(n, a_i, b_i, a, b)$ are obtained. To evaluate $N(n, a_i, b_i, a, b)$, two cases are considered. First, when $a_i, b_i\neq 0$, $1\leqq i\leqq n$, a, b arbitrary, and second, when a_i, b_i are all arbitrary.

5. *On Ramanujan's sum*, by G. S. Donovan, University of Colorado.

6. *Compound stochastic processes*, by S. A. Patil and M. M. Siddiqui, Colorado State University.

7. *A simple remark on Waring-type problems and linear Diophantine equations*, by A. J. Kempner, University of Colorado.

"Obvious, but not trivial": $1x_1+2x_2+3x_3+\cdots+kx_k+\cdots=n$, $n, x_1, x_2, x_3, \cdots$ integers $\geqq 0$, has for all n solutions $3\geqq x_1\geqq x_2\geqq x_3\geqq\cdots$ (0 from same point on). Similarly $1x_1+3x_2+5x_3+\cdots+(2k-1)x_k+\cdots=n$ has for all n solutions $4\geqq x_1\geqq x_2\geqq x_3\geqq\cdots$ (0 from same point on). Corresponding statements are given for Waring's general theorem, for Fermat's $x^n+y^n=F^n$, etc.

8. *A problem on integral operators*, by G. H. Meisters, University of Colorado.

Because everywhere-defined linear transformations of Hilbert space are bounded whenever they (1) are closed, (2) have adjoints with dense domains, (3) have a matrix representation—and for other reasons, the author conjectures first (vaguely) that linear transformations which are "constructively" defined everywhere on Hilbert space (or any B-space) are necessarily bounded, and second (precisely) that everywhere-defined (absolutely convergent Lebesgue) integral operators on Hilbert space are necessarily bounded. The author proves that if there exists a measurable set $E_1\subset E(=\text{measurable }\subset R^n)$ such that $m(E-E_1)=0$ and that for all $x\in E_1$ and all $f\in L_2(E)$, $Kf(x)=\int_E k(x, y)f(y)dy$ exists and belongs to $L_2(E)$, where k is measurable, then $K:L_2(E)\to L_2(E)$ is bounded.

9. *Differential inequalities with exceptional sets*, by J. W. Bebernes and G. H. Meisters, University of Colorado.

The following two theorems are slightly generalized versions of results of G. H. Meisters and this author. THEOREM 1: If $D_+u(t)\leqq f(t, u(t))$ a.e., and $D_+u(t)<+\infty$ nearly everywhere on $[a, b]$ (n.e. allows a countable exceptional set) where u and f are continuous, then any maximal solution ϕ_m of $x'=f(t, x)$ with $\phi_m(a)\geqq u(a)$ satisfies $u(t)\leqq\phi_m(t)$ on its interval of existence. THEOREM 2: Suppose there is a nonnegative continuous function $h(t, u)$ such that $u'=h(t, u)$, $u(t_0)=0$, has zero as its unique (right) solution and that $x_1\leqq x_2$ implies $f(t, x_2)-f(t, x_1)\leqq h(t, x_2-x_1)$ a.e. on $[a, b]$. If $D^+u(t)<+\infty$, $D^+v(t)>-\infty$ n.e., and if $D^+u(t)-f(t, u(t))\leqq D^+v(t)-f(t, v(t))$ a.e., for continuous u and v with $u(a)\leqq v(a)$, then $u(t)\leqq v(t)$ on $[a, b]$.

10. *The approximate solution of Riccati's equation*, by Ronald Huffstutler and F. M. Stein, Colorado State University.

This paper discusses the approximate solution of the Riccati equation $L(y)\equiv y'-P(x)y-Q(x)y^2=R(x)$ over $[0, 1]$ by a sum of zero-th order Bessel functions $S_n(x)=\sum_{m=1}^{n}B_mJ_0(\lambda_m x)$,

satisfying $y_0=y(x_0)$, that is the *best approximation* in the sense that $\int_0^1 |R(x)-L[S_n(x)]|^m dx$, $m>0$, is a minimum. Particular use is made of the fact that $J_0'(x)=-J_1(x)$, and thus both the solution $y(x)$ and its derivative $y'(x)$ can be uniformily approximated by $S_n(x)$ and $S_n'(x)$ respectively throughout $[0, 1]$.

11. *On triples of quasi-conjugate matrices*, by L. S. Johnson and V. J. Varineau, University of Wyoming.

The concept of quasi-conjugate n-tuples of matrices is re-examined and some of its most interesting properties are discussed. A similarity relation for quasi-conjugate n-tuples is defined. The set of quasi-conjugate triples over a finite field is examined with respect to the similarity partition and the number of triples in each class is determined. The number of similarity classes for a finite field with m elements is found to be m.

12. *Geometry and vision: A plea for perspective geometry in senior high school*, by A. J. Kempner, University of Colorado.

Few of our students realize that we live our daily lives in two mutually contradictory worlds of geometry: the tactile (Euclidean) and the optical (perspective). An understanding of this situation—apparently totally lacking under our present high-school training—would be of marked scientific, cultural and philosophical value. To mention only one aspect: it would prepare the ground for psychological acceptance of the various non-Euclidean geometries and of the Einstein-Minkowski geometry of relativity, etc. A course including a satisfactory foundation of projective geometry could well be fitted into our present high school set-up.

13. *The statistics program at Colorado State University*, by J. S. Williams, Colorado State University.

14. *Implementation of CUPM Recommendations, Levels I and III*, Panel on Teacher Training, by J. J. Fisher, Colorado State Department of Education.

15. *The junior college mathematics curriculum*, by T. D. Cavanagh, Colorado State College.

The speaker discussed a study he had made of the junior college mathematics curriculum. The study was primarily a questionnaire study. A survey of the catalogues available from the junior colleges in the sample was included as a part of the study. The speaker discussed the present mathematics offerings of the junior colleges, the ways in which these offerings are changing, and the factors which influence such changes. He also made some recommendations for change in the junior college mathematics curriculum.

16. *Use of modern language techniques in the teaching of mathematics*, by Miss Ann Pape, Lakewood, Colorado.

Over a four year period, the audio-lingual methods used in the teaching of languages, including extensive use of tape recorders, have been applied in mathematics classes. Postulates and theorems are taught like grammar drills in a language situation. There are a number of advantages to this method. Necessary repetition is attained with less effort; slow learners and chronic absentees are helped without infringing upon teacher time; novelty and variety are added. The progress of groups using audio-lingual methods shows a significant improvement in accomplishment over control groups.

17. *Information feedback for mathematics student teachers*, by J. M. Moser, University of Colorado.

The paper discussed a study of the mathematics student teaching experience. The study attempted to minimize the subjective nature of evaluations of student teaching through the medium of feeding back objective information to the student teacher by means of audio tape recordings and discussions of teaching behavior matrices which are part of the Minnesota System of Interaction Analysis. Observations made during a year indicated that student teachers tend to become fairly

rigid in the pattern of their teaching behavior once they have found one which suits their personality. The greatest amount of student participation and teacher-student interaction was found in those classes which used SMSG or UICSM text materials.

W. N. SMITH, *Secretary-Treasurer*

CALENDAR OF FUTURE MEETINGS

Forty-eighth Summer Meeting, University of Toronto, Toronto, Ontario, Canada, August 28–30, 1967.

Fifty-first Annual Meeting, San Francisco, California, January 25–27, 1968.

ALLEGHENY MOUNTAIN, West Virginia University, Morgantown, May 6, 1967.

ILLINOIS, University of Illinois, Urbana, May 12–13, 1967.

INDIANA, Wabash College, Crawfordsville, May 13, 1967.

IOWA, Drake University, Des Moines, April 21, 1967.

KANSAS, Fort Hays State College, Hays, April 22, 1967.

KENTUCKY, Murray State University, Murray, April 1, 1967.

LOUISIANA-MISSISSIPPI, Jung Hotel, New Orleans, Louisiana, March 4–5, 1967.

MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, University of Virginia, Charlottesville, April 22, 1967.

METROPOLITAN NEW YORK, Long Island University, Brooklyn Division, March 18, 1967.

MICHIGAN, University of Michigan, Ann Arbor, March 18, 1967.

MINNESOTA, St. John's University, Collegeville, May 6, 1967.

MISSOURI, Northeast Missouri State Teachers College, Kirksville, April 29, 1967.

NEBRASKA, University of South Dakota, Vermillion, May 6, 1967.

NEW JERSEY

NORTHEASTERN, Mt. Allison University, Sackville, New Brunswick, June 23–24, 1967.

NORTHERN CALIFORNIA

OHIO, Ohio State University, Columbus, April 22, 1967.

OKLAHOMA-ARKANSAS, Northeastern State College, Tahlequah, Oklahoma, April 1–2, 1967.

PACIFIC NORTHWEST, University of Montana, Missoula, June 16–17, 1967.

PHILADELPHIA, University of Delaware, Newark, November 18, 1967.

ROCKY MOUNTAIN, Western State College of Colorado, Gunnison, May 12–13, 1967.

SOUTHEASTERN, Florida Presbyterian College, St. Petersburg, Florida, March 31–April 1, 1967.

SOUTHERN CALIFORNIA, San Diego State College, San Diego, March 11, 1967.

SOUTHWESTERN, University of Arizona, Tucson, March 31–April 1, 1967.

TEXAS, Austin College, Sherman, April 14–15, 1967.

UPPER NEW YORK STATE, State University College, Plattsburgh, May 20, 1967.

WISCONSIN, St. Norbert College, DePere, May 6, 1967.

FUTURE MEETINGS OF OTHER ORGANIZATIONS

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, New York, N. Y., December 26–31, 1967.

AMERICAN MATHEMATICAL SOCIETY, Toronto, Ontario, Canada, Aug. 29–Sept. 1, 1967.

AMERICAN SOCIETY FOR ENGINEERING EDUCATION, Michigan State University, June 19–23, 1967.

ASSOCIATION FOR COMPUTING MACHINERY, Sheraton-Park, Washington, D. C., August 29–31, 1967.

ASSOCIATION FOR SYMBOLIC LOGIC

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS, Chicago, November 23–25, 1967.

INSTITUTE OF MATHEMATICAL STATISTICS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, Convention Center, Las Vegas, Nevada, April 16–20, 1967.

OPERATIONS RESEARCH SOCIETY OF AMERICA, New York Hilton Hotel, May 31–June 2, 1967.

PI MU EPSILON

SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS, Shoreham Hotel, Washington, D. C., June 12–15, 1967 (Symposium on applied probability and fluid dynamics).

MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The fiftieth annual meeting of the Rocky Mountain Section of the MAA was held at Western State College, Gunnison, Colorado, on Friday and Saturday, May 12 and 13, 1967.

There were 95 people registered for the meeting including Professor F. M. Stein of Colorado State University, Sectional Governor, and Professor W. E. Dorgan of Western State College, Section Chairman.

The invited address was delivered on Friday afternoon by Dr. John Gary, National Center for Atmospheric Research, Boulder, Colorado. He spoke on "Large Scale Numerical Simulation of Atmospheric Processes."

Professor W. E. Dorgan presided at the banquet Friday night. The Section was welcomed by Dean D. H. Cummins, Dean of Faculties of Western State. Following the banquet mathematical movies were shown.

The following 16 papers were presented at the meeting:

1. *Semi-continuity and linear transformation*, by J. A. Jensen, University of Wyoming.

This paper presents several theorems which show that, with appropriate conditions on the topological spaces, if a function is semicontinuous, then it is continuous.

2. *Some equations in a finite field*, by A. D. Porter, University of Wyoming.

3. *On a cyclide and its associated circular cubic*, by N. X. Vinh, University of Colorado.

This paper discusses the cyclide defined by the equation $y(x^2+y^2+z^2)=a^2x$ and the circular cubic, intersection of the surface and the plane $z=0$. The cyclide contains three real and two imaginary families of circles. They are the intersections of the surface and the families of spheres having their centers on the xy plane, and on two paraboloids of revolution. Properties of the circular cubic were derived and its connections with the lemniscate of Bernoulli and the rectangular hyperbola were shown.

4. *A note on a class of generating functions*, by J. E. Faulkner, Brigham Young University.

5. *Some results on the Mikusiński convolution ring*, by Daryl Kreiling and S. Johnson, University of Wyoming.

It is shown that the Mikusiński convolution ring C is a Jacobson radical ring in which the descending chain condition on ideals does not hold. It is also shown that C can be expressed as a subring of the direct sum of some family of rings and that C has a family of ideals whose intersection is the zero ideal. Such a representation and family of ideals is given.

6. *Some results on the number of rings of order n*, by Clyde Martin and A. D. Porter, University of Wyoming.

It is shown that the number of rings of order n may be obtained by determining the number of rings of prime power order. An upper bound for rings of order p^a, p prime, is then obtained. A second approach to the problem shows that finding the number of rings of order p^a is equivalent to determining the number of solutions to a certain set of n^3 congruences.

7. *Similarity and orthogonal similarity in a finite field*, by John Adams and A. D. Porter, University of Wyoming.

Classical results for similarity, orthogonal similarity, and unitary similarity of square matrices over the real and complex fields are paralleled in finite fields. Necessary and sufficient conditions are given for these relations to hold between certain classes of square matrices and diagonal matrices.

8. *Matrix notations for the Taylor series expansion*, by C. A. Halijak, University of Denver.

9. *The mathematics program at the USAF Academy*, by Lt. Colonel R. C. Rounding, USAF Academy.

10. *Evaluation and placement system at the USAF Academy*, by Colonel Archie Higdon, USAF Academy.

11. *A survey course for business majors*, by H. H. Frisinger, Colorado State University.

12. *Criteria for positive definiteness of large multi-diagonal matrices*, by E. L. Allgower, Colorado State University.

The approach is to seek conditions on the elements such that sequences of symmetric multi-diagonal matrices obtained by bordering are positive definite independent of the size. Such sequences of matrices are also characterized in terms of monotonicity of the sequences of the corresponding matrices having 1's down the main diagonal. This result is related to the nature of the solutions of the recursion or difference equations which describe the determinants of multi-diagonal type matrices.

13. *Robust estimation of location I*, by E. L. Crow, Environmental Science Services Administration, Boulder, Colorado, and M. M. Siddiqui, Colorado State University.

14. *Robust estimation of location II*, by M. M. Siddiqui, Colorado State University, and E. L. Crow, Environmental Science Services Administration, Boulder, Colorado.

15. *Recurrency in the integer solutions of quadratic equations*, by E. I. Emerson, Boulder, Colorado.

16. *Some investigations on partially balanced arrays*, by D. V. Chopra, Southern Colorado State College.

At the business meeting, which was held on Saturday morning, May 13, with Professor Dorgan presiding, the following officers were elected for 1967/68:

Chairman—Kenneth Noble, University of Denver; Vice-Chairman—Jerrold Bebernes, University of Colorado; Secretary-Treasurer—C. R. Wylie, Jr., University of Utah.

W. N. Smith, *Secretary-Treasurer*

OCTOBER MEETING OF THE OHIO SECTION

A special meeting of the Ohio Section of the MAA was held on October 20–21, 1967, at Stouffer's University Inn, Columbus, Ohio. This was a joint meeting of the Ohio Section and the Committee on the Undergraduate Program in Mathematics (CUPM) and was entitled "Conference on Collegiate Mathematics in Ohio." Professor Daniel Finkbeiner, Chairman of the Section, and Professors Arnold Ross and H. D. Lipsich presided at the general sessions. There were two hundred and eight registered in attendance including one hundred fifty-four members of the Association.

The following program was presented:

1. *A Brief Survey of CUPM Activities*, by R. D. Anderson, Louisiana State University, Chairman, CUPM.

This talk discussed the current status of CUPM's activities, particularly in the areas of the Teacher Training Panel, the College Teacher Preparation Panel, the Panel on Two Year Colleges, and the three panels on Applications of Mathematics: the Panel on Mathematics in the Life Sciences, the Panel on Computing, and the Panel on Statistics.

2. *A General Curriculum in Mathematics for Colleges*, by G. B. Price, University of Kansas.

5. *Nonlinear boundary value problems and the equation of first variation*, by Lynn Erbe, University of Nebraska.

6. *Some aspects of the theory of extensors*, by P. S. Morey, Jr., University of Omaha.

7. *The dependence of solutions of second order differential equations on their boundary conditions*, by G. A. Klassen, University of Nebraska.

8. *Unsolved problems in intuitive geometry*, by Victor Klee (invited lecture).

HENRY M. COX, *Secretary*

MAY MEETING OF THE ILLINOIS SECTION

The forty-seventh annual meeting of the Illinois Section of the MAA was held on the Edwardsville campus of Southern Illinois University on May 10–11, 1968. Dr. R. D. Boswell, Section Chairman, presided. There were eighty-five persons in attendance.

Professor Daniel Zelinsky of Northwestern University spoke on the topic, "Algebra is the study of functions too" and this address was followed by the presentation of five papers:

1. *Homomorphism topologies on abelian groups*, by B. F. Hobbs, Olivet Nazarene College.

2. *Abelian surfaces*, by Nancy Fincke, Western Illinois University.

3. *Approximation of Banach-valued multidimensional complex functions*, by W. J. Neath, Northern Illinois University.

4. *On a special linear transformation*, by Carl Townsend, Southern Illinois University (Carbondale).

5. *Why not divide by zero?* by William Bennewitz, Southern Illinois University (Edwardsville).

The dinner address was presented by Dr. H. K. Farahat, Senior Lecturer, University of Sheffield, England, and currently Visiting Associate Professor of Mathematics at the University of Illinois. His topic was "Mathematical education in England." Professor A. A. Albert, Dean of the Division of Physical Sciences, University of Chicago, spoke on Saturday morning on "Finite Projective Planes." The meeting concluded with a panel on "Computers in the Undergraduate Curriculum" in which the participants were Professors Alphonso DiPietro, Eastern Illinois University, Donald Herrick, Northern Illinois University, Jurg Nievergelt, University of Illinois, and William Rippenberger, Knox College.

At the annual business meeting following sessions on Friday afternoon the following officers of the Section were elected for 1968–1969: Chairman, Professor Arnold Wendt, Western Illinois University; Vice-Chairman, Professor Hiram Paley, University of Illinois; and Secretary-Treasurer, Professor Howard Saar, Shawnee Community College.

HOWARD SAAR, *Secretary-Treasurer*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The fifty-first annual meeting of the Rocky Mountain Section of the MAA was held at the University of Denver, Colorado, on May 10 and 11, 1968. There were 110 registrants, including Professor F. M. Stein of Colorado State University, the Sectional Governor, and Professor Kenneth Noble of the University of Denver, the Section Chairman. The invited address was delivered by Dr. W. S. Dorn, Watson Research Center, IBM Corporation, who spoke on 'Computer Extended Instruction'. Chancellor M. B. Mitchell of the University of Denver welcomed the Section at the banquet on Friday evening.

At the business meeting, the nominating committee was instructed to consider the advisability of amending the By-Laws of the Section to provide for the election of a second vice chairman to look after the interests of the junior colleges. The committee is to make its recommendation at next year's meeting of the Section. The Section also voted to

sponsor a program of high school lecturers, and a committee consisting of Professor Robert McKelvey, University of Colorado, Chairman; Professor William Scott, University of Utah and Professor Verne Varineau, University of Wyoming, was set up to implement the proposal.

The following officers were elected: Chairman, Jerrold Bebernes, University of Colorado, Boulder, Colorado; Vice Chairman, Ray Hanna, University of Wyoming, Laramie, Wyoming; Secretary-Treasurer, C. R. Wylie, Jr., University of Utah, Salt Lake City, Utah.

The following papers were read at the meeting:

1. *Numerical invariants in noncommutative orders*, by D. W. Ballew, South Dakota School of Mines and Technology.

2. *The Duplication of the sphere*, by Robert Bitts, Arapahoe Junior College.

3. *Edge diffraction for parabolic differential equations*, by Jack Cohen and David Hector, University of Denver.

4. *Student grades as a multiple Markov chain*, by Major R. L. Eisenman, USAF Academy.

5. *Continuous dependence for two-point boundary-value problems*, by R. E. Gaines, Colorado State University.

6. *Eigenvalue studies for second order differential equations using invariant bedding*, by Frank Hagin, University of Denver.

7. *Closed factors of Chebyshev polynomials* $S_n(x)$, by C. A. Halijak, University of Denver.

8. *An* L_q *approximate solution of the Ricatti equation*, by M. S. Henry and F. M. Stein, Colorado State University.

9. *A maximal ideal radical class*, by T. L. Jenkins, University of Wyoming.

10. *When is a curve a curve?*, by A. J. Kempner, University of Colorado.

11. *The spin—an algebraic and probabilistic toy*, by Jean-Paul Marchand, University of Geneva, Visiting Professor in Mathematics and Physics, University of Denver.

12. *A senior seminar topic*, by D. C. B. Marsh, Colorado School of Mines.

13. *Generalized quadratic forms in a finite field*, by A. D. Porter, University of Wyoming.

14. *Nonlinear difference methods for ordinary differential equations*, by D. P. Squier, Colorado State University.

15. *Some random hydrodynamics*, by J. W. Thomas, University of Wyoming.

16. *A class of positive definite functions on noncommutative groups*, by R. C. Weger, South Dakota School of Mines and Technology.

17. *A model for projective spaces with three points on every line*, by A. Zirakzadeh, University of Colorado.

In addition to these papers, the program included a panel discussion on 'Computer Extended Mathematics' in which the participants were W. S. Dorn, Watson Research Center, IBM Corporation, E. R. Kreuger, University of Colorado, Ruth Hoffman, University of Denver, John Skelton, University of Denver, Larry Blevins, Northeastern Junior College.

C. R. WYLIE, JR., *Secretary-Treasurer*

At the business meeting in the afternoon, the Secretary-Treasurer reported that Mr. Stephen Helmreich of Valparaiso University and Mr. Eric Isaacson of Indiana University had each been awarded a one-year membership in the Association in recognition of their achievement in the 29th Putnam Mathematical Competition. Officers for 1969–70 were elected as follows: Chairman, Professor N. B. Haaser, University of Notre Dame; Vice-Chairman, Professor W. C. Swift, Wabash College; Secretary-Treasurer, Professor M. J. Mansfield, Purdue University at Fort Wayne.

Following the business meeting Professor G. S. Young, President of the MAA, addressed the group on "Topology and Analysis."

M. J. MANSFIELD, *Secretary-Treasurer*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The fifty-second annual meeting of the Rocky Mountain Section of the MAA was held at the University of Colorado, Boulder, Colorado, on May 9 and 10, 1969. There were 153 persons registered for the meeting, including Professor F. M. Stein of Colorado State University, Sectional Governor, and Professor J. W. Bebernes of the University of Colorado, Section Chairman. The invited address was delivered by Professor V. L. Klee, Jr., of the University of Washington, who spoke on "Shapes of the Future—Unsolved Geometric Problems for Science and Technology." Professor W. E. Briggs, Dean of the College of Arts and Science of the University of Colorado, welcomed the Section at the banquet on Friday evening.

At the business meeting, the Report of the Nominating Committee recommending that the By-Laws of the Section be amended to provide for the election of a Second Vice-Chairman to look after the interests of the junior colleges, was approved. Professor T. D. Cavanagh, Contest Chairman of the Section, reported that 8610 students from 141 high schools participated in the 1968 MAA mathematics contest. Professor Robert McKelvey reported for the High School Lecturer Program inaugurated last year, and his recommendation that the program be continued was approved. The present committee, consisting of Professor Robert McKelvey, University of Colorado, Chairman, Professor W. R. Scott, University of Utah, and Professor Verne Varineau, University of Wyoming, was reappointed to continue the administration of this program.

The following officers were elected: Chairman, Ray Hanna, University of Wyoming, Laramie, Wyoming; First Vice-Chairman, George Stratopoulos, Weber State College, Ogden, Utah; Second Vice-Chairman, James Davis, Mesa Junior College, Grand Junction, Colorado; Secretary-Treasurer, D. J. Sterling, Colorado College, Colorado Springs.

The following papers were read at the meeting:

1. *Construction of projective ideals*, by D. W. Ballew, South Dakota School of Mines and Technology.
2. *A relationship of perfect fields to compact classes*, by A. J. Boes, Colorado School of Mines.
3. *Surfaces in three-dimensional euclidean space*, by C. E. Burgess, University of Utah.
4. *A property of perfect groups*, by Harold Finkelstein, University of Colorado.
5. *Uniform structures from abstract spaces*, by G. C. Gastle, University of Wyoming.
6. *Similarity of normal matrices in $GF(q)$*, by Mrs. Leslie Hanson and A. D. Porter, University of Wyoming.
7. *On uniform distribution of sequences in $GF(q, x)$ and $GF(q, x)$*, by J. H. Hodges, University of Colorado.
8. *Minimum and maximum topological spaces*, by R. E. Larson, University of Colorado.
9. *Perturbation of the poles of the scattering matrix*, by James LaVita, University of Denver.
10. *On computing the dimensions of spaces of automorphic functions*, by G. L. Loudner, South Dakota School of Mines and Technology.
11. *Quasi-local rings with Noetherian filtrations*, by Sylvia Chin-Pi Lu, University of Colorado (Denver Center).

12. *The recent discovery of infinitesimal analysis*, by Gary Meisters, University of Colorado.

13. *The continuum hypothesis and the axiom of choice—remarks on the practical relevance of recent independence results*, by Donald Monk, University of Colorado.

14. *Some matric equations over $GF(q)$*, by A. D. Porter, University of Wyoming.

15. *Quasi symmetric functions*, by T. J. Reed, University of Colorado.

16. *Lagrange's version of Lagrange's theorem on groups, 1771*, by R. L. Roth, University of Colorado.

17. *The number of solutions of polynomial equations*, by L. E. Shader, University of Wyoming.

18. *CSU closed circuit television mathematics courses*, by F. M. Stein and R. H. Niemann, Colorado State University.

19. *Applicability of mathematics*, by Stanislaw Ulam, University of Colorado.

C. R. WYLIE, *Secretary-Treasurer*

MAY MEETING OF THE WISCONSIN SECTION

The annual meeting of the Wisconsin Section of the MAA was held at Wisconsin State University-Oshkosh, on May 2 and 3, 1969. Chairman J. A. Raab, Wisconsin State University-Oshkosh, presided. Approximately 110 persons attended.

After registration on Friday afternoon, May 2, the following papers were presented:

1. *Matrix norms composed from norms of submatrices of a partitioned matrix*, by N. E. Nirschl, St. Norbert College, West DePere.

2. *Pseudo quasi metric space*, by Y. W. Kim, Wisconsin State University, Eau Claire.

3. *Generalized characteristic exponents of linear homogeneous systems of differential equations*, by H. S. Gunderson, Wisconsin State University, Oshkosh.

4. *How important is the normal distribution*? by D. Lund, Wisconsin State University, Eau Claire.

A banquet was held Friday evening. After the banquet, several films were shown including "Challenge in the Classroom: The Methods of R. L. Moore."

The Saturday morning session was devoted to a short business meeting and the presentation of three more papers. During the business meeting, Dr. Marshall Wick, Wisconsin State University, Eau Claire, was elected Chairman; Mr. Warren White, University of Wisconsin Extension Center-Sheboygan, was elected Vice-Chairman; and Dr. R. W. Christensen, Wisconsin State University-La Crosse, was elected Secretary-Treasurer. The remainder of the morning session was devoted to the presentation of the following papers:

1. *Nearly antiflexible division algebras*, by L. W. Davis, Wisconsin State University, White water.

2. *Not solving differential equations*, by F. Brauer, University of Wisconsin, Madison.

3. *Commutative subrings of the ring of $n \times n$ matrices over a finite field*, by C. B. Henneken, Marquette University, Milwaukee.

After a luncheon break, a panel consisting of F. Brauer, University of Wisconsin, Madison, J. Lakin, Wisconsin State University-Oshkosh, L. Wahlstrom, Wisconsin State University, Eau Claire, and E. Wilde, Beloit College, presented a discussion of "Geometry and the Undergraduate Program." The Saturday afternoon session concluded with a presentation of the following papers:

1. *CL products of CL spaces*, by C. C. Braunschweiger, Marquette University, Milwaukee.

2. *Complex number algebra as a simple case of Heavyside operational calculus*, by D. Moore, University of Wisconsin, Green Bay.

R. W. CHRISTENSEN, *Secretary-Treasurer*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The University of Wyoming, Laramie, Wyoming, hosted the fifty-third annual meeting of the Rocky Mountain Section of the MAA on May 8 and 9, 1970 and the meeting of the Junior College Cooperative Mathematics Program. There were 140 registrants, including Professor W. N. Smith, the Sectional Governor, and Professor J. R. Hanna, the Section Chairman, both of the University of Wyoming. The invited address delivered by Professor S. A. Jennings of the University of Victoria, Second Vice-President of the Association, was entitled "Some Generalizations of Absolute Value." W. D. Carlson, President of the University of Wyoming, welcomed the Section at the banquet Friday evening.

At the business meeting the By-Laws of the Section were amended to provide for:

(1) additional members who had petitioned the Association to join the Rocky Mountain Section;
(2) an increase in funds available for the annual meeting;
(3) the instigation of special meetings upon petition of the Section membership;
(4) an increase in the registration fee charged at the annual meeting.

The following officers were elected: Chairman, R. W. Irvine, Weber State College; Vice-Chairman, C. A. Swanson, Southern Colorado State College; Second Vice-Chairman, W. J. Bonini, Western Wyoming Community College; Secretary-Treasurer, D. J. Sterling, The Colorado College.

The following six papers were read at the invitation of the program committee:

1. *Computers and Computer Applied Mathematics*, by R. Hoffman, University of Denver.
2. *Providing Materials for Classroom Use*, by B. Nolsle, Mathematics Research Center, University of Wisconsin.
3. *Orthogonal Similarity in Finite Fields*, by A. D. Porter, University of Wyoming.
4. *Finite Simple Groups*, by W. Scott, University of Utah.
5. *The Mathematics Curriculum*, by D. J. Sterling, The Colorado College.
6. *Mathematical Foundations of Electromagnetic Theory*, by C. H. Wilcox, University of Denver

Nine papers were contributed and read on the program:

1. *The Structure of Invertible Ideals*, by D. W. Ballew, South Dakota School of Mines and Technology.
2. *The Matric Equation* $U_1 \cdots U_n A V_1 \cdots V_s = B$, by A. D. Porter and R. Dalla, University of Wyoming.
3. *Arithmetic Series*, by G. S. Donovan, Metropolitan State College.
4. *A Conjecture in Consecutive Composite Numbers*, by C. A. Grimm, South Dakota School of Mines and Technology.
5. *A Conjecture Concerning the Equation* $x^n = a$ *in a Finite Group*, by D. W. Ballew and R. Higgins, South Dakota School of Mines and Technology.
6. *Invariantive Properties of the Class of Formal Series* $\sum a_k e^{i\omega_k}$, $k = 1, \cdots, \infty$, $a_k > 0$, $\omega_k \not\equiv 0 \mod \pi$, by A. J. Kempner, University of Colorado.
7. *Crisis in the Classroom—Teachers Who Can't Teach*, by W. D. Popejoy, Colorado State College.
8. *Investigating Periodic Ternary Continued Fractions on the Computer*, by J. A. Raab, Metropolitan State College.
9. *An Existence Theorem for a Generalized Integral Transform*, by D. M. Rognlie, South Dakota School of Mines and Technology.

In addition to the above papers, six mathematical films were shown through the cooperation and generosity of Modern Learning Aids Inc.: *What is Area, Area Under a Curve, The Definite Integral, Infinite Acres, The Fundamental Theorem of Calculus, The Theorem of the Mean.*

D. J. STERLING, *Secretary-Treasurer*

MAY MEETING OF THE MISSOURI SECTION

The annual meeting of the Missouri Section of the MAA was held at Missouri Southern College, Joplin, on April 30 and May 1, 1971; seventy-five persons were in attendance.

Professor Charles Stuth, Section Vice-Chairman, presided at the Friday afternoon session, during which Professor A. B. Willcox gave the invited address, "England was Lost on the Playing Fields of Eton: A Parable for Mathematics," and the following papers were presented:

1. *On Schauder decompositions, two norm spaces and pseudo reflexivity*, by P. K. Subramanian, Missouri Southern College.
2. *The lattice of faces of a convex cone II*, by G. P. Barker, University of Missouri, Kansas City.
3. *A note on topology*, by Troy Hicks, University of Missouri, Rolla.
4. *A geometric introduction to stability theory and Liapunov functions*, by Stephen Bernfeld, University of Missouri, Columbia.
5. *Indefinite Finsler spaces*, by J. K. Beem, University of Missouri, Columbia.
6. *Criteria involved in the formulation of definitions involving sets*, by Henry Polowy, Lincoln University.

Professor Rochelle Boehning, Section Chairman, presided at the Saturday session, during which Professor J. W. Keesee gave the invited address, "Weakly Continuous Cohomology Theories." Also a panel discussion on Accreditation and Certification was presented by: Professor Glen Haddock, moderator, and panel members, Paul Burcham, University of Missouri, Columbia; L. T. Shiflett, Southwest Missouri State College; Ray Balbes, University of Missouri, St. Louis; and Charles Stuth, Stephens College.

At the business meeting, the following officers were elected for 1971–1972: Professor Charles Stuth, Stephens College, Chairman; Professor Fred Wilke, University of Missouri, St. Louis, Vice-Chairman; and Professor Troy Hicks, University of Missouri, Rolla, Secretary-Treasurer.

JACK JOLLY, *Secretary-Treasurer*

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

Weber State College, Ogden, Utah, hosted the fifty-fourth Annual Meeting of the Rocky Mountain Section of the MAA on May 7 and 8, 1971. There were 65 registrants, including Professor W. N. Smith, of the University of Wyoming, the Sectional Governor, and Professor R. W. Irvine of Weber State College, the Section Chairman. The invited address, "Paths on Polyhedra," was delivered by Professor Victor Klee of the University of Washington, President of the Association. H. P. Hofmann, Academic Vice-President of Weber State College, welcomed the Section at the banquet Friday evening.

The following officers were elected at the business meeting: Chairman, C. A. Swanson, Southern Colorado State College; Vice-Chairman, Robert Gutzman, Colorado School of Mines; Second Vice-Chairman, C. N. Podraza, Northeastern Junior College; Secretary-Treasurer, D. J. Sterling, Colorado, College.

The following four papers were read at the invitation of the program committee:

1. *Recent developments in geometric topology*, by L. C. Glaser, University of Utah.
2. *Calculus: CUPM's unused version*, by Ben Roth, University of Wyoming.
3. *Accreditation and certification*, by D. J. Sterling, Colorado College.
4. *Computer graphics and the head-mounted display*, by D. L. Vickers, University of Utah.

Ten papers were contributed and read on the program:

1. *A relation between π and greatest common divisors*, by David Ballew, South Dakota School of Mines and Technology.
2. *A generalization of a conjecture of Erdös*, by R. B. Crittendon, Portland State University.
3. *A generalized Riemann-Stieltjes integral*, by M. L. Klasi, South Dakota School of Mines and Technology.
4. *Self-directed study in mathematics*, by K. F. Klopfenstein*, Wilson Brumley, Darrell Perkins, Colorado State University.
5. *Partitions of a matrix*, by A. D. Porter, University of Wyoming.
6. *Generalized inverses of group homomorphisms*, by D. W. Robinson, Brigham Young University.
7. *Categorical methods applied to Pontryagin duality*, by D. W. Roeder, Colorado College.
8. *Incidence algebra and GF* $[q, x]$, by L. E. Shader, University of Wyoming.
9. *On an existence theorem for boundary value problems*, by W. G. Sutton*, South Dakota School of Mines and Technology, J. H. George, University of Wyoming.
10. *Arc length and the mean value theorem*, by S. G. Wayment, Utah State University.

In addition to the above papers, an exhibit of textbooks for use in the junior and community college curriculum was presented with the generous help of the following publishers: Addison-Wesley; Harcourt Brace Jovanovich; Harper and Row; Prindle, Weber and Schmidt; Scott Foresman; and Van Nostrand Reinhold.

D. J. STERLING, *Secretary-Treasurer*

CUPM AND THE MATHEMATICAL SOCIAL SCIENCE BOARD

The Mathematical Social Science Board and the Committee on the Undergraduate Program in Mathematics are seeking interesting problems or illustrative examples, from each of the social sciences, whose solutions and study make use of ideas and techniques from one or more of the following topics in undergraduate mathematics: sets and relations, differential and integral calculus, matrices and linear algebra, and probability.

We propose to collect such examples into a book mainly to be used by mathematics teachers and students as a source (1) of current social science applications of mathematics and (2) of material for textbook and classroom exercises to illustrate how topics in collegiate mathematics arise in a social science context. We also plan to include annotated bibliographies of articles and books involving applications of mathematics to the various social sciences.

The most preferred contribution would be an exposition giving (a) the social science problem and its background, (b) the reduction of the problem to mathematical form, (c) the mathematical analysis, perhaps with associated numerical results obtained on a computer, and (d) the meaning and insights provided by the mathematical analysis when related back to the original social science problem. Less desirable, but still very welcome, would be a reprint including material from which such an exposition could be extracted. References to the literature would also be helpful.

The CUPM-MSSB Project Committee presently consists of the following persons: D. W. Bushaw, Samuel Goldberg, Harold Kuhn, R. D. Luce, Henry Pollak.

Please send contributions to: CUPM-MSSB Project, Post Office Box 1024, Berkeley, California 94701.

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The fifty-sixth Annual Meeting of the Rocky Mountain Section of the MAA was held on the campus of the University of Northern Colorado, May 5 and 6, 1973. There were 143 registrants including Dean C. Benson of South Dakota School of Mines and Technology, the Sectional Governor, and Professor F. N. Fisch of the University of Northern Colorado, the Section Chairman. The invited address, *Consequences of Continuity*, was delivered by Professor R. P. Boas of Northwestern University, President of the Association. R. R. Bond, President, University of Northern Colorado, welcomed the Section at the banquet Friday evening. The banquet address, *Developing Mathematics in India and Latin America*, was delivered by Professor Emeritus B. W. Jones of the University of Colorado.

The following officers were elected at the business meeting: Chairman, F. N. Fisch, University of Northern Colorado; Vice-Chairman, J. C. Davis, Mesa College; Meeting Chairman, R. R. Gutzman, Colorado School of Mines.

The following panels and papers were read at the invitation of the program committee:

1. Innovations in Teaching College Mathematics: Terry Cleveland, Cragmor Center, University of Colorado, *Precalculus Mathematics as Self-Paced Study*. Hal Moore, Brigham Young University, *Precalculus Mini-Courses — Self Pacing with Constraints*. John Skelton, University of Denver, *Interaction among Art, Computer Science and Mathematics*. D. J. Sterling, Colorado College, *Mathematics under a Modular Scheme*. Don Tucker, University of Utah, *Experiments with Pacing in Precalculus and Calculus Mathematics*.

2. Innovative Practices in the Mathematics Preparation of Prospective Elementary Teachers: Ruth Hoffman, University of Denver, *Multi-Media for the Mathematics Training of Elementary Teachers*. Charles McNerney, University of Northern Colorado, *The Preparation of Prospective Elementary Teachers — A Multi-Facet Approach*. Harry Rosenberg, Fort Lewis College, *Use of a Concept Development Approach in the Training of Elementary Teachers*.

3. *MAA-CUPM Recommendations on Statistics at Undergraduate Level*, by Robert Heiny, University of Northern Colorado.

4. *Think Metric — The Process of Change*, by Robert Johnson, University of Northern Colorado.

5. *The Mathematics Component of the University of Colorado Upstep Program*, by Marc Swadener, University of Colorado.

Sixteen papers were contributed and read on the program:

1. *Cylic Ideals in Orders*, by David Ballew, South Dakota School of Mines and Technology.

2. *A Classification Procedure Based on Principal Components*, by Dennis Brady, South Dakota School of Mines and Technology.

3. *Decompositions of Finitely Generated Cotorsion Modules*, by Stephen Bronn, Southern Colorado State College.

4. *The Fish Story, a Population Model*, by Ross Fraker, Utah State University.

5. *The First Known Mathematician in America*, by Howard Frisinger, Colorado State University.

6. *An Axiomatic Trigonometry*, by Joe Frommer, Boulder, Colorado.

7. *Splitting Rings of Generalized Triangular Type*, by John Fuelberth and James Kuzmanovich, University of Northern Colorado.

8. *General Proximities and Pairs of Spaces*, by George Castl, University of Wyoming.

9. *Patterns in Problem Solving*, by Richard Gibbs, Fort Lewis College.

10. *Stochastic Models for Surpluses and Deficits and Approximations*, by Robert Heiny, University of Northern Colorado.

11. *Remarks on Reconstruction and n-Reconstruction Graphs*, by Bennet Manvel, Colorado State University.

12. *Permutation Groups and Determinants*, by Peter Murray, Westminster College.

13. *The Evolution of a Function*, by Duane Porter, University of Wyoming.

14. *Gauss, Least Squares and Generalized Inverses*, by Donald Robinson, Brigham Young University.

15. *A Strategy for the Game of SIM*, by Leslie Shader, University of Wyoming.

16. *A Note on Fermat's Last Theorem*, by Richard Swaller, South Dakota School of Mines and Technology.

In addition to the above papers and addresses, a textbook exhibit was presented with the generous assistance of numerous publishers.

D. J. STERLING, *Secretary-Treasurer*

MAY MEETING OF THE SEAWAY SECTION

The Spring Meeting of the Seaway Section of the MAA was held at Rosary Hill College, Buffalo, N. Y., on May 12, 1973, in recognition of the twenty-fifth anniversary of Rosary Hill College. Chairman Erik Hemmingsen presided at the meeting which had a registered attendance of 105 people.

The annual Harry M. Gehman Invited Lecture was given by K. O. May, University of Toronto, whose topic was *Communication Problems in Mathematics*.

At the business meeting the following officers were elected: Chairman, William Stone, Union College; First Vice-Chairman, D. O. McKay, University of Western Ontario; Second Vice-Chairman, C. A. Lathan, Monroe Community College. The new Governor of the Seaway Section, Malcolm Pownall, Colgate University, was introduced.

Paul Lemke, student at Rensselaer Polytechnic Institute, was recognized as receiving the highest score in the Seaway Section in the 1972 William Lowell Putnam Mathematical Competition.

H. B. Foisy, State University College at Postdam, gave the Annual Report on the MAA High School Mathematics Contest.

Papers presented at the morning session were:

The Computer Oriented Calculus Course at Clarkson, by Gustave Rabson, Clarkson College.

Convergence Criteria for Rational Series, by Jennifer Kevins, student at St. Bonaventure University.

A Net Characterization of the E-transformation, by J. H. Tsai, State University College at Geneseo.

Models of Ordinals, by D. S. Martin, State University College at Brockport.

Papers presented at the afternoon session were:

A Note on Graphs Whose Neighborhoods are n-cycles, by B. L. Chilton, State University College at Fredonia.

The Permanent Function of a Symmetric Matrix, by E. T. Hoefer, Rosary Hill College.

Transformation Semi-groups Acting on Totally Ordered Spaces, by Dennis Anson, Alfred University.

An Application of Logic, by R. G. Van Meter, State University College at Oneonta.

EMMET STOPHER, *Secretary-Treasurer*

14. *Alternatives to Taylor's theorem in proving analyticity*, by J. A. Eidswick, University of Nebraska-Lincoln.

15. (Invited Address) *Distributions and their uses*, by G. H. Meisters, University of Nebraska-Lincoln.

16. *What is spectral analysis*?, by C. F. Masters, Doane College.

17. *Branching, bounding, and stopping*, by G. F. Haddix, University of Nebraska-Omaha.

H. M. Cox, *Secretary-Treasurer*

APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The fifty-eighth Annual Meeting of the Rocky Mountain section of the MAA was held on the campus of Mesa College, April 11 and 12, 1975. There were 108 registrants including D. C. Benson, Governor of the Section, and J. C. Davis, Chairman of the Section. The invited address was presented by Professor Ivan Niven, University of Oregon, First Vice President of the Association. Dr. Carl Wahlberg, Acting President of Mesa College, welcomed the Section after the banquet Friday evening.

The following papers were contributed and read on the program:

1. *Contractive mappings on complete metric spaces*, By John Ausink and Douglas James, USAF Academy.

2. *Survey of approximations to pi*, by David Ballew, South Dakota School of Mines and Technology.

3. *Lower math and what you can do with it*, by R. J. Bitts, Arapahoe State College.

4. *Accessibility of the boundary of a plane region*, by C. E. Burgess, University of Utah.

5. *An upper bound on the dimension of the space of solutions A of the matrix equation*, by Bruce Collings, Brigham Young University.

6. *Introductory statistics: a module-course demanding student responsibility and student performance*, by W. P. Cooke, University of Wyoming.

7. *What are the opportunities for a B.S. degree with a math major*, by Allan David, University of Utah.

8. *Exploiting some ideas in computing to teach mathematics*, by W. S. Dorn, Denver University.

9. *Limiting distributions for immigration branching processes with decomposable mean matrix: the strictly critical case*, by J. H. Foster, Denver University.

10. *Applying the "shepherd's principle,"* by R. A. Gibbs, Fort Lewis College.

11. *A generalization of certain corresponding continued fractions*, by John Gill, Southern Colorado State College.

12. *Existence, uniqueness and continuability of solutions of second order functional differential equations*, by Gary Grefsrud, Fort Lewis College.

13. *Characterizations of strictly convex normed linear spaces*, by Stan Gudder, Denver University.

14. *Models and misfits*, by J. R. Hanna, University of Wyoming.

15. *Geometry as mathematics: an approach to undergraduate geometry*, by Z. R. Hartvigson, Colorado University at Denver.

16. *Matric analogues of number theory*, by E. E. Hasz, Metropolitan State College.

17. *Las Vegas: springboard to mathematics study — fact or fiction*?, by L. S. Johnson, Fort Lewis College.

18. *For to think metric: an operational model for teacher re-education*, by Bob Kansky, University of Wyoming.

19. *Dividing the area of a circle into two parts in the ratio s: t by certain curves*, by Hung C. Li, Southern Colorado State College.

20. *Mastery learning — consider it*, by M. D. McClenahan, University of Wyoming.

21. *On sigma-ideals of sets*, by C. G. Mendez, Metropolitan State College.

22. *Projectile targets revisited, or — can I hit it twice*?, by Roger Opp, South Dakota School of Mines and Technology.

23. *Programmed instruction*, by L. M. Orman, Southern Colorado State College.

24. *Mini-courses for mathematics education*, by A. D. Porter, University of Wyoming.

25. *The generalization of a Putnam problem*, by W. C. Ramaley, Fort Lewis College.

26. *Ulm's theorem without addition*, by Laurel Rogers, Colorado University.

27. *Watson's lemma in several variables*, by D. M. Rognlie, South Dakota School of Mines and Technology.

28. *Estimates for factorial effects in constrained randomization models*, by R. L. Schwaller, South Dakota School of Mines and Technology.

29. *Plane coloring problems*, by Leslie Shader, University of Wyoming.

30. *Mathematical analysis of computer system performance*, by Don Warner, Mesa College.

31. *A qualitative study of ordinary differential equations at the U. S. Air Force Academy*, by Major Williams, USAF Academy.

In addition to the above papers, a textbook exhibit was presented with the assistance of numerous publishers.

D. J. Sterling, *Secretary*

11. *Honor roll students, 1974 and 1975 Annual High School Mathematics Examination*, by H. M. Cox, University of Nebraska-Lincoln.

12. *Report on Nebraska-South Dakota Mathematics Contest for High School Students*, by L. J. Stephens, University of Nebraska at Omaha.

13. *Invited address: Relationship between the applications of mathematics and teaching of mathematics*, by H. O. Pollak, Bell Laboratories, Murray Hill, New Jersey.

H. M. Cox, *Secretary-Treasurer*

APRIL-MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The Fifty-ninth Annual Meeting of the Rocky Mountain Section of the MAA was held on the campus of Ft. Lewis College, Durango, Colorado, April 30 and May 1, 1976. There were eighty-six registrants, including Forest Fisch, Governor of the Section, and A. D. Porter, Chairman of the Section. Dean Larry Johnson, Director of School of Arts and Sciences, welcomed the Section to Ft. Lewis College. The banquet address "Whither Mathematics", was delivered by Professor R. D. Anderson of Louisiana State University. The keynote address, "Yes, 'Virginia', There is such a Thing as a Liberal Arts Course in Mathematics", was given by Professor John Hodges, University of Colorado, and two invited addresses were given: "When Is a Sequence Random", by Dr. Gus Simmons, Sandia Corp., and "Algorithmically Defined Functions", by Professor R. D. Anderson, Louisiana State University.

There were thirty-one contributed papers read at the meeting:

1. *An actuarial option in the traditional mathematics program*, by D. C. Benson, South Dakota School of Mines and Technology.

2. *What mathematics skills do prospective elementary teachers command?*, by Milfried Olson, The University of Wyoming.

3. *Minicourses in mathematics education — a year later*, by A. D. Porter, The University of Wyoming.

4. *A variation on a problem of Davenport and Diophantus*, by B. W. Jones, University of Colorado.

5. *On the Oxtoby-Ulam theorem*, by C. G. Mendez, Metropolitan State College.

6. *Reversible multiples*, by David Ballew and C. A. Grimm, South Dakota School of Mines and Technology.

7. *On internal gravity wave generation by a stationary oscillating source*, by Ed Adams, University of Southern Colorado.

8. *Ill posed linear-operator equations*, by O. N. Strand, National Oceanic and Atmospheric Administration.

9. *Families of fourth order Sturm-Liouville systems*, by F. M. Stein, Colorado State University.

10. *A note on the matrix equation* $\Phi = P\Sigma P^T$, *where P is a projection*, by J. O. Kork, Colorado School of Mines.

11. *On imitating the Euclidean metric*, by Ira Rosenholtz, The University of Wyoming.

12. *Several conjectures on commutativity in algebraic structures*, by K. S. Joseph, Metropolitan State College.

13. *Solving the Gauss equation via exterior algebra*, by Jack Vilms, Colorado State University.

14. *The integral closure of a ring from a topological point of view*, by C. H. Brase, Littleton, Colorado.

15. *Categorical properties of algebraic structures*, by D. D. Cox and R. E. Prather, University of Denver.

16. *Computer use in teaching mathematics at Utah State University*, by J. D. Watson, Utah State University.

17. *Irreducible self-adjoint unbounded representations of* $*$*-algebras*, by W. M. Scruggs, University of Denver.

18. *An inverse diffraction technique*, by R. D. Mager, University of Denver.

19. *Estimation of parameters in acceleration models by use of ranks*, by Raymond Williams, Fort Lewis College.

20. *Five problems for freshmen and sophomores*, by F. M. Stein, Colorado State University.

21. *Some uses of the programmable calculator in the teaching of probability and statistics*, by J. H. Foster, Weber State College.

22. *Training for prospective and inservice teachers on the role of the hand-held calculator in the mathematics curriculum*, by Steven Kerr, Weber State College.

23. *Undergraduate mathematics education — a user's view*, by William Orth, USAF Academy.

24. *The Jacobian of a certain transformation*, by Hung C. Li, University of Southern Colorado.

25. *Evaluation of the zeta function by Pascal's triangle*, by C. A. Grimm, South Dakota School of Mines and Technology.

26. *A generalized Euclidean algorithm and n-ary continued fractions*, by J. A. Raab, Metropolitan State College.

27. *Converging factors for sequences of linear fractional transformations*, by John Gill, University of Southern Colorado.

28. *The Fibonacci pseudo-group and tri-diagonal matrices*, by H. R. P. Ferguson, Brigham Young University.

29. *Mathematics on the air: making a video tape*, by Corrinne Brass, Littleton, Colorado.

30. *Conversion to metric — a Canadian approach*, by Len Orman, University of Southern Colorado.

31. *Combinations via functional equations*, by Don Snow, Brigham Young University.

The business meeting was convened Saturday morning, May 1, at 8:00 A. M. by Professor Porter who presided. Thirty-eight members attended.

New officers for 1976–77 are: Chairman, Donald Bushnell, Ft. Lewis College; Chairman-Elect, Vern Nelson, Metropolitan State College; Second Vice Chairman, Robert Bitts, Arapahoe Community College; Co-Program Chairmen, Nancy Warren and Freida Holley, Metropolitan State College.

The officers of the new Intermountain Section are: Chairman, E. A. Davis, University of Utah; First Vice Chairman, Stephen Parker, Idaho State University; Second Vice Chairman, Leslie Glaser, University of Utah; and Secretary-Treasurer, Donald Snow, Brigham Young University. The splitting of the Rocky Mountain Section into the Intermountain Section and the Rocky Mountain Section is now official.

DAVID BALLEW, *Secretary-Treasurer*

APRIL MEETING OF THE TEXAS SECTION

The annual spring meeting of the Texas Section was held at Texas A & M University in College Station on April 2–3, 1976. There were 231 registered persons in attendance. This meeting was one of the Centennial Academic Assemblies of Texas A & M University.

Presenting invited addresses were: Professor H. L. Alder, President-Elect of the MAA, who spoke on "Recent Developments in the Theory of Partition Identities"; Professor P. R. Halmos who spoke on "Bounded Matrices"; and Professor Paul Erdös, Centennial Professor of Mathematics at Texas A & M, who spoke on "Combinatorial Problems in Elementary Geometry." Professor Charles Chui of Texas A & M arranged a special session on Approximation theory. Professor Larry Guseman of Texas A & M arranged a special session on Mathematical Pattern Recognition. Professor Howard Rolf, Baylor, organized a panel of representatives of business, industry, and government to discuss the needs of undergraduate programs. Professor Norman Fletcher of Mountain View College organized a special pedagogical session relating to Mathematics in the first two years.

Officers for 1976–77 are: Chairman: G. R. Blakley, Texas A & M University; First Vice Chairman: H. L. Rolf, Baylor University; Second Vice Chairman: R. G. Dean, Stephen F. Austin State University; Past Chairman: J. E. Hodge, Angelo State University; Level I Director: B. D. Langston, Tarrant County Jr. College, Northeast Campus; Level II Director: Archie Brock, East Texas State University; Director at Large: R. H. Cranford, Texas Eastern University; Secretary-Treasurer: J. C. Bradford, Abilene Christian University; MAA High School Contest: J. R. Boone, Texas A & M University.

Contributed papers were:

Two theorems characterizing increasing k-set contraction mappings, by K. L. Singh, Texas A & M University.

Banach lattices whose duals are l_1, by H. E. Lacey, University of Texas at Austin.

Closed subsets of compact groups, by A. Y. W. Lau, North Texas State University.

Orthogonal complements in $C(K)$ *spaces*, by R. G. Bilyeu, North Texas State University.

Further results on expansive mappings, by R. K. Williams, Southern Methodist University.

Absolute summability and stretchings of sequences, by T. A. Keagy, Wayland Baptist College.

On the summation of infinite series and the gamma function, by Russell Cowan, Lamar University.

Summability of sequences determined by certain difference properties, by D. F. Dawson, North Texas State University.

On the space $BSV^m[a,b]$, by F. N. Huggins, University of Texas at Arlington.

Endomorphism rings of totally projective Abelian groups, by R. K. O'Callaghan, University of Texas of the Permian Basin.

Critical maximal ideals, by R. W. Yeagy, Stephen F. Austin State University; H. S. Butts, Louisiana State University; N. H. Vaughn, North Texas State University.

Automata that recognize free monoids, by Tom Head, University of Alaska and Department of Mathematical Sciences, Rice University.

On consecutive primitive roots, by M. G. Monzingo, Southern Methodist University.

Monogenic near-rings, by Henry Heatherly, University of Southwestern Louisiana.

The coloring of graphs, by R. R. Korfhage, Serban Constantin, Judith Feld, and Fred Reagor, Southern Methodist University.

Characterizations of normed algebras with multiplicative norms, by R. S. Doran, Texas Christian University.

A unified method of solving second order linear partial differential equations with constant coefficients, by W. S. McCulley, Texas A & M University.

A new computer oriented development of linear algebra, by Robert Ducharme, University of Texas at San Antonio.

The rug cutting paradox, by John Lamb, Jr., East Texas State University.

Graffiti on teacher evaluations, by J. W. Strain, Midwestern State University.

Proofs by coloration, by George Berzsenyi, Lamar University.

The program consisted of the contributed papers, an invited address by Professor R.P. Boas, Governor's report by Elsie Muller, and the business session. D.V. Meyer, Central College, Pella, was elected as the Chairman-elect.

The program, arranged by Ellen Oliver, consisted of the following:

1. *Reciprocals of integers as repeating decimals*, by B.E. Gillam, Des Moines.
2. *Modular instruction in Mathematics at Central College*, by Leland Graber, Pella.
3. *The right constructs for programming*, by R.F. Keller, Ames.
4. *Staying alive as a mathematician while teaching at a two-year college*, by R.H. Lambertson, Boone.
5. *The programmable calculator as a teaching device*, by Don Benbow, Norlin Rober, Don McVay, Marshalltown.
6. *A mathematical analysis of the metropolitan government in Polk County, Iowa*, by A.F. Kleiner, Jr., Des Moines.
7. *Positive invariant closed sets for delay differential equations*, by Y.F. Chang, Ames.
8. *Find polynomial inverses of matrices of rational functions*, by James Bruening, LeMars.

Invited Address: *The Harmonic series and some of its applications*, by R.P. Boas, Past-President, MAA, Editor, American Mathematical Monthly, Northwestern University, Evanston, Illinois.

B.E. GILLAM, *Secretary-Treasurer*

APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The Sixtieth Annual Meeting of the Rocky Mountain Section of the MAA was held on the campus of Metropolitan State College, Denver, Colorado, April 29-30, 1977. There were 143 registrations, including Forest Fisch, Governor of the Section, Donald Bushnell, Chairman of the Section, and H.O. Pollak of Bell Laboratories. The Banquet address "On the Relationship between Applications of Mathematics and the Teaching of Mathematics" was delivered by Dr. Pollak. Invited addresses were: "A Problem-Solving Course in an Industrial Setting", by Armando Gingras, Metropolitan State College; "A Transition Course Between Calculus and Upper Division Mathematics", by Kent Goodrich, University of Colorado; and "On Shortest Connecting Networks", by H.O. Pollak.

There were twenty-six contributed papers:

1. *Those ubiquitous Catalan numbers*, by R.A. Gibbs, Fort Lewis College.
2. *On the conjugates of* $1-2|x|$, by Jan Mycielski, University of Colorado.
3. *Puzzles and graphs*, by Stan Gudder, Denver University.
4. *A radical theory for hemirings and an open problem*, by D.M. Olson, Cameron University.
5. *Certain matrix congruences and* $(\mathrm{mod}\, p^n)$, by T.P. Donovan, University of Colorado.
6. *The conjecture* $\pi(x+y) < \pi(x) + \pi(y)$, by David Ballew, South Dakota School of Mines and Technology.
7. *Ranked solutions of some matrix equations*, by Nick Mousouris, Humboldt State University.
8. *An extremum problem in two independent variables*, by R.S. Fisk, Colorado School of Mines.
9. *Some forgotten topics in analytic geometry*, by D.W. Hardy, Colorado State University.
10. *Sex differences in the learning of mathematics*, by Nancy Angle, University of Colorado at Denver.
11. *On residue class cryptography*, by Ron Whittekin, Metropolitan State College.
12. *Another construction of a strictly increasing, continuous, singular function on* $[0,1]$, by Paul O'Meara, University of Colorado at Denver.
13. *Computer arithmetic and abstract algebra*, by Keith Joseph, Metropolitan State College.
14. *Problem-solving techniques usable in secondary mathematics*, by Melfried Olson, University of Wyoming.
15. *An assessment of the processes used by community college students in mathematical problem-solving with suggestions for teaching*, by Beverly Gimmestad, Metropolitan State College.
16. *Emerging trends in secondary teacher education*, by Earl Hasz, Metropolitan State College.
17. *Some simple applications of statistics in industry*, by M.F. Flynn, Coors Container Company.
18. *Research suggestions in Lanchester combat theory*, by P.M. Ellis, Utah State University.
19. *Mathematical programming solutions to identification problems in mass spectroscopy*, by D.W. Fausett, Colorado School of Mines.

20. *On infinite games, the law of large numbers, and Baire category*, by Celestino Mendez, Metropolitan State College.

21. *A cooperative audio-visual project in mathematics and electrical engineering*, by David Ballew and Ronald Schmitz, South Dakota School of Mines and Technology.

22. *Research results with implications for the teaching of general math*, by Bill Juraschek, University of Colorado at Denver.

23. *Matrices can be useful (and fun)!!!*, by A.D. Porter, University of Wyoming.

24. *The curve parallel to a parabola is not a parabola*, by F.M. Stein, Colorado State University.

25. *Problems in teaching the history of mathematics*, by Burnett Meyer, University of Colorado.

26. *Bye-bye slide rule*, by L.M. Orman, University of Southern Colorado.

In addition to the above papers and addresses, exhibits were presented by the Association for Women in Mathematics, McGraw-Hill Book Company, Macmillan Publishing Company, Inc., Prindle, Weber & Schmidt, Inc., MAA, and Worth Publishers, Inc.

The business meeting was convened on Saturday morning, April 30, 1977, at 8:00 A.M. by Professor Bushnell who presided. Thirty-eight members attended. The minutes of the 1976 meeting were circulated and approved. The Treasurer's report for 1975 and an interim report for 1976 were circulated and approved.

Professor Dean Benson presented the Nominating Committee's slate of nominees; the following officers were elected: Vern Nelson, Metropolitan State College, Chairman; John Hodges, University of Colorado, Chairman-Elect; C.A. Grimm and Dale Rognlie, South Dakota School of Mines and Technology, Program Chairmen.

Christopher Bretherton, of the University of Colorado, and Peter Li, of the University of Southern Colorado, were given memberships to the MAA for their high placement on the Putnam Examination.

Professor Forest Fisch, University of Northern Colorado, gave the Governor's report from the Toronto and St. Louis meetings. Professor George Donovan, Metropolitan State College, Chairman of the High School Lectureship program, presented that committee's report. The Section contributed $100 to the continuance of that program. A written report from the High School Mathematics Contest had been submitted by Professor Robert Vunovich of the University of Southern Colorado.

Professor Vunovich was commended for his service to the Mathematics Contest.

The Section voted to start a newsletter edited by Professor David Ballew, South Dakota School of Mines and Technology.

The meeting concluded with a "rap session" conducted by Dr. H.O. Pollak, Past President of the MAA.

DAVID BALLEW, *Secretary-Treasurer*

APRIL MEETING OF THE WISCONSIN SECTION

The 1977 annual meeting of the Wisconsin Section of the MAA was held on the campus of the University of Wisconsin at Oshkosh on April 29 and 30, 1977. There were 135 registered attendants, including 112 MAA members and 23 students.

At the business meeting some revisions in the section by-laws were passed. The section voted to begin a yearly section newsletter. Based on the success of our first fall workshop on mathematical models, the Section voted to hold a second fall workshop.

The two principal speakers at the meeting were Professor Wolfgang Haken of the University of Illinois and Professor Leonard Gillman of the University of Texas. Professor Haken's talk was: "The Four-Color Problem"; Professor Gillman's talk was: "Choosing a Wife."

The contributed presentations were:

The human side of Gauss, by Merrill Barnaby, University of Wisconsin-LaCrosse.

Sylvester at Hopkins: an American centennial, by John Finch, Beloit College.

The misnamed Gaussian (normal) curve, by Carroll Rusch, University of Wisconsin-Superior.

The pocket calculator in higher education, by Eli Maor, University of Wisconsin-Eau Claire.

Returning theory of equations to the undergraduate curriculum, by C.W. Schelin, University of Wisconsin-LaCrosse.

Trees and maximal outplanar graphs, by A.E. Barkauskas, University of Wisconsin-LaCrosse.

APRIL MEETING OF ROCKY MOUNTAIN SECTION

The 61st Annual Meeting of the Rocky Mountain Section of the MAA was held April 28-29, 1978 at the South Dakota School of Mines and Technology, Chairman Vern Nelson, Metropolitan State College, presiding. There were 81 in attendance, including Professor Henry Alder, President of the MAA, Professor Erdös, and Professor Forest Fisch, Section Governor.

Dr. Alder presented the banquet address *Prime Generating Functions and Congruences* and an address on *Recommendations for the Preparation of High School Students for College Mathematics Courses.* Professor Erdös presented *Unconventional Problems in Number Theory and Problems on Consecutive Integers.*

There were two panel discussions:

Ways to Encourage More Women to take Mathematics Courses, Professor Ruth Struik, moderator; Professor B. Gimmestad, University of Colorado at Colorado Springs; Professor J. Hodges, Colorado U; Ms. Ann Pape, Denver.

Employment Problems of the Undergraduate B.A. and B.S. in Mathematics, Professor Dale Rognlie, moderator; Professor V. Nelson, Metro State; Professor D. Elliott, UNC; Professor A. Magnus, CSU; Professor David Ballew, SDSM&T.

The following papers were contributed:

The Approximation of π, Professor Arne Magnus, CSU.

Some Matrix Models in Accounting and Finance and some Exponential and Logarithmic Models in Economics, Professor C. G. Mendez, Metro State.

A Lemma Useful in Multivariate Analysis, Professor Hung C. Li, USC.

When are $a^2 + b^2$ *and* $a^2 + (b\text{-}na)^2$ *Perfect Squares,* Professor Gerald E. Bergum, SDSU.

The Probability of a Number Being Prime, Professor David Ballew, SDSM&T.

A Problem Concerning Polygons in Networks, Professor B. Jones, CU.

Mathematics Competition for High School Students, Professor D. W. Hardy, CSU.

Avoiding Variation of Parameters, Professor Roger Opp, SDSM&T.

Geometrical Modification of Continued Fractions, Professor John Gill, USC.

Color Symmetry and Group Theory, Professor R. L. Roth, CU.

A Geometrical Problem in Graph Theory, Professor Laurel Rogers, CU at Colorado Springs.

Boundary Value Problem at Resonance, Professor M. Martelli, CU (University of Florence).

Hand Held Calculator Supplement for Calculus Courses, Professor Lewis Huff, CSU.

In addition to the above papers and addresses, exhibits were presented by the MAA, The Association for Women in Mathematics, Worth Publishers, Inc.

The business meeting was convened on Saturday, April 29 by Professor Nelson who presided. Thirty-five members attended. New officers for 1978-79: Professor John Hodges, University of Colorado, Chairman; Professor Laurel Rogers, University of Colorado at Colorado Springs, Chairman Elect; Professor Allan Skillman, Casper College, Vice Chairman; Professor David Ballew, Secretary-Treasurer; Professor David Hector, Denver University, Program Chairman. Professor Duan Porter has been elected Governer of the Section.

It was noted that Professor Clarence Swanson had passed away. Professor Swanson has been a long-time member of the Section and had served as Chairman. Professor George Donovan of Metropolitan State was formally recognized for his long service as Chairman of the High School Lectureship Program. Professor Ballew reported on prospective by-law changes to increase registration fees at the meetings. Professor Forest Fisch gave the Governor's report and discussed the proposed new building for MAA headquarters. Christopher Bretherton of the University of Colorado was given a membership in MAA for his high placement on the Putnam examination.

The 1979 meeting will be in April at the University of Denver; the 1980 meeting will be March 29-30 at the University of Colorado.

David Ballew, *Secretary-Treasurer*

NEW JERSEY SECTION MEETING

The New Jersey Section of Mathematical Association of America held its annual spring meeting in cooperation with AMTNJ (Association of Mathematics Teachers of New Jersey) on Saturday, April 29, 1978 at Steinert High School, Trenton, New Jersey.

During the two morning sessions of the conference, attendees could choose among six simultaneous presentations. Professor Anneli Lax of Courant Institute of Mathematical Sciences spoke on Linear Transformations and Function Theory. MAA-sponsored short student talks were given by Mark Kleiman, winner of the International Mathematics Olympiad (*Generating Functions*); Brian Farrell, St. Peter's College (*A Model for Pedestrian Crossing*); Jose Fernandez, St. Peter's College (*A Model for Music Theory*); Betsey Taylor, Douglass College (*Surfaces of Crystals*); and Erick Schweber, Monmouth College (*Maxwell Functions*).

The afternoon session was a panel discussion inspired by the MAA publication *Recommendations for the Preparation of High School Students for College Mathematics Courses.* The panel was moderated by Section Chairman, B. Melvin Kiernan, and included Daniel Flegler, Waldwick High School; Dr. John K. Reckzeh, Jersey City State College; and Dr. Francis T. Rush, St. Peter's College. A lively discussion among MAA and AMTNJ members followed. Slides of the new national headquarters for MAA in Washington were shown, and the advantages of the purchase were discussed.

The November meeting of the New Jersey Section will take place on November 4, 1978 at St. Peter's College, Jersey City, New Jersey.

Jean Lane, *Secretary*

information about the overall program is available from AAAS Congressional Science Fellow Program, 1776 Massachusetts Avenue, N.W., Washington, C.D. 20036; telephone (202) 467-4475.

The AMS-MAA-SIAM Congressional Science Fellowship is to be awarded competitively to a mathematically trained person at the postdoctoral to mid-career level without regard to sex, race, or ethnic group. Selection will be made by a panel of the AMS-MAA-SIAM Joint Projects Committee for Mathematics with the cooperation and advice of the overall AAAS Program. *Applications should be sent to the Conference Board of the Mathematical Sciences, 1500 Massachusetts Avenue, N.W., Suite 457-458, Washington, D.C. 20005. The deadline for receipt of applications is 15 February 1980.* It is anticipated that the award will be made by around 1 April 1980.

In addition to demonstrating exceptional competence in some areas of the mathematical sciences, an applicant for the AMS-MAA-SIAM Congressional Science Fellowship should have a rather broad scientific and technical background and a strong interest in the uses of the mathematical and other sciences in the solution of societal problems. He or she would also be articulate, literate, flexible and able to work effectively with a wide variety of people. An application should state why the applicant wants to be a Congressional Science Fellow, should summarize his or her qualifications, and should be accompanied by a resume. Also, CBMS should receive by 15 February 1980 three letters from persons knowledgeable about the applicant's competence and suitability for the award.

MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

MAY MEETING OF THE INTERMOUNTAIN SECTION

The meeting was held at Idaho State University, Pocatello, on May 4-5, 1979. Chairman was Steven Parker, Idaho State University with committee members T. L. Williams, Idaho State University; S. S. Terry, Ricks College; S. J. Leon, Weber State College; and M. P. Windham, Utah State University. Sessions for presented papers were held Friday afternoon and Saturday morning. Titles and authors are listed below.

The invited guest of honor for the banquet was Dr. James D. Murray, Professor of Mathematics at Oxford University and currently a Visiting Professor at the Massachusetts Institute of Technology, University of Utah and California Institute of Technology. His invited address was: *How the Leopard Got Its Spots and Some Other Biological and Ecological Stories: A Mathematician's View.*

The invited address on Saturday morning was given by Dr. Keith Reed, University of Utah, titled: *The Mathematics of Chance—A Mathematics Course in a Liberal Education Program.*

JUNE MEETING OF THE PACIFIC NORTHWEST SECTION

The Annual Meeting of the Pacific Northwest Section was held June 15-16, 1979 at the University of British Columbia, Vancouver, B.C. in conjunction with meetings of the AMS and SIAM. Highlight of the meetings was Constance Reid's dinner talk, "The Answer to the Question Everyone Asks."

In addition to Mrs. Reid's talk, the following invited addresses were given

An Introduction to Cognitive Mapping, T. Pletcher, Vancouver Community College, Langara, B.C.

Furstenberg's Proof of Szemeredi's Theorem (or, Ergodic Theory Strikes Again), Douglas A. Lind, University of Washington, Seattle, Washington

Mathematics in the Open University, B. Coates, British Open University and J. Koumi, British Broadcasting Company

Cooperative Education in Mathematics, D. Dale Olesky, University of Victoria

Geometrical Theorems in Slides—An Innovative Approach for Teaching Geometry, Ved P. Madan, Red Deer College, Red Deer, Alberta

Compartmentalization of Mathematical Cognition, Hazel Jo Reed, The Evergreen State College, Olympia, Washington

A Probabilistic Approach to Studying Groups, Kenneth A. Ross, University of Oregon, Eugene, Oregon

The AMS program included two invited addresses.

One-Dimensional Transformations, Oscar E. Lanford III, University of California, San Diego, California

Some Geometrical Aspects of General Relativity, Theodore T. Frankel, University of California, San Diego, California

In addition there were Special Sessions on Mathematical Physics, Probability, Analysis, Representations and Ring Theory, and Algebra and Set Theory.

JOHN HERZOG, *Secretary-Treasurer*

APRIL MEETING OF ROCKY MOUNTAIN SECTION

The 62nd Annual Meeting of the Rocky Mountain Section was held April 27-28 at the University of Denver, Chairman John Hodges, University of Colorado presiding. There were 117 in attendance including Professor Peter Hilton, First Vice President of MAA and Professor Duane Porter, University of Wyoming, Section Governor.

Professor Hilton presented the banquet address on "Teaching Applied Mathematics." The theme of the 62nd meeting was Applied Mathematics. There was a panel discussion on this theme moderated by Professor Ottis Rechard of Denver University. Participants were: Professor Peter Hilton; Professor John Maybee, University of Colorado; and Herbert Greenberg, University of Denver.

The following papers were contributed:
Generalized Convexity and Applications, Stanley Gudder, DU
Measuring the Pointedness of a Convex Body, James Fickett, CU
How to Impress the Administration, David Ballew, SDSM
The Cubic Equation Revisited, F. Max Stein, CSU
Math Anxiety in College Students, Nancy Angle, UCD
A Mixed Integer Programming Model of Biological Systems, Wilbur Miller, USC
Problems with Potential, Roger Opp, SDSM
Computer Graphics in the Calculus Classroom, Austin R. Brown, CSM
The Mathematics Students Study and Need in Wyoming, Melfried Olson, UW
Confessions of an Applications Addict, William Ramaly, Ft. Lewis College
On Cooking a Turkey, Robert Fisk, CSM
A Stochastic Model for Resource Allocation, S. K. Sengupta, SDSM
Early History of the Mathematics Department at the University of Colorado, Burton Jones, CU
Doubly Stochastic Matrices in Accounting, Dean Lucas, Martin Marietta
Personalized Instruction Systems, Matthew Hassett, AFA & ASU

The Business Meeting was convened on Saturday, April 28 by Professor Hodges with forty-one in attendance. The officers for 1979-80 are: Chairperson, Professor Laurel Rogers, University of Colorado at Colorado Springs; Chairperson-elect, Professor William Ramaly, Ft. Lewis College; Vice Chairperson, Allan Skillman, Casper College; Program Chairperson, David Rearick, University of Colorado; Past Chairperson, Professor John Hodges, University of Colorado; Secretary-Treasurer, David Ballew, South Dakota School of Mines and Technology.

The minutes and treasurer's report were approved. The proposed changes in the Bylaws to increase registration fees and free more funds for the Annual Meeting passed. Professor Duane Porter presented the Governor's report, discussed the High School Mathematics Contest and the MAA Headquarters building. Professor Robert Vunovich presented the report on the High School Mathematics Contest. The Section voiced its appreciation to Professor Vunovich for his work on the contest. Professor Peter Hilton represented the MAA at the meeting and discussed future joint meetings of MAA and AMS. Professor Rebekka Struik of the University of Colorado presented the report on the High School Lectureship Program.

DAVID BALLEW, *Secretary-Treasurer*

SPRING MEETING OF THE NORTH CENTRAL SECTION

The spring meeting of the North Central Section was held at the College of St. Teresa in Winona, Minnesota, on April 27-28, 1979. Professor Joe Konhauser presided at the business meeting. Reports were given on the 1979 Summer Seminar on Statistics at the University of Minnesota, Duluth and on the MAA High School Contest results in Minnesota and North Dakota. Recognition was given to the high scorers on the Putnam Examination from the North Central Section.

The membership elected Steve Hilding, Gustavus Adolphus College President-Elect, Roger Avelsgaard, Bemidji State University, Member at Large on Executive Committee, and re-elected Charles Heuer, Concordia College, Moorhead as Secretary-Treasurer.

An invited address entitled *The Ideal Situation for Continuous Functions*, was given on Friday evening by Professor Don Mattson of Moorhead State University. On Saturday an invited address was given by Professor R. P. Boas of Northwestern University. His talk was entitled *The Harmonic Series*.

The following contributed papers were given:
A Minimum Competency Requirement in Mathematics for College Students, Francis Hatfield, Mankato State University
Matrix Examples for an Abstract Algebra Course, John Schue, Macalester College
A Converse Problem in Differential Equations, Warren Shreve, North Dakota State University
Elementary Integrals, Hubert Walczak, College of St. Thomas
Dissipation in Lotka-Volterra Systems, Tom Haigh, St. Johns University
Some Geometrical Aspects of the Fundamental Theorem of Calculus, Helen Skala, College of St. Teresa
The Greatest Integer Function and the Golden Ratio, Gerald Bergum (presented by Kenneth Yocom), South Dakota State University
A Generalization of Cramer's Rule, Sylvan Burgstahler, University of Minnesota, Duluth

CHARLES V. HEUER, *Secretary-Treasurer*

APRIL MEETING OF THE ALLEGHENY MOUNTAIN SECTION

The annual spring meeting of the Allegheny Mountain Section of the MAA was held at Westminster College, New Wilmington, Pennsylvania, on April 27-28, 1979. There were approximately 75 members of the Association and 35 students present from the member schools. The host was Barbara Faires, Mathematics Department of Westminster College.

Friday evening, April 27, 1979, was highlighted by two invited presentations and a contributed papers session for both students and faculty. The invited speakers were Anthony LoBello, Allegheny College, and Fred Roberts, Rutgers University. The title of Professor LoBello's presentation was *Observations on the History of Mathematics on the Occasion of the Einstein Centennial*. The invited talk by Professor Roberts was *Interval Graphs, Traffic Phasing, Ship Building, and Mobile Radio Frequency*.

There were three invited talks Saturday morning. Caulton Irwin, West Virginia University, presented a paper entitled *Mathematical Aspects of Energy Modeling*. T. R. Nealigh, Westminster College, presented a paper, designed primarily for students, entitled *The Trials and Tribulations of the*

ROCKY MOUNTAIN SECTION MEETING

The 63rd Annual Meeting of the Rocky Mountain Section of MAA was held March 28-29, 1980, on the campus of the University of Colorado in Boulder with 122 members of MAA in attendance. Professor Dorothy Bernstein, President of the Mathematical Association of America, gave the annual banquet address, *A Differential Equation of Literary Criticism.*

The program included two panel discussions:

Transferability of College Credits, moderated by Professor Allan Skillman of Casper College and including as panelists, Professor Ardel Boes of Colorado School of Mines; Professor Corrine Brase of Arapahoe Community College; Professor Burnett Meyer of the University of Colorado at Boulder; and Professor Duane Porter of the University of Wyoming.

Nontraditional Uses of Computers, moderated by Professor Laurel Rodgers and including as panelists Professor Dorothy Bernstein, Brown University; Dr. Bertram Herzog, University of Colorado at Boulder; and Professor Austin Brown, Jr., Colorado School of Mines.

There were fourteen 20-minute papers contributed.

On the Teaching of Projective Geometry, Professor Arne Magnus, CSU
Second-chance Examinations: an Evaluation, Professor David Ballew, SDSM&T
Computer Graphics in the Calculus Classroom, Professor Austin B. Brown, Jr. and Professor Robert S. Fisk, CSM
Some Parallels Between Digital Computers and the Human Mind, Dr. Ira Becker, Ball Brothers
Optimal Channel Assignment and Chromatic Graph Theory, William K. Hale, National Telecommunications and Information Administration, U.S. Department of Commerce
A Conjecture on Consecutive Composites, II, Professor C. Albert Grimm, SDSM&T
Uses of Woodbury's Formula, Professor Dale Rognlie, SDSM&T
An Effective Interactive Introduction to Mathematics for Elementary Teachers (and Others), Professor Earl Hasz, MSC
Cooperative Education in Mathematics, Professor W. E. Brumley, CSU
The Hungarian Magic Cube — A Search for a Solution, Professor Les Shader, UW
Modules in the Mathematics Classroom, Professor Joan R. Hundhausen, CSM
Which is the Most Beautiful Polygon? Professor John H. Hodges, UCB
Four Mutually Tangent Circles — the Second Circle Varying, Professor Hung C. Li, USC
A Locus Determined by the Three Real Roots of the Reduced Cubic Equation, Professor F. Max Stein, CSU

Professor Laurel Rodgers of the University of Colorado at Colorado Springs, Chairman of the Section, presided at the Annual Business Meeting. The new officers for 1980-81 are

Chairperson: Professor *William Ramaley,* Ft. Lewis College
Chairperson-Elect: Professor *John Gill,* University of Southern Colorado
Vice-Chairperson: Professor *Aubrey Owen,* Community College of Denver
Program Chairperson: Professor *Stephen Shiffman,* Colorado College

Professor *Rebekka Struik* of the University of Colorado reported on the Section's High School Lectureship Program, and Professor *Bernstein* discussed such programs at the national level. Professor *Bernstein* also encouraged MAA membership promotion for those students who plan on college careers. Professor *Duane Porter*'s Governor's Report was read.

DAVID BALLEW, *Secretary*

MARCH MEETING OF THE WISCONSIN SECTION

The Spring Meeting of the Wisconsin Section of the MAA was held at the University of Wisconsin at Milwaukee, March 28-29, 1980, with 83 MAA members and 17 non-members registering. Chairman Norbert J. Kuenzi (US-Oshkosh) presided. The program had been drawn up by Chairman-Elect Anthony E. Barkauskas (UW-LaCrosse) and local arrangements were handled by Robert Hall (UW-Milwaukee).

The invited addresses were:

Modeling Within Mathematics, Peter Hilton (Battelle Research Institute)
Hadamard, Hotelling, Harwit: A New Application of Some Old Mathematics, Neal J. A. Sloane (Bell Telephone Laboratories). Unfortunately, Professor Sloane was too ill to attend and present his address.

The following additional talks were given:

Velocity Changes and Weak Mixing in Ergodic Flows, David Yurchak (UW-Milwaukee)
The Missionaries and Cannibals Learn Graph Theory, Timothy V. Fossum (UW-Parkside)
The Programmable Calculator and the Bored High School Student, Florence N. Greville
Developing Number Concepts in Young Children, Kathleen L. Briggs (UW-Milwaukee)
Some Results in Reducibility in Ordinary Differential Equations, M. M. Subramaniam (UW Center-West Bend)
Auto-homeomorphisms of a 2-manifold Induced by 'Twists', Norman Frisch (UW-Oshkosh)
Two Variable Power Series--A Topic for Undergraduates, Jonathan Kane (UW-Madison)
Mathematics in the Social and Biological Sciences: Applications from Project UMAP, Philip D. Straffin (Beloit College)
Mathematical Models of Energy Economics, William Holahan (UW-Milwaukee Economics Department)
Folds, Pleats, and Halos—A Slide Presentation, Walt Tape (UW-Eau Claire)
The Inverse Prediction Problem in Regression, Paul J. Campbell (Beloit College)
A Description of MATH TIPS as Used in a Precalculus Course, Frederic Tufte (UW-Platteville)
Torsion-Theoretic Generalization of Semisimple Modules, William Lau (UW-Milwaukee)
Individualizing the Intermediate Algebra Course—A Report, J. D. Wine (UW-LaCrosse)
The Logarithmic Spiral—A Slide Presentation, Eli Maor (UW-Eau Claire)
Conservative Graphs, David W. Bange (UW-LaCrosse)

SPRING MEETING OF THE MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA SECTION

The spring meeting of the Maryland-District of Columbia-Virginia Section of MAA was held on Saturday, April 11, 1981 at the College of William and Mary, Williamsburg, Virginia. Eighty-two registrants attended the meeting. *Marcia Sward*, Associate Executive Director of the MAA, gave the invited address. Her topic was "Death on the Highways: Can Mathematics Help?"

Section Chairman *John Smith* presided at the business meeting. *Ernest Mabrey* (Department of Energy) was elected Vice-Chairman for Membership. Other section officers for the coming year are: Chairman, *John Schmeelk* (Virginia Commonwealth University); Immediate Past Chairman, *John Smith* (George Mason University); Vice-Chairman for Programs, *Patrick Hayes* (Federal Reserve); Treasurer, *Arthur Charlesworth* (University of Richmond); AHSME Exam Coordinator, *Edward Bender* (J. Sargeant Reynolds Community College); Newsletter Editor, *Howard Penn* (U. S. Naval Academy); Governor, *Ronald Davis* (Northern Virginia Community College); and Secretary, *Robert Hanson* (James Madison University). Reports on the Annual High School Mathematics Contest, the national newsletter, summer workshops, and future meetings were given. Hearty thanks were expressed to *John Smith* for his leadership as section chairman for the past several years.

The following talks were presented: "Teaching Technical Mathematics," *Ronald Davis*, Northern Virginia Community College; "Computer Use in the Classroom," *James Newsom*, Tidewater Community College; "Teaching Remedial Mathematics," *Betty Weissbecker*, J. Sargeant Reynolds Community College; "Computer Assisted Instruction," *Edward Huff*, Northern Virginia Community College; "Math Labs in Two Year Colleges: An Informal Discussion," *John Massey*, Tidewater Community College; "Least-Cost Livestock Feeding Using Ethanol Byproducts," *Christopher Reed*, TRW, Inc., and *Douglas Samuelson*, Federal Aviation Administration; "An Algorithm to Route Jets for Transporting Checks Among Federal Reserve Banks," *David McCarthy*, Bureau of Labor Statistics and *Y. C. Park*, Naval Sealift Command; "A Model for Scheduling the Production of Currency," *Hosain Ali Mahan*, The George Washington University; "The Applied Mathematics Laboratory at Towson State University," *John Morrison* and *Martha Siegel*, Towson State University; "Recommendations of the CUPM Subpanel on Computing," *George Engel*, Christopher Newport College; "Recommendations of the CUPM Subpanel on Modeling," *Ralph Disney*, Virginia Polytechnic Institute and State University; "Teaching Computer Graphics," *Caren Diefenderfer*, Hollins College; "A Solution for Rubik's Cube," *Howard Penn*, U.S. Naval Academy; "Group Norms and the Grading of Chronologies," *William Wardlow*, U.S. Naval Academy; "On Sets with Distinct Subset Sums," *Paul Stockmeyer*, College of William and Mary; "Discrete Optimization Using Incremental Analysis," *John Drew* and *Margaret Schaefer*, College of William and Mary; "MAA Placement Testing," *George Lowerre*, Northern Virginia Community College (Woodbridge); "Current and Future MAA Activities: An Informal Discussion," *Marcia Sward*, MAA; "Findings of the MAA Committee on Improving Remediation Efforts in the Colleges," *Eleanor Green Jones*, Norfolk State University; "Production and Use of Video Tapes in Developmental Mathematics," *Calude Moore*, Danville Community College.

APRIL MEETING OF THE INDIANA SECTION

The spring meeting of the Indiana Section of the MAA was held at Indiana University-Purdue University at Indianapolis on Saturday, April 11, 1981, with 59 members present. The Indiana Small College Math Competition was held in conjunction with the meeting.

The following papers were presented at the meeting: "Girard Triangles," *Rodney T. Hood*, Franklin College; "A tiling of the Plane with Triangles," *Paul T. Mielke*, Wabash College; "Counting Matrices of Given Rank," *Ralph P. Grimaldi*, Rose-Hulman Institute of Technology; "Let Us Teach Concepts," *Herman Rubin*, Purdue University; "Differential Games," *Leonard D. Berkovitz*, Purdue University; "Computer Graphics in Teaching Mathematics," *Gerald J. Porter*, University of Pennsylvania; "The Mathematics Curriculum in the 80's," *William A. Marion*, Valparaiso University.

At the business meeting led by Chairman *Duane Deal*, the chairman of the nominating committee *Paul Mielke* presented the following slate of officers for 1981-82, which was unanimously approved: Chairman, *M. Mundt*, Valparaiso University; Vice-Chairman, *C. Cowen*, Purdue University; Secretary-Treasurer, *R. Patterson*, Indiana University-Purdue University at Indianapolis. *Meyer Jerison* of Purdue University was elected Governor. Memberships in the MAA were awarded to *Michael Call*, Rose-Hulman Institute of Technology, and *David Dwyer*, Purdue University, in recognition of their performance on the Putnam examination.

MAY MEETING OF THE ROCKY MOUNTAIN SECTION

The sixty-fourth annual meeting of the Rocky Mountain Section of MAA was held on May 1-2, 1981 on the campus of Colorado College, Colorado Springs, Colorado, with 120 members in attendance. Professor *Leonard Gillman*, Treasurer of the MAA, gave the annual banquet address, "We Can't Teach Our Way out of a Paper Bag," and the invited address, "Optimal Strategies in Sports."

The program included three panel discussions: "Employment Opportunities for Students," moderated by Professor *John Hodges* of the University of Colorado; speakers were *Robert Frost*, NOAA; Professor *Kent Goodrich*, University of Colorado; Professor *John Sopka*, Ft. Lewis College; and Professor *S.W. Wilson*, University of Colorado. "How Do Your Start a Computer Science Program?" moderated by *W.S. Dorn* of Denver University. The speakers included Professor *James Davis* of Mesa College; Professor *Charlotte Murphy*, Metro State College; and Professor *Ron Prather* of Denver University. "How Do We Encourage the Exceptional Student?" moderated by Professor *David Ballew* of South Dakota School of Mines and Technology. The participants were Professor *Stephen Bronn*, University of Southern Colorado and Professor *Paul Perlmutter* of Colorado College.

Twenty-three papers were contributed, eight of them by undergraduate students. "Some Unorthodox Thoughts About Quantum Mechanics," by *Jan Mycielski*, University of Colorado, Boulder; "Can a Mathematician Find Happiness in the Computing Field?" by *William Marion*, Valparaiso University; "Eigenvalues

of Tridiagonal Matrices," by *Dale M. Rognlie*, South Dakota School of Mines and Technology; "A Critique of Axiomatic Systems," by *George S. Donovan*, Metro State College; "Unsolved Mathematical Problems Arising from Spectrum Management Issues," by *William K. Hale*, US Department of Commerce, National Telecommunications and Information Administration, Institute for Telecommunications Sciences, Boulder; "A Recent Result in Continued Fraction Theory," by *John P. Gill*, University of Southern Colorado; "Some Elementary Models in Business and Economics Part I," by *C. G. Mendez*, Metro State College; "It's not What *You Say*, It's What *They Do* (A Brief Report on a Modularized Approach to Calculus)," by *William R. Astle* and *Thomas E. Kelley*, Colorado School of Mines; "Extremals of $\int$Ldt and of $\int$F(L)dt." by *Roger Opp* and *Ron Weger*, South Dakota School of Mines and Technology; "An Elementary Proof That $(1 + 1/n)^n$ Tends to e," by *Lee Badger*, Ft. Lewis College; "The Evolution of Calculus Teaching and Texts," by *Wolfgang Thron*, University of Colorado, Boulder; "Pancakes and Permutations," by *Leslie E. Shader*, University of Wyoming; "The Evolution of a Generalization," by *Richard Gibbs*, Ft. Lewis College; "The Pi Theorem of Buckingham--Dimensional Analysis and Some Examples," by *Robert S. Fisk*, Colorado School of Mines; "How to Cut a Triangle," by *Alexander Soifer*, University of Colorado, Colorado Springs.

Papers presented by undergraduate students: "Analysis of a Trisection of the Angle Algorithm" by *Leon Nelson*, South Dakota School of Mines and Technology; "Robots--Are They Here to Stay?" by *Janet Potts*, South Dakota School of Mines and Technology; "The Fibonacci Sequence and Evolution," by *Terri Bush*, Ft. Lewis College; "An Application of Game Theory in Taking Tests," by *Dean Mogck*, South Dakota School of Mines and Technology; "The Theory of Evolution Applied to Programming Language," by *Colleen Quatier*, South Dakota School of Mines and Technology; "Computer Graphics of Parametric Equations," by *Gary Ricard*, South Dakota School of Mines and Technology; "Monte Carlo Methods for Testing Large Primes," by *Anthony Wakely*, South Dakota School of Mines and Technology; "The Relations of Differentiable Functions and the Power Series," by *Brian Bunsness*, South Dakota School of Mines and Technology.

Professor *William Ramaley* of Ft. Lewis College, Chairperson of the Section, presided at the annual business meeting. The new officers elected for 1981-82 are as follows: Chairperson, Prof. *John Gill*, University of Southern Colorado; Chairperson-elect, Prof. *George Donovan*, Metro State College; Vice-Chairperson, Prof. *Aubrey Owen*, Community College of Denver; Program Chairman, Prof. *Gale Nash*, Western State College; Secretary-Treasurer, Prof. *David Ballew*, SD School of Mines and Technology. The section heard the minutes, treasurer's report, Prof. *Duane Porter's* Governor's report, and a report from Professor *Gillman* on "super-section" meetings. The section recognized Prof. *Dean C. Benson* of SD School of Mines and Technology and Prof. *Ray Hanna* of the University of Wyoming on the occasion of their retirement and commended them for their long service to the Section.

SPRING 1981 MEETING OF THE NORTH CENTRAL SECTION

The spring meeting of the North Central Section was held at Mankato State University, Mankato, Minnesota, on May 1 and 2. Section President, *Stephen Hilding* of Gustavus Adolphus College, presided at the business meeting. Items of business included the presentation of a citation by Governor *Dale Varbert* to *A. Wayne Roberts* of Macalester College and to *John Sitcomb* of the University of North Dakota for their long service as coordinators of the MAA high school exams in Minnesota and North Dakota. the 21 students from the North Central Section scoring in the top 500 on the 1980 Putnam Exam were also honored. New officers elected were: *Sabra Anderson*, University of Minnesota, Duluth, Chairperson-elect; *Alan Kirch*, Macalester College, Secretary-treasurer; and *James Rue*, University of North Dakota, member-at-large.

Invited addresses were given by *Steve Galovich* of Carleton College, "Arithmetic in Characteristic p" and by Chairman-elect of the MAA, *R. D. Anderson*, who spoke on "Algorithmically Defined Functions."

Contributed papers were: "The Arithmetic of Convex Polytopes" by *Walter Sizer* of Moorhead State University; "Probability Distribution Fitting for Large Loss Insurance Data" by *Bruce Binzel, Kari Johnson, Jennifer Smith, Karen Thompson, Carol Veum*, St. Olaf College (students); "Parameter Estimation in Linear Combinations of Exponentials" by *Dale Larson, Kathryn Lenz, Barry Mason, Richard Selby*, St. Olaf College (students); "A Note on Factoring Differential Equations" by *Clayton Knoshaug*, Bemidji State University; "Field Structures on the Natural Numbers" by *Jon Shreve*, Carleton College (student); "An Interdisciplinary Course Based on Godel, Escher, Bach" by *H. B. Coonce*, Mankato State University; "Squares and Non-Squares in Finite Fields" by *Roger Avelsgaard*, Bemidji State University; "Graphing Non-Real Branches of Garden Variety Equations" by *Ron Rietz*, Gustavus Adolphus College.

ALLEGHENY MOUNTAIN SECTION MEETING

The Allegheny Mountain Section of MAA met May 15-16, 1981 at Duquesne University, Pittsburgh, PA. Invited lectures were given by *George E. Andrews* of Penn State University, "More on Ramanujan's Lost Notebook," and by *Alfred B. Willcox*, Executive Director, MAA, "Some Bridges to and From Mathematics or There is a Mathematician Loose in the Supermarket."

There was a panel discussion of "Job Opportunities in Mathematics." Another panel consisting of *Donald Platte*, Mercyhurst College, Coordinator; *James E. Allison*, Bethany College; *Charles Cable*, Allegheny College; and *Richard MacCamy*, Carnegie-Mellon University, discussed "Trends in College Mathmatics Curricula."

Four undergraduate students gave talks: *Dave Plottell* of Allegheny College on "Desarguesian and Non-Desarguesian Projective Planes;" *Don Kocher* of Allegheny College on "The Projective Plane and Incidence Matrices" *Theresa Presecan McChesney* of Westminster College on "Optimal Control;" and *Dan Duh* of Allegheny College on "Optimal Harvesting and Dynamic Programming."

Contributed papers by faculty members were "Micro-computers as an Instructional Aid in Mathematics," *Roy Meyers* and *Richard Reynolds*, Penn State University at New Kensington and McKeesport; "Improving the Learning Strategies of the Non-Major," *Mary Kay Hudspeth*, Penn State University at University Park; "Perturbation Solutions of Non-Linear Difference Equations," *Richard F. Melka*, University of Pittsburgh at Bradford; "On the Asymptotic Behavior of Solutions of Ordinary Differential Equations," *M. M. Subramaniam*, Penn State University, Delaware County Campus.