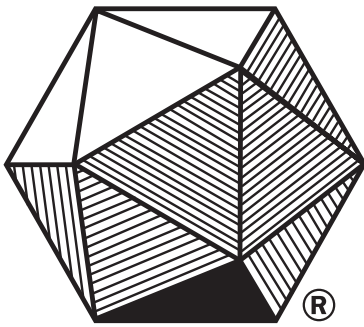


Annual Meeting  
of the  
***Pacific Northwest Section***  
of the

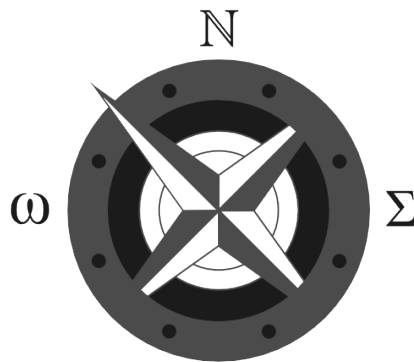


**MAA**

**MATHEMATICAL ASSOCIATION OF AMERICA**

together with the

**8th Annual  
Northwest  
Undergraduate  
Mathematics  
Symposium**



**NUMS**

Oregon State University  
Corvallis, Oregon

*April 1–2, 2016*



Annual Meeting  
of the  
Pacific Northwest Section  
of the  
Mathematical Association of America  
and the  
Northwest Undergraduate Mathematics Symposium

Sponsored & Hosted by

*Oregon State University  
Department of Mathematics  
College of Science  
Corvallis, Oregon*

*April 1–2, 2016*

Friday, April 1

8:00	<p><b>Project NExT Meeting</b> <i>Willamette Seminar Rooms, Valley Library 3rd floor</i></p>	
2:45	<p><b>Mini-course Registration</b> <i>Willamette Seminar Rooms, Valley Library 3rd floor</i></p>	
3:00	<p><b>Mini-course: David Pengelley</b> <i>Teaching and Learning Mathematics from Primary Historical Sources [1]</i> <i>Willamette Seminar Room East</i></p>	<p><b>Mini-course: Stuart Boersma &amp; Cheryl Beaver</b> <i>Codes and Ciphers [2]</i> <i>Willamette Seminar Room West</i></p>
5:30	<p><b>Student Activity: Math Wrangle (pizza provided)</b></p>	
7:30	<p><b>Registration</b> <i>LINC Lobby</i></p>	<p><i>Willamette Seminar Rooms Valley Library 3rd floor</i></p>
8:00	<p><b>Public Invited Lecture: David Pengelley</b> <i>How efficiently can one untangle a double-twist? Waving is believing! [3]</i> <i>LINC 128</i></p>	
9:00	<p><b>Reception</b> <i>LINC Lobby</i></p>	
10:00		

## Saturday, April 2

8:00		<b>Reception</b> <i>LINC Lobby</i>
9:00	<b>Registration</b> <i>LINC Lobby</i>	<b>Introduction and Welcome: Sastry Pantula</b> Dean, College of Science, Oregon State University
9:10		<b>Invited Lecture: Stuart Boersma</b> <i>Cryptologic Tidbits [4]</i> <i>LINC 128</i>
10:05		
10:15	<b>Registration, Book Sales &amp; Exhibit</b>  <i>STAG 270</i>	<b>Contributed Talks</b> <i>STAG 160–163 &amp; 260–263</i>
12:15		
12:20		<b>Lunch (provided)</b> <i>Horizon Room, Memorial Union</i>
12:30		<b>Panel Discussion: Jobs and Grad School [7]</b> <i>Horizon Room, Memorial Union</i>
1:40		
1:45		<b>Contributed Talks</b> <i>STAG 160–163 &amp; 260–263</i>
3:25		<b>Afternoon Coffee Break &amp; Poster Session</b> <i>STAG 270</i>
4:05		
4:15		<b>Invited Lecture: Brian Hopkins</b> <i>The Symmetric Group and Fair Division: Does Knowledge Matter? [6]</i> <i>LINC 128</i>
5:10	<b>Business Meeting</b> <i>LINC 128</i>	<b>Panel Discussion: Math Clubs [8]</b> <i>LINC 368</i> <span style="background-color: red; color: white; padding: 2px;">Updated Room Number</span>
5:40		<b>Social Hour</b> <i>Horizon Room Lobby, Memorial Union</i>
6:45		<b>Banquet Dinner - Section Awards</b> <i>Horizon Room Lobby, Memorial Union</i>
7:45		<b>Invited Lecture: Karen Saxe</b> <i>Measuring Inequality [5]</i> <i>Horizon Room Lobby, Memorial Union</i>
9:00		

## Program of Special Sessions and Contributed Papers

The program of contributed papers appears on the following pages. In some cases, titles or other information are abbreviated for reasons of space; please see the full abstract for more information.

A dagger (†) indicates which contributor(s) will present when multiple contributors are listed and fewer are presenting the work. An asterisk (\*) indicates the contributor is an undergraduate student. Double asterisks (\*\*) indicate the contributor is a graduate student.

### Sessions and Organizers

- *Algebra and Number Theory:*  
Tom McKenzie and Jeff Wand, Gonzaga University
- *Research in Undergraduate Mathematics Education:*  
Elise Lockwood, Oregon State University
- *Teaching Tricks, Techniques and Discoveries:*  
Jeffrey Stuart, Pacific Lutheran University
- *Thinking Outside the Circle: Alternate Outreach:*  
Annie Raymond and Matt Junge, University of Washington
- *Junior Faculty Research:*  
Katharine Shultis and Kate Kearney, Gonzaga University
- *General Contributed Papers:*  
Tevian Dray, Oregon State University
- *NUMS Student Presentations and Poster Session:*  
Nathan Gibson, Oregon State University
  - *Topology*
  - *ODEs and Dynamical Systems*
  - *Combinatorics*
  - *Mathematics Applications*
  - *Computing and Statistics*
  - *Algebra and Geometry*
  - *History of Mathematics and Math Ed*
  - *Poster Session*

Please contact the session organizers with any questions about a session.

Moderators: Please start each talk on time, but **not** early. Meeting participants often move between sessions and will want to be there when the talks are scheduled to begin.

<b>Saturday Morning Concurrent Sessions</b>		
	<b>Junior Faculty Research</b> <i>Chairs: Katharine Shultis &amp; Kate Kearney</i>	<b>General Papers I</b> <i>Chair: Tom Dick</i>
	<i>STAG 160</i>	<i>STAG 161</i>
10:15	<i>Geometric Invariants of Knots</i> [64] <b>Christian Millichap, Linfield College<sup>†</sup></b>	<i>Infinity (Still) Blows My Mind</i> [65] <b>Aaron Montgomery, Central Washington University<sup>†</sup></b> <b>Doug DePrekel, Central Washington University</b>
10:35	<i>An Obstruction to Knots Bounding Mobius Bands in <math>B^4</math></i> [49] <b>Kate Kearney, Gonzaga University<sup>†</sup></b>	<i>An Active STEM-Prep Curriculum</i> [79] <b>Francisco Savina, University of Texas at Austin</b>
10:55	<i>Introduction to permutation patterns</i> [46] <b>Masaki Ikeda, Western Oregon University<sup>†</sup></b>	<i>If You Must Gamble: Minimizing Expected Loss</i> [37] <b>Curtis Feist, Southern Oregon University<sup>†</sup></b> <b>Jake Scott, Southern Oregon University</b>
11:15	<i>The Difficulty of Classifying Decomposable Torsion-Free Abelian Groups</i> [77] <b>Kyle Riggs, Eastern Washington University<sup>†</sup></b>	<i>First to Toss <math>N</math> Heads Wins</i> [39] <b>Chris Hallstrom, University of Portland<sup>†</sup></b>
11:35	<i>Dispersion reduction schemes for the wave equations</i> [19] <b>Yajun An, Pacific Lutheran University<sup>†</sup></b>	<i>Propagation of transient behaviour in linear flocking models</i> [40] <b>David Hammond, Oregon Institute of Technology - Wilsonville<sup>†</sup></b>
11:55	<i>Using Eigenvalues to Investigate Numerical Oscillations</i> [41] <b>Corban Harwood, George Fox University<sup>†</sup></b>	<i>Hypothesis Testing with Person-Time Data</i> [88] <b>Terri Torres, Oregon Institute of Technology<sup>†</sup></b>

<b>Saturday Morning Concurrent Sessions</b>		
	<b>NUMS I: Mathematics Applications</b>  <i>Chair: TBD</i>  <b>STAG 162</b>	<b>Thinking Outside the Circle: Alternate Outreach</b>  <i>Chairs: Annie Raymond &amp; Matt Junge &amp;</i>  <b>STAG 163</b>
10:15	<i>The process of representing food webs as interval competition graphs</i> [89]  <b>Kayla Vincent, Western Oregon University*†</b>	<i>Mathday at University of Washington</i> [67]  <b>James Morrow, University of Washington†</b>
10:35	<i>Insects and Spirals</i> [25]  <b>Kaylee Church, Western Oregon University*†</b>	<i>Girls' Day or Mdchen-Zukunftstag</i> [75]  <b>Annie Raymond, University of Washington†</b>
10:55	<i>Eigensystems of Electrical Networks</i> [24]  <b>Nikki Carter, University of Portland*†</b>	<i>The Diversity of the national Math Circle movement</i> [94]  <b>Brandy Wieggers, Central Washington University, National Association of Math Circles†</b>
11:15	<i>Starved? Let's Solve That Math Hunger!</i> [69]  <b>TJ Norton, Klamath Community College*</b>	
11:35	<i>Theatrical! Practical! Mathematical!</i> [47]  <b>Thad Joachim, Klamath Community College*†</b>	<i>Inside the box: college in prison</i> [48]  <b>Matthew Junge, University of Washington**†</b>
11:55	<i>Fractals in Geology: Measuring Intricate Forms</i> [53]  <b>Will Kugler, Klamath Community College*†</b> <b>Cancelled</b> <i>The Math Behind The Rubiks Cube</i>  [44] <b>Garrison Iams, Klamath Community College*†</b> <b>Moved here from Session: NUMS VII—History/Math Ed</b>	

<b>Saturday Morning Concurrent Sessions</b>		
	<b>Algebra and Number Theory I</b> <i>Chairs: Tom McKenzie &amp; Jeff Wand</i>	<b>NUMS II: Combinatorics</b> <i>Chair: TBD</i>
	<i>STAG 260</i>	<i>STAG 261</i>
10:15	<i>A Game of 1s and 2s: Constructing level 2 Demazure Filtrations of level 1 Demazure modules [91]</i> <b>Jeffrey Wand, Gonzaga University<sup>†</sup></b>	
10:35	<i>Visualizing p-adic numbers [29]</i> <b>Dibyajyoti Deb, Oregon Institute of Technology<sup>†</sup></b>	
10:55	<i>Hamiltonian Properties of Toroidal Zero Divisor Graphs [72]</i> <b>Shannon Overbay, Gonzaga University<sup>†</sup></b>	
11:15	<i>Decomposable Cyclically Presented Groups and Shift Dynamics [63]</i> <b>Kirk McDermott, Oregon State University<sup>**†</sup></b>	<i>Examining Ramsey Numbers [36]</i> <b>Amanda Evola, Western Oregon University<sup>*†</sup></b>
11:35	<i>The subregular part of Lusztig's asymptotic Hecke algebra [96]</i> <b>Tianyuan Xu, University of Oregon<sup>**†</sup></b>	<i>Math and Sudoku: Exploring Sudoku boards through graph theory, group theory, and combinatorics [70]</i> <b>Kyle Oddson, Portland State University<sup>*†</sup></b>
11:55	<i>G-Sets and Sequences Associated with nth Order Linear Recurrences Modulo Primes [74]</i> <b>Robert Ray, Gonzaga University<sup>†</sup></b>	<i>On the Turan-number of forests [50]</i> <b>Omid Khormali, University of Montana<sup>**†</sup></b> <b>Cancelled</b>



<b>Saturday Morning Concurrent Sessions</b>		
	<b>Research in Undergraduate Mathematics Education I</b> <i>Chair: Elise Lockwood</i> STAG 262	<b>NUMS III: Computational/Statistical</b>  <i>Chair: TBD</i> STAG 263
10:15	<i>Improving algebra skills of university students through participation in academic service-learning</i> [97]  <b>Ekaterina (Katya) Yurasovskaya, Seattle University<sup>†</sup></b>	<i>Algorithmic Variants of QR</i> [73]  <b>Sally Peck, Western Oregon University<sup>*†</sup></b>
10:35	<i>Reasoning with "dx" in the integral</i> [33]  <b>Robert Ely, University of Idaho<sup>†</sup></b>	<i>Perturbing Equilibria of the Three-Body Problem</i> [83]  <b>Joseph Stauss, Gonzaga University<sup>*†</sup></b>
10:55	<i>What do students attend to when first graphing planes in <math>R^3</math>?</i> [31]  <b>Allison Dorko, Oregon State University<sup>**†</sup></b>	<i>Looking for a (Super)resolution to an Image Processing Problem.</i> [61]  <b>Alleeta Maier, Linfield College<sup>*†</sup></b>
11:15	<i>Student Generalizations of Distance and the Cauchy Property</i> [76]  <b>Zackery Reed, Oregon State University<sup>**†</sup></b>	<i>Applying a Multilinear Regression Model to Predict Air Quality in Burns, Oregon</i> [59]  <b>Travis Lowe, Eastern Oregon University<sup>*†</sup></b> <b>Sydney Nelson, Eastern Oregon University<sup>*†</sup></b> <b>Amy Yielding, Eastern Oregon University</b>
11:35		<i>Use and Misuse of Regression for Chemical Analysis</i> [27]  <b>Bridget Daly, Pacific University<sup>*†</sup></b>
11:55		<i>Logging with Markov chains</i> [21]  <b>Dora Bixby, Southern Oregon University<sup>*†</sup></b>

<b>Saturday Afternoon Concurrent Sessions</b>		
	<b>NUMS IV: Topology</b> <i>Chair: Ally Stacy</i>	<b>General Papers II</b> <i>Chairs: Tevian Dray</i>
	<i>STAG 160</i>	<i>STAG 161</i>
1:45	<i>A Basis for the Space of Order 5 Chord Diagrams</i> [82] <b>Allison Stacey, Oregon State University</b> **†	<i>An Investigation of Subtraction Algorithms Utilized in the US during the 18th and 19th Centuries</i> [93] <b>Nicole Wessman-Enzinger, George Fox University</b> †
2:05	<i>From Manipulatives to Theory in Knot Theory</i> [42] <b>Brett Hegge, Western Oregon University</b> *†	<i>Pathological Continuity: A Zoo of Nowhere-Differentiable Functions</i> [85] <b>Nathan Taylor, University of Montana, Missoula</b> **† <b>Cancelled</b> <i>Numerical Inversions of the Broken Ray Transform with fixed initial and terminal directions</i> [80] <b>Brian Sherson, Oregon State University</b> **† <b>Moved here</b>
2:25	<i>Separability and the Cantor Set</i> [30] <b>Anthony Dominquez, Western Oregon University</b> *†	<i>Numerical Inversions of the Broken Ray Transform with fixed initial and terminal directions</i> [80] <b>Brian Sherson, Oregon State University</b> **† <b>Moved up one</b> <i>Breaking Bad Symmetries</i> [87] <b>Gerald Todd, University Of Montana</b> **† <b>Moved here</b>
2:45	<i>Klein links versus torus links, part I</i> [56] <b>Ryan Lattanzi, Gonzaga University</b> *† <b>Bryan Strub, Gonzaga University</b> *† <b>Hayley Olson, Gonzaga University</b> *	<i>Breaking Bad Symmetries</i> [87] <b>Gerald Todd, University Of Montana</b> **† <b>Moved up one</b> <i>Examining Matroids with Unique Addresses</i> [60] <b>Roger MadPlume, University of Montana</b> **† <b>Moved here</b>
3:05	<i>Klein links versus torus links, part II</i> [71] <b>Hayley Olson, Gonzaga University</b> *† <b>Bryan Strub, Gonzaga University</b> *† <b>Ryan Lattanzi, Gonzaga University</b> *	<i>Examining Matroids with Unique Addresses</i> [60] <b>Roger MadPlume, University of Montana</b> **† <b>Moved up one</b>

<b>Saturday Afternoon Concurrent Sessions</b>		
	<b>NUMS V: ODEs and Dynamical Systems</b> <i>Chair: TBD</i> STAG 162	<b>Teaching Tricks, Techniques and Discoveries</b> <i>Chair: Jeffrey Stuart</i> STAG 163
1:45	<i>Equations for Bacteria Growth</i> [32] <b>Svetlana Dyachenko, Western Oregon University</b> <sup>*†</sup>	<i>Creating Websites about Math - Not Just for Mathematicians</i> [54] <b>Nick Lassonde, Klamath Community College</b> <sup>†</sup>
2:05	<i>An Elementary Analysis of Chua's Circuit</i> [20] <b>Steven Beres, Gonzaga University</b> <sup>*†</sup> <b>Caleb Tjelle, Gonzaga University</b> <sup>*†</sup>	<i>Beyond Email: Reaching Students Where They Are</i> [55] <b>Kristin Lassonde, Klamath Community College</b> <sup>†</sup>
2:25	<i>Dynamic of Love</i> [45] <b>Aubrey Ibele, Gonzaga university</b> <sup>*†</sup> <b>Audrey Gomez, Gonzaga University</b> <sup>*†</sup>	<i>Wanna Play? Gamification of STEM Courses in Higher Education</i> [43] <b>John Hossler, Seattle Pacific University</b> <sup>†</sup>
2:45	<i>How to Take a Hot Bath, if You Can Spare the CPU Cycles</i> [23] <b>Thomas Burns, Southern Oregon University</b> <sup>*†</sup> <b>Jacob Schultz, Southern Oregon University</b> <sup>*†</sup>	<i>Stimulating Students to Succeed with Standards Based Grading</i> [57] <b>Jean Marie Linhart, Central Washington University</b> <sup>†</sup>
3:05	<i>Mathematical Modeling of Heat Through a Hot Bath</i> [38] <b>McKenzie Garlock, Eastern Oregon University</b> <sup>*†</sup> <b>Jeremy Bard, Eastern Oregon University</b> <sup>*†</sup> <b>Zach Nilsson, Eastern Oregon University</b> <sup>*†</sup>	<i>Specific Examples, Generic Elements and Size Tuning - Overcoming Student Roadblocks in Linear Algebra</i> [84] <b>Jeffrey Stuart, Pacific Lutheran University</b> <sup>†</sup>

<b>Saturday Afternoon Concurrent Sessions</b>		
	<b>Algebra and Number Theory II</b> <i>Chairs: Tom McKenzie &amp; Jeff Wand</i>	<b>NUMS VI: Algebra/Geometry</b> <i>Chair: TBD</i>
	<i>STAG 260</i>	<i>STAG 261</i>
1:45	<i>Systems of parameters and the Cohen-Macaulay property</i> [81] <b>Katharine Shultis, Gonzaga University</b> <sup>†</sup>	<i>Laplacians &amp; Laplace Transforms with Respect to Geometric Analysis of 1-Manifolds of Constant Curvature</i> [35] <b>Jonathan David Evenboer, Oregon State University</b> <sup>*†</sup>
2:05	<i>Categories of Representations of the Lie superalgebra <math>q(2)</math></i> [28] <b>Nicholas Davidson, University of Oregon</b> <sup>**†</sup>	<i>Regular Stars, Polygons, and Musical Scales</i> [52] <b>Mackenzie Koll, Western Oregon University</b> <sup>*†</sup>
2:25	<i>Overpartition Statistics</i> [66] <b>Thomas Morrill, Oregon State University</b> <sup>**†</sup>	<i>Analyzing Transition Matrices of Chutes and Ladders Game Variant</i> [22] <b>Bryce Boyle, George Fox University</b> <sup>*†</sup> <b>Matt DeBiao, George Fox University</b> <sup>**†</sup>
2:45		<i>Commuting Pairs in Finite Nonabelian Groups</i> [62] <b>Tyler McAfee, Western Oregon University</b> <sup>*†</sup>
3:05		<i>Kummer subspaces of Central Simple Algebras</i> [68] <b>Nhan Nguyen, University of Montana</b> <sup>**†</sup>

<b>Saturday Afternoon Concurrent Sessions</b>		
	<b>Research in Undergraduate Mathematics Education II</b> <i>Chair: Elise Lockwood</i> STAG 262	<b>NUMS VII: History/Math Ed</b> <i>Chair: TBD</i> STAG 263
1:45	<i>Impacts of Inquiry Pedagogy on Undergraduate Students Conceptions of the Function of Proof</i> [26] <b>Emily Cilli-Turner, University of Washington Tacoma<sup>†</sup></b>	<i>Infinitesimal-Based Calculus</i> [78] <b>Eric Rogers, Gonzaga University<sup>*†</sup></b> <b>Ethan Snyder, Gonzaga University<sup>*†</sup></b>
2:05	<i>Student Conceptions of Isomorphism</i> [90] <b>Kristen Vroom, Portland State University<sup>**†</sup></b> <b>Kate Melhuish, Teachers Development Group</b>	<i>The Math Behind The Rubiks-Cube</i> [44] <b>Garrison Iams, Klamath Community College<sup>*†</sup></b> <b>Moved to Session: NUMS I— Mathematics Applications</b>
2:25	<i>Deconstructing and Reconstructing the Multiplication Principle</i> [58] <b>Elise Lockwood, Oregon State University</b> <b>John Caughman, Portland State University<sup>†</sup></b> <b>Zackery Reed, Oregon State University<sup>**</sup></b>	<i>Rearrangement in an Infinite Series</i> [86] <b>Lane Thomason, Southern Oregon University<sup>*†</sup></b>
2:45	<i>Student Attitudes Toward Listing as a Strategy for Solving Counting Problems</i> [34] <b>Sarah Erickson, Oregon State University<sup>**†</sup></b>	<i>Reading <math>\pi</math>: Helping children move from symbol to meaning</i> [95] <b>Emma Winkel, Pacific University<sup>*†</sup></b>
3:05	<i>Prospective Teachers Reasoning about Integers and Temperature</i> [93] <b>Nicole Wessman-Enzinger, George Fox University<sup>†</sup></b>	<i>Math Anxiety: The Common Plight</i> [51] <b>Tiffany Klink, Klamath Community College<sup>*†</sup></b>

## Social Events

### **Friday Project NExT Dinner**

(For Project NExT Participants Only, No Host)

6:00

### **Friday Student Activity: Math Wrangle**

(with pizza provided)

Organized by Brandy Wiegers, Central Washington University

5:30 Willamette Seminar Room, Valley Library 3rd floor

### **Friday Evening Invited Public Lecture**

*How efficiently can one untangle a double-twist? Waving is believing!*

David Pengelley [3]

8:00 LINC 128

### **Friday Evening Reception**

9:00 LINC Lobby

### **Saturday Morning: Coffee, Tea & Pastries**

8:00–10:00 LINC Lobby

### **Saturday Lunch**

12:20 Horizon Room, Memorial Union

### **Saturday Afternoon: Coffee Break and Poster Session**

3:25–4:05 STAG 270

### **Saturday Evening Social Hour**

Co-sponsored by Pearson Publishing 5:50 Horizon Room, Memorial Union

### **Saturday Evening Awards Ceremony and Banquet Dinner**

6:45 Horizon Room, Memorial Union

Introduction of new Section Project NExT Fellows

Recognition of 25- and 50-year MAA membership

Announcement of NUMS Student Presenter Awards

PNW MAA Meritorious Service Award

PNW MAA Distinguished Teaching Award

### **Saturday Evening Invited Lecture**

*Measuring Inequality*

Karen Saxe [5]

7:45 Horizon Room, Memorial Union

# Campus Information

## Campus Map:

<http://oregonstate.edu/campusmap/>

**Building Codes:** LINC Learning Innovation Center  
STAG Strand Agricultural Hall  
MU Memorial Union

**WiFi** is widely available on campus; select the “Visitor” network (no password is necessary).

## Parking:

- Friday 7 am — 5 pm: Parking permits are required to park on campus. To buy one online, go to <https://my.parking.oregonstate.edu/sales/daily/>.  
See the parking maps on that page as well. There is a choice of A or BC, A is better, \$12 for the whole day and you can park anywhere (except at a meter).
- After 5 pm and on Saturday: Parking is free on campus.
- Several convenient parking lots are indicated on the conference map  
<http://math.oregonstate.edu/pnwmaa2016/directions>.

## Emergency Information:

- Corvallis Police/Fire/Medical Emergency: 911
- OSU Public Safety: Emergency 541-737-7000; Non-Emergency 541-737-3010

## Acknowledgments

This meeting would not be possible without the hard work and generous contributions of the following:

- Special Session organizers Tom McKenzie, Jeff Wand, Elise Lockwood, Jeffrey Stuart, Annie Raymond, Matt Junge, Katharine Shultis and Kate Kearney
- PNW MAA Executive Committee Nancy Ann Neudauer, Brian Blitz, Tom McKenzie, Jennifer Nordstrom, Kelly McKinnie, Aaron Montgomery and Dominic Klyve for thoughtful advice.
- NUMS Steering Committee Naomi Cameron (Lewis & Clark), Tom Edgar (Pacific Lutheran), David Perkinson (Reed), and Colin Starr (Willamette).
- Program Co-Chairs Tom Dick, Tevian Dray and Elise Lockwood
- Project NExT Coordinator Jenny McNulty and Project NExT Local Arrangements Chair Holly Swisher for organizing the Section NExT meeting.
- Alysun Burns with Pearson Publishing, the OSU Department of Mathematics, the OSU College of Science, MAA and Pi Mu Epsilon for financial support.
- Students Dionysus Birnbaum, Allison Dorko, Ricardo Reyes Grimaldo, Dwight Holland, Sarah Erickson, Paul Duffin, Puttha Sakkaplangkul, Charles Camacho, Lida Bentz, Samantha McGee, Joe Umhoefer, Thomas Morrill, Rebecca Ott, Lucas Estabrook, Claire Gibbons, Zackery Reed for helping with registration, book sales, and session chairs everything else.
- Matthew Brabham of Cosine IT Services for significant website support.
- Department of Mathematics office staff Deanne Murray, Lisa Rogers and Stephanie Wise for handling printing and reservations.
- All the conference speakers, panelists and participants for making the conference great!

Thanks for coming. Nathan Gibson, Local Arrangements Chair



## Mini-course Descriptions

Friday, April 2

### 1 *Teaching and Learning Mathematics from Primary Historical Sources*

**David Pengelley, Oregon State University**

Why would one consider having students learn their mathematics directly from studying primary historical sources? In this hands-on workshop, participants will collaboratively engage in both the student and instructor sides of studying primary sources, in the form of guided student project modules that include mathematical context, original source excerpts, tasks for students, and biography. Multiple project modules from lower division courses for mathematics majors will be tackled, with discussion and evaluation from both the student and instructor points of view. More background via links at <https://www.math.nmsu.edu/history/#discrete>.

### 2 *Codes and Ciphers*

**Stuart Boersma & Cheryl Beaver**

The codebreaking competition, KRYPTOS, is the week following the PNW-MAA meeting. Come and hear about KRYPTOS, learn about some ciphers, and test your skills at cryptanalysis! Helpful online cryptanalysis tools will be shared with participants together with some tried and true pencil and paper techniques. This minicourse is designed primarily for students, so please encourage your students to attend!

## Abstracts of Invited Lectures

(in chronological order)

### 3 *How efficiently can one untangle a double-twist? Waving is believing!*

**David Pengelley, Oregon State University**

Can you do the Philippine candle dance, the Dirac belt trick, or the Feynman plate trick? Whether your answer is yes or no, this event will engage you in this and far beyond in discovering and understanding the most mathematically efficient way to untangle a double-twist in 3-space. Limber up!

### 4 *Cryptologic Tidbits*

**Stuart Boersma, Central Washington University**

Cryptology offers an ideal setting to give students a powerful and successful experience in mathematics. Cryptologic examples allow students to connect new content with prior knowledge, and provide students the opportunity for productive struggle with challenging material. Cryptologic contexts offer students multiple entry points to rich questions and provide all students an opportunity to engage with the material, to propose unique problem solving strategies, and to build upon the ideas and conjectures of their peers. Cryptologic applications allow students to take ownership of their problem solving strategies and increase their confidence as learners. Oh yeah...cryptology is fun! In addition to providing some justification for the above claims, I will present a variety of examples from cryptology that could easily find their way into existing courses in probability, statistics, quantitative reasoning, linear algebra, and abstract algebra.

### 5 *Measuring Inequality*

**Karen Saxe, Macalester College**

Whether a resource – such as income – is distributed evenly among members of a population is often an important political or economic question. The Occupy Movement has recently drawn more attention to the fact that income inequality in the United States is increasing. How can we measure this inequality? How can we decide whether the distribution of wealth in this country is becoming more or less equitable over time? How can we decide which country has the most equitable income distribution? This talk describes one tool, the Gini index, used to answer these questions. Aimed at students, will use integral calculus. Karen is principal investigator on the NSF-funded Common Vision project, an initiative aimed at improving undergraduate learning in the mathematical sciences, especially in courses typically taken in the first two years. At the end of her talk, she will briefly describe the project. She will be happy to answer questions about the talk, Common Vision or both!

### 6 *The Symmetric Group and Fair Division: Does Knowledge Matter?*

**Brian Hopkins, Saint Peter's University**

Sports drafts and divorce settlements are examples of situations where players take turns selecting indivisible goods. Like other topics in fair division, the situation is made more interesting because people may value the goods in different ways. In this talk, we focus on the case of two players, where the machinery of permutations is surprisingly applicable. How many possible outcomes are there? In what circumstances do both players get their best possible outcomes? How can one best take advantage of knowing the other's preferences? What happens when a player's motivation switches from greed to spite, the common good, or selfless altruism? In this colorful talk, we'll sample some applied algebraic combinatorics and address these issues along with the provocative question of the title.

## Panel Descriptions

### 7 *Career Panel*

**Manny Hur, Oregon State University, Matthew Sottile, Galois, Elise Lockwood, Oregon State University**

You're about to earn a degree in mathematics, now what? You may be surprised to know that teaching isn't your only option; in the "real world" mathematical knowledge is a valued commodity and there are many interesting job opportunities for mathematicians in non-academic settings. So, whether you are a mathematics student looking for a job once you graduate or an advisor looking for advice to give to future job-seeking students, this session will help you gain new perspectives on career experiences and what employers value in their employees. Panelists will share their paths to their current positions, the ways in which they utilize their mathematical background, and offer advice to others looking for employment in similar venues. Questions that may be addressed include: How do you find which jobs are available? How do you choose which jobs you want to apply for? What are academic and other employers looking for in the materials that you send? How should you tailor your application materials for the job that you are applying for? How do employers conduct interviews?

### 8 *Leading a Successful Math Club*

**Kelsey Jewell, Western Washington University, Maureen Sturgeon, Western Washington University**

This session will be an open discussion led by the student leaders of Western Washington University's math club. The topic of discussion will be leading academic clubs on campus. The goal of the session is to allow clubs throughout the Pacific Northwest to collaborate and improve our clubs, as well as to encourage students to start a new club. Kelsey and Maureen have a lot of ideas to share but encourage other clubs to share their successes, suggestions and questions. Some of the topics for the discussion are: increasing attendance, hosting interesting events, successful fund-raising, participating in outreach events, and promoting an inclusive community. Other topic ideas are welcome!

## Abstracts of Contributed Posters

(in alphabetical order, by presenter)

### 9 *Understanding the Traveler's Dilemma*

**Carter Bedsole, George Fox University**\*†

**Gary Buckley, George Fox University**\*†

In the game theory problem The Traveler's Dilemma, the theoretical and experimental results differ greatly. This discrepancy is explained through evolutionary game theory.

### 10 *Rat Game Expanded*

**David Calkins, George Fox University**\*†

Mathematician Aviezri S. Fraenkel wrote a paper on a game called The Rat Game, which involves mathematical moves and strategies. I plan to inform and expand on some of these moves and add to the game.

### 11 *Mathematical Modeling of Competition and Coexistence of Phytoplankton Species*

**Luke Campbell, Central Washington University**\*Student†

Microscopic phytoplankton form the basis of the food chain in the earth's oceans. A system of differential equations relates phytoplankton population and nutrient concentration in an isolated environment. The equations were modeled with MATLAB. I conducted sensitivity analyses to determine the relationships between the system of differential equations, and their respective parameters, on phytoplankton behavior. I explored the dynamics of competition and coexistence between multiple species with multiple nutrients in the system. I was able to predict competitive exclusion, and when multiple species could reach states of coexistence when the equations reach equilibrium.

Initially left out of program.

**12** *Triangular Numbers and Squared Numbers***Tyler Chin, George Fox University**<sup>\*†</sup>**Ben Van Vliet, George Fox University**<sup>\*†</sup>

Visual demonstrations of the relationships between squared numbers and centered squared numbers.

**13** *A Visual Expansion of the Pythagorean Theorem***Viv Diebel, George Fox University**<sup>\*†</sup>**Ian Johnson, George Fox University**<sup>\*</sup>

The poster is a visual expansion of the Pythagorean Theorem.

**14** *Predicting Wins and Losses in Division III Women's Basketball***Charles Haneberg, George Fox University**<sup>\*†</sup>

A method is presented for predicting future wins and losses in tournament-style games based on previous game outcomes. This method is applied to the recent Division III women's basketball season.

**15** *Adding Color to Combinatorics***Emily Hiscox, George Fox University**<sup>\*†</sup>**Ellen Pearson, George Fox University**<sup>\*</sup>

Taking the conclusions found in Candy Crush Combinatorics by Dana Rowland, and expanding them by one color to see how many possible combinations can be found using two rows of candy's and 3 colors.

**16** *7 Variables***Mimi Miller, George Fox University**<sup>\*†</sup>

Proving the number of unique fair games that can be played with seven variables and showing that they are isomorphic.

**17** *Graph Products and Colorings in Relation to the Hedetniemi Conjecture***Crystal Susbauer, Pacific University**<sup>\*†</sup>

There are many types of product operations defined in graph theory. These products use the vertices and edges of two (not necessarily distinct) graphs to create a new graph. We focus on four important types of graph products: the Cartesian product, the direct product, the strong product, and the lexicographic product. Additionally, we explore ways of labeling graphs through graph coloring and list coloring, and investigate the chromatic number of a graph. Finally, we relate both topics to one another by examining the Hedetniemi conjecture and researching what work as been done to try to prove or disprove the conjecture.

18 *A Mad Math Tea Party*

**Jillian Welk, George Fox University**<sup>\*†</sup>

This poster presentation will focus on the research done in an article on cyclic partitions involving an Alice in Wonderland theme.

## Abstracts of Contributed Talks

(in alphabetical order, by presenter)

### 19 *Dispersion reduction schemes for the wave equations*

**Yajun An, Pacific Lutheran University<sup>†</sup>**

(*Session: Junior Faculty Research*)

Finite Difference (FD) schemes have been used widely in computing approximations for partial differential equations for wave propagation, as they are simple, flexible and robust. However, even for stable and accurate schemes, waves in the numerical schemes can propagate at different wave speeds than in the true medium. This phenomenon is called numerical dispersion error. Traditionally, FD schemes are designed by forcing accuracy conditions, and in spite of the advantages mentioned above, such schemes suffer from numerical dispersion errors. Traditionally, two ways have been used for the purpose of reducing dispersion error: increasing the sampling rate and using higher order accuracy. More recently, Finkelstein and Kastner (2007, 2008) propose a unified methodology for deriving new schemes that can accommodate arbitrary requirements for reduced phase or group velocity dispersion errors, defined over any region in the frequency domain. Such schemes are based on enforcing exact phase or group velocity at certain preset wavenumbers. This method has been shown to reduce dispersion errors at large wavenumbers. In this talk, we discuss the construction and behaviors of FD schemes designed to have reduced numerical dispersion error. We prove that the system of equations to select the coefficients in a centered FD scheme for second order wave equations with specified order of accuracy and exact phase velocity at preset wavenumbers can always be solved. Furthermore, from the existence of such schemes, we can show that schemes which reduce the dispersion error uniformly in an interval of the frequency domain can be constructed from a Remez algorithm. In these new schemes we propose, we can also specify wavenumbers where the exact phase or group dispersion relation can be satisfied. For an incoming signal consisting of waves of different wavenumbers, our schemes can give more accurate wave propagation speeds. Furthermore, when we apply our schemes in two dimensional media, we can obtain schemes that give small dispersion error at all propagation angles.

### 20 *An Elementary Analysis of Chua's Circuit*

**Steven Beres, Gonzaga University<sup>\*†</sup>**

**Caleb Tjelle, Gonzaga University<sup>\*†</sup>**

(*Session: NUMS V—ODEs and Dynamical Systems*)

Prior to the invention of Chua's circuit by Leon Chua in 1983, it was generally believed that it was not possible to design an electronic oscillator which exhibited chaos. In this talk, we provide an overview and analysis of Chua's chaotic circuit. Principally, we will show the nondimensionalization of the model and a bifurcation analysis for the case where all passive circuit elements have positive values. This analytical approach will be further illustrated by numerical solutions to the state equations for selected parameter values. Finally, we will compare our theoretical results with the output of a circuit simulator and also with measured results from a physical implementation of Chua's circuit. Previous experience with circuit analysis is not necessary to understand this talk.

**21** *Logging with Markov chains***Dora Bixby, Southern Oregon University**<sup>\*†</sup>*(Session: NUMS III—Computational/Statistical)*

Oregon is the U.S.'s top lumber producer and the industry makes up a large portion of the jobs in the Oregon workforce. I will be discussing the use of absorbing Markov chains to model the growth of trees in a stand, then analyzing the model to develop a reliable and steady harvesting schedule.

**22** *Analyzing Transition Matrices of Chutes and Ladders Game Variant***Bryce Boyle, George Fox University**<sup>\*†</sup>**Matt DeBiao, George Fox University**<sup>\*†</sup>*(Session: NUMS VI—Algebra/Geometry)*

Imagine each directional chute and ladder in the classic board game replaced with bidirectional portals. How does the game change? Expected game play and probabilities of each position are determined by the location and type of eigenvalues. We analyzed the transition matrix and discovered Gershgorin eigenvalue bounds for each board layout were constructed from a limited set of Gershgorin disks. Further, we discovered a minimum bounding region for the eigenvalues, independent of location and number of portals on the board.

**23** *How to Take a Hot Bath, if You Can Spare the CPU Cycles***Thomas Burns, Southern Oregon University**<sup>\*†</sup>**Jacob Schultz, Southern Oregon University**<sup>\*†</sup>*(Session: NUMS V—ODEs and Dynamical Systems)*

We discuss multiple methods for modelling the temperature of a bathtub taking into account conductance, evaporation, and turbulence. One model uses a system of ordinary differential equations to represent the temperature with respect to time of multiple materials in the system, while another discretizes the space into uniform cubes propagating heat at discrete time steps.

**24** *Eigensystems of Electrical Networks***Nikki Carter, University of Portland**<sup>\*†</sup>*(Session: NUMS I—Mathematics Applications)*

The eigenvectors of an electrical network are voltages that, when placed at the boundary vertices, produce boundary currents that are a scalar multiple of the boundary voltage. The objective of this research is to gain information about a given electrical network using eigensystems. In particular, eigensystems are useful when an edge or multiple edges in a network have zero resistance or infinite resistance. Using the eigensystems of an electrical network which are based on the resistance and connectivity of edges in the network, one can determine the location of a dysfunctional resistor in a network or determine the expected number of dysfunctional resistors.

**25 *Insects and Spirals*****Kaylee Church, Western Oregon University<sup>\*†</sup>***(Session: NUMS I—Mathematics Applications)*

The logarithmic spiral, also known as the growth spiral, is an interesting form in mathematics that happens to be very applicable to the natural world. We explore the structure of this curve, and how this spiral can be used to model the flight pattern of a moth. Specifically, we investigate assumptions used to construct a model for insect flight. Analysis of these underlying assumptions gives insight into possible improvements to the model and limitations of specific models.

**26 *Impacts of Inquiry Pedagogy on Undergraduate Students Conceptions of the Function of Proof*****Emily Cilli-Turner, University of Washington Tacoma<sup>†</sup>***(Session: Research in Undergraduate Mathematics Education II)*

Mathematicians and mathematics educators agree that proof has many different roles in mathematics beyond that of verifying the truth of a statement. For instance, some proofs can not only show that a statement is true, but also explain why it must be true. However, students may not appreciate these other functions of proof as they are not explicitly taught in the classroom. This report outlines an inquiry-based teaching intervention in an introduction to proof course and its impacts on students' appreciation of other functions of proof. Results show that exposure to inquiry pedagogy changed students' conceptions of the functions of proof and increased their recognition of the explanatory and communication power of proof.

**27 *Use and Misuse of Regression for Chemical Analysis*****Bridget Daly, Pacific University<sup>\*†</sup>***(Session: NUMS III—Computational/Statistical)*

It is standard practice in analytical chemistry to use linear regression, particularly to calibrate analytical instruments. If a regression line were used to estimate the output of the instrument for a known concentration of analyte, all would be well. However, chemists use this line in reverse, estimating the concentration of an unknown analyte from the output of the instrument. In this talk we explore how this misuse of the regression line influences the estimation of concentration.

**28 *Categories of Representations of the Lie superalgebra  $q(2)$*** **Nicholas Davidson, University of Oregon<sup>\*\*†</sup>***(Session: Algebra and Number Theory II)*

I will give an overview of the representation theory of the queer Lie superalgebra  $q(n)$ , focusing in particular on the representations of  $q(2)$  in its BGG category  $\mathcal{O}$ .



**29** *Visualizing p-adic numbers***Dibyajoti Deb, Oregon Institute of Technology<sup>†</sup>***(Session: Algebra and Number Theory I)*

P-adic numbers after its introduction by Kurt Hensel more than a century ago, has been a mainstay in the field of number theory. An abstract concept by itself, in this talk we will look at how we can visualize p-adic numbers with a tree structure and look at some basic questions in p-adic analysis that can be answered with this visualization.

**30** *Separability and the Cantor Set***Anthony Dominquez, Western Oregon University<sup>\*†</sup>***(Session: NUMS IV—Topology)*

The Cantor Set is a famous set in point-set topology. There are a wide variety of types of Cantor Sets. However, we will only cover the ternary, or standard Cantor Set. In this talk, we will define and discuss what it means for a set to be separable and prove that the Cantor Set satisfies this definition.

**31** *What do students attend to when first graphing planes in R3?***Allison Dorko, Oregon State University<sup>\*\*†</sup>***(Session: Research in Undergraduate Mathematics Education I)*

This talk considers what students attend to as they first encounter R3 coordinate axes and are asked to graph  $y = 3$ . Graphs are critical representations in single and multivariable calculus, yet findings from research indicate that students struggle with graphing functions of more than one variable. We found that some students thought  $y = 3$  in R3 would be a line, while others thought it would be a plane. In creating their graphs, students attended to equidistance, parallelism, specific points, and the role of  $x$  and  $z$ . Students use of these ideas was often generalised from thinking about the graphs of  $y = b$  equations in R2. A key finding is that the students who thought the graph was a plane always attended to the  $z$  variable as free.

**32** *Equations for Bacteria Growth***Svetlana Dyachenko, Western Oregon University<sup>\*†</sup>***(Session: NUMS V—ODEs and Dynamical Systems)*

Bacteria growth is really important in our life. Some bacteria cells help us overcome different diseases, while others bring those diseases to us. We have learned to produce medicine with help of bacteria growth, like insulin, to help those who are ill. Modeling bacteria growth is an important part of understanding it. Consider a situation of bacteria doubling every half an hour starting with one cell, how much time will it take to fill all the oceans on Earth? Questions like this are answered by developing mathematical models of bacteria growth. We examine modeling bacteria growth using differential equations. Our focus is on model construction and building realistic models that match empirical data.

**33 Reasoning with "dx" in the integral****Robert Ely, University of Idaho<sup>†</sup>***(Session: Research in Undergraduate Mathematics Education I)*

Research indicates that in order for students to be able to successfully interpret and flexibly model with definite integrals, they must conceptualize the integral as "adding up the pieces" of a quantity, rather than as (a) a symbolic template, (b) an anti-derivative, or even just (c) an area under a curve. Task-based interviews with students in my recent experimental calculus class indicate that the "adding up the pieces" conceptualization of  $\int_a^b f(x)dx$  requires the student to be able to view the " $f(x)dx$ " as a small bit of the total quantity. This understanding in turn relies on being able to treat " $dx$ " as an increment of  $x$ , which has a size, across which the quantity to be summed accumulates (at a rate of  $f(x)$ ). I argue that this incremental understanding of  $dx$  was also crucial in allowing Leibniz to develop calculus where his predecessors (e.g. Cavalieri and Torricelli), who used indivisibles that have no "width", did not.

**34 Student Attitudes Toward Listing as a Strategy for Solving Counting Problems****Sarah Erickson, Oregon State University<sup>\*\*†</sup>***(Session: Research in Undergraduate Mathematics Education II)*

While researchers have found that students at a variety of levels struggle to solve counting problems correctly, listing has been shown to be a potentially effective remedy to student difficulties. Motivated by its importance in helping students solve counting problems, this presentation describes a research study conducted to investigate attitudes that undergraduate students have about listing. This study found that undergraduate students seem to agree that listing is a useful activity, but they may not have a strong understanding of how to productively use a list of outcomes to solve counting problems. Implications for pedagogy and future research directions are discussed.

**35 Laplacians & Laplace Transforms with Respect to Geometric Analysis of 1-Manifolds of Constant Curvature****Jonathan David Evenboer, Oregon State University<sup>\*†</sup>***(Session: NUMS VI—Algebra/Geometry)*

This presentation continues on the course begun in past NUMS presentations. Properties of the Laplacian values of the Circle and Aster, and the Laplacian's role in construction of the heat kernel on both manifolds, are covered. Also covered will be properties of the Laplace Transform values of the circle and the aster. A sheet of fundamental/basic notation and a copy of the paper on which this talk is based are/will be available before March at <http://jonathandavidevenboer.weebly.com/blog>

**36 Examining Ramsey Numbers****Amanda Evola, Western Oregon University<sup>\*†</sup>***(Session: NUMS II—Combinatorics)*

This paper explores the work of Frank P. Ramsey who founded Ramsey's Theorem and is centered on the fact that complete disorder is impossible. The goal is to dig into Ramsey's Theory

by examining various Ramsey Numbers and bounds. Through this examination of Ramsey Numbers we will begin to see how we can reach structures arbitrarily large and be able to guarantee substructure through the proof of Ramsey's Theorem.

### **37** *If You Must Gamble: Minimizing Expected Loss*

**Curtis Feist, Southern Oregon University<sup>†</sup>**

**Jake Scott, Southern Oregon University**

(*Session: General Papers I*)

We consider a typical gambling situation such as red/black roulette bets of a fixed size, with a limited bankroll, a fixed goal (such as doubling one's money), and a maximum time of play. Through the use of Markov chains, we analyze the expected value of this game for various bet sizes. Unlike in the classical Gamblers Ruin problem with no time constraint, we will see there really are good and (very!) bad bet sizes, which depend on the values of the various constraints.

### **38** *Mathematical Modeling of Heat Through a Hot Bath*

**McKenzie Garlock, Eastern Oregon University<sup>\*†</sup>**

**Jeremy Bard, Eastern Oregon University<sup>\*†</sup>**

**Zach Nilsson, Eastern Oregon University<sup>\*†</sup>**

(*Session: NUMS V—ODEs and Dynamical Systems*)

No one likes a cold bath. When the bath water starts to get cold a person might turn on a constant trickle of water to keep the water tepid. We attempted to model, and optimize, this behavior with differential equations. To do this we simplified the situation and started with a bathtub that had only water in it. We were able to model the heat flow through the bath water using a simple lumped system analysis that broke the tub up into 10 equal plates of water. The first of these plates was assumed to stay at a constant temperature because of the continual introduction of hot water from the tap. That first plate would then pass some of its heat to the next plate according to thermal conductance and Newton's Law of Cooling. This model assumed that no heat was lost anywhere in the system except to the next plate of water. To add an element of realism we then added heat loss to the air above each plate. We then attempted to add a person to the tub. We treated the person as if they were a Heat Sink. Meaning that they take in heat but their temperature never changes. The rate of heat transfer between the water and the person required different physics from those that we had been using to model the heat flow through the homogeneous tub. We approximated this different rate by using the ratio of the thermal conductance's of water and a person. We also distributed the person's mass evenly in each plate. This model worked reasonably well. Showing that the water in the tub would all reach the temperature of the incoming water. We then attempted to distribute the person's mass throughout the tub in a more realistic way, with more mass towards the back of the tub. To do this we introduced a mass ratio term to our equations. This ratio of water mass to person mass did not have the desired results. The water at the back of the tub would continue to decline in temperature, seemingly unaffected by the incoming hot water. This discrepancy is mostly likely caused by the person absorbing more heat than they actually would in reality. The effect of the person's movements was not included in our models. The shape of the tub was also not explored; our models assume either a perfectly rectangular tub or a perfectly hemispherical tub.

**39** *First to Toss  $N$  Heads Wins***Chris Hallstrom, University of Portland**<sup>†</sup>*(Session: General Papers I)*

Two players play a game, taking turns tossing a coin; the winner is the first to reach  $n$  heads for some agreed upon value of  $n$ . What is the probability that the player to go first wins? In this talk, we will consider this question as well as its application to your lunch-time half-court basketball game.

**40** *Propagation of transient behaviour in linear flocking models***David Hammond, Oregon Institute of Technology - Wilsonville**<sup>†</sup>*(Session: General Papers I)*

This talk will introduce a set of models for describing the behaviour of linear flocks. One application of these models is to describe groups of autonomous automobiles on a one-lane road, where each automobile controls its acceleration based on the differences of its own position and velocity from those of its neighbors. In my talk I will discuss recent theoretical results describing the propagation of perturbations across the flock, occurring when one vehicle is initially given a velocity different than all of the others. We study systems with asymmetric coupling, in which case the transients may travel with different velocities in two directions.

**41** *Using Eigenvalues to Investigate Numerical Oscillations***Corban Harwood, George Fox University**<sup>†</sup>*(Session: Junior Faculty Research)*

In numerically solving linear partial differential equations, carefully formed matrices are powered up as the algorithm iterates the solution through time. Depending upon the eigenvalues, the solution either blows up to non-numerical values, stabilizes with bounded oscillations, or stabilizes free of oscillations. This talk presents comparisons of eigenvalue conditions through extended von Neumann stability analysis of standard numerical methods for foundational partial differential equations. Known lower eigenvalue bounds for stability are not optimal, so computational testing is necessary. We will compare several implementations to automate detection of solution behavior for testing theoretical conjectures. Along the way, we will pause to enjoy some surprisingly beautiful figures resulting from such analysis and computation.

**42** *From Manipulatives to Theory in Knot Theory***Brett Hegge, Western Oregon University**<sup>\*†</sup>*(Session: NUMS IV—Topology)*

Knot Theory is the study of simple closed curves in three dimensions. Complicated structures can be analyzed using three basic moves and knots can be shown to be equivalent. We discuss ways of using physical manipulation to get middle school students interested in mathematics. We also, explore the basic mathematical postulates and theoretical foundation of Link/Knot Theory.

### 43 *Wanna Play? Gamification of STEM Courses in Higher Education*

**John Hossler, Seattle Pacific University<sup>†</sup>**

(*Session: Teaching Tricks, Techniques and Discoveries – Share what works for you*)

While the word "gamification" may sound like it means playing games in class, it means something entirely different: the infusion of game principles into an otherwise non-game situation. Gamification is the addition of game elements, mechanics, and principles to non-game contexts—the classroom, for example. Gamified settings are becoming more and more popular in non-classroom contexts, and this research specifically looks at what it might take to gamify an undergraduate STEM course, including advantages, disadvantages, and challenges.

### 44 *The Math Behind The Rubiks Cube*

**Garrison Iams, Klamath Community College<sup>\*†</sup>**

(*Session: NUMS I—Mathematics Applications*)

Moved from NUMS VII at 14:05 to NUMS I at 11:55

Come hear about the intricacies of the Rubik's Cube! I will discuss the history of God's number, which is the least amount of moves it takes to solve a scrambled Rubiks Cube. There are many techniques to solve a scrambled Rubik's Cube. One solution technique which can be modeled by group theory will also be discussed in a way that is understandable to college students who have not yet taken abstract algebra. As an added bonus, I will demonstrate solving a Rubiks Cube in 10 seconds!

### 45 *Dynamic of Love*

**Aubrey Ibele, Gonzaga university<sup>\*†</sup>**

**Audrey Gomez, Gonzaga University<sup>\*†</sup>**

(*Session: NUMS V—ODEs and Dynamical Systems*)

Dynamical systems both linear and non-linear have the power to describe intricate behavior and provide analysis. In this paper, linear and non-linear models are employed to replicate the interaction between individuals with varying romantic styles. Using traditional analysis methods the goal was to examine the models laid out in Sprott of the dynamic phenomena of love. The graphical outputs and implications between the simple linear and two-dimensional non-linear models were compared; despite identical initial conditions, results varied. This showcases the impact that a variation of parameters has on the system. The non-linear model had an additional logistic function, which made the system more realistic by adding the possibility for emotional reactions. Predictions suggest that with the addition of more parameters, the dynamical system will diverge to chaos.

### 46 *Introduction to permutation patterns*

**Masaki Ikeda, Western Oregon University<sup>†</sup>**

(*Session: Junior Faculty Research*)

In enumerative combinatorics, the study of permutation patterns blossomed in the 1980s with the Stanley-Wilf conjecture. In this talk, I will introduce the basic concept of permutation patterns and some approachable examples as well as the final result of my doctoral research.

**47** *Theatrical! Practical! Mathematical!***Thad Joachim, Klamath Community College**<sup>\*†</sup>*(Session: NUMS I—Mathematics Applications)*

My talk will focus on how mathematics can be used in the real world relating to theatre arts, and I will support my discussion by giving real-world examples. Actors and other theatre individuals should be able to take math seriously when the math is involved with a theatrical performance. I will also share some personal stories about theatre regarding how I have found math useful in theatre for budgets, costume designs, ticket sales, and stage design.

**48** *Inside the box: college in prison***Matthew Junge, University of Washington**<sup>\*\*†</sup>*(Session: Thinking Outside the Circle: Alternate Outreach)*

Prisoners that receive college education in prison have drastically lower recidivism rates. Mathematicians are in short supply here, and can make a great difference. I will discuss my experience designing and teaching the first-ever for-credit math course in the Washington Corrections Center for Women.

**49** *An Obstruction to Knots Bounding Mobius Bands in  $B^4$* **Kate Kearney, Gonzaga University**<sup>†</sup>*(Session: Junior Faculty Research)*

The relationship between embedded surfaces and their knotted boundaries has been one of the main topics of knot theory for much of the last half century. This talk focuses on a particular case, namely whether a given knot in the three-sphere can be the boundary of a Mobius band embedded in the four-ball,  $B^4$ . We will discuss a new example of a knot which does not bound a Mobius band in  $B^4$ , and describe how the d-invariant of Heegaard-Floer theory is used to obstruct this and other knots from bounding Mobius bands in  $B^4$ .

**50** *On the Turán number of forests (Cancelled)***Omid Khormali, University of Montana**<sup>\*\*†</sup>*(Session: NUMS II—Combinatorics)*

A well-known conjecture of Erdős and Sós states that the Turán number for paths is enough for any tree i.e. a graph  $G$  on  $n$  vertices and more than  $\frac{k-2}{2}n$  edges contains any tree on  $k$  vertices. A natural extension of the problem is the determination of the Turán number of forests. Erdős and Gallai considered the graph  $H$  consisting of  $k$  independent edges. Brandt generalized Erdős and Gallai's result by proving that the Turán number for  $k$  independent edges is enough for any forest on  $k$  edges without isolated vertices. Bushaw and Kettle found the Turán number and extremal graph for the forest with components, which are paths, of the same order  $l > 2$ . Recently, Lidický, Liu and Palmer generalized these results by finding the Turán number and extremal graph for the forests with components of paths with arbitrary length. Also, they investigated Turán number for a star forest  $F = \cup_{i=1}^k S_i$  where  $d_i$  is the maximum degree of  $S_i$  and  $d_1 \geq \dots \geq d_k$ , and the Turán number for forests with all components of order 4.

In this paper, we determine the Turán number of the forest  $F = a \cdot P_\ell \cup b \cdot S_t$  i.e.  $a$  disjoint copies of path  $P_\ell$  and  $b$  disjoint copies of star  $S_t$ . In addition, we obtain the Turán number for forest with all components of order 5.

**51** *Math Anxiety: The Common Plight***Tiffany Klink, Klamath Community College**<sup>\*†</sup>*(Session: NUMS VII—History/Math Ed)*

There are multiple causes to Math Anxiety ranging from actual facts to myths about math. Mathematics is a subject that many people struggle with daily. Not just mathematics involved in engineering, but the basic math skills needed for leaving a tip. Some topics discussed will be: how to think about math differently, the fact that there are multiple methods to solve any given math problem, how making mistakes is an important way to learn mathematics, and common causes of Math Anxiety according to research. I will also present several different solutions to help reduce Math Anxiety so that you can help your friends and family today!

**52** *Regular Stars, Polygons, and Musical Scales***Mackenzie Koll, Western Oregon University**<sup>\*†</sup>*(Session: NUMS VI—Algebra/Geometry)*

Edge scales are musical scales constructed from the edges and vertices of a regular polygon. Regular polygons are polygons that have specific structure and they can be constructed from regular stars. We will discuss this structure using elements of rational trigonometry and discuss regular stars of order  $n$ . A star of order 6 will be used to construct a regular hexagon and motivate the construction of an edge scale. Similar constructions can be used to make other types of musical scales such as stellation scales.

**53** *Fractals in Geology: Measuring Intricate Forms (Cancelled)***Will Kugler, Klamath Community College**<sup>\*†</sup>*(Session: NUMS I—Mathematics Applications)*

First I will give an introduction to fractals, a naturally occurring mathematical set that repeats a detailed pattern visible at any scale. Then I will connect fractals to real-world applications in Geology and Earth Science. I will discuss how fractals are used by geologists with the use of mathematical equations to find the dimensions of detailed forms ranging from bigger surfaces such as coastlines and islands to smaller forms such as leaves. Come find out how scientists relate these findings to tectonic plate slipping and the formation of mountain ranges!

**54** *Creating Websites about Math - Not Just for Mathematicians***Nick Lassonde, Klamath Community College**<sup>†</sup>*(Session: Teaching Tricks, Techniques and Discoveries – Share what works for you)*

Even undergraduate students can create websites with beautiful mathematics displays. MathJax is a free tool that can be used to render math on web pages, and CodeCogs is a free online equation editor with LaTeX translation and HTML embedding of images. Teach your students how to create their own websites with fully-rendered math using MathJax, CodeCogs, and free hosting sites. This simple-to-use technique empowers students to create more than they ever imagined and can spark an interest in learning more about how technology works.

**55 *Beyond Email: Reaching Students Where They Are***  
**Kristin Lassonde, Klamath Community College<sup>†</sup>**

(*Session: Teaching Tricks, Techniques and Discoveries – Share what works for you*)

Email is not always the most effective way to reach our students. To improve instructor-student rapport and ultimately student success, consider implementing alternate methods of contact in your classes. Google Voice for texting and social media for other messaging are two powerful tools available for free which you could start using today! Stop sending unanswered emails and start actually communicating!

**56 *Klein links versus torus links, part I***  
**Ryan Lattanzi, Gonzaga University<sup>\*†</sup>**  
**Bryan Strub, Gonzaga University<sup>\*†</sup>**  
**Hayley Olson, Gonzaga University<sup>\*</sup>**

(*Session: NUMS IV—Topology*)

We will examine the relationship between Klein links and torus links, using both diagrammatic techniques and link invariants. We begin with definitions of these links and some basic results about Klein links.

**57 *Stimulating Students to Succeed with Standards Based Grading***  
**Jean Marie Linhart, Central Washington University<sup>†</sup>**

(*Session: Teaching Tricks, Techniques and Discoveries – Share what works for you*)

Students often don't master required prerequisite material in foundational courses, and then they struggle in later courses that require mastery of earlier concepts. To move students towards mastery learning and full proficiency, I implemented Standards Based Grading in Discrete Mathematics, requiring them to fully master certain areas in order to pass the class. Students can retake evaluations as needed to pass. Their grade is based on the number of learning objectives mastered over the course of the quarter.

**58 *Deconstructing and Reconstructing the Multiplication Principle***  
**Elise Lockwood, Oregon State University**  
**John Caughman, Portland State University<sup>†</sup>**  
**Zackery Reed, Oregon State University<sup>\*\*</sup>**

(*Session: Research in Undergraduate Mathematics Education II*)

The multiplication principle ("MP") is fundamental to combinatorics, underpinning many standard formulas and providing justification for counting strategies. Given its importance, the way it is presented in textbooks is surprisingly varied. In this talk, we identify key elements of the principle and present a categorization of statement types found in a textbook analysis. We incorporate excerpts from a reinvention study that shed light on how students reason through key elements of the principle. We conclude with a number of potential mathematical and pedagogical implications of the categorization.



**59** *Applying a Multilinear Regression Model to Predict Air Quality in Burns, Oregon*

**Travis Lowe, Eastern Oregon University**<sup>\*†</sup>

**Sydney Nelson, Eastern Oregon University**<sup>\*†</sup>

**Amy Yielding, Eastern Oregon University**

(*Session*: NUMS III—Computational/Statistical)

The City of Burns, Oregon has a serious air quality issue. The city frequently experiences very high levels of PM<sub>2.5</sub>. PM<sub>2.5</sub> consists of a variety of particulates whose size is less than 2.5 micrometers. Such particulates can be inhaled and generally accumulate in the lungs of humans, displaying a strong positive correlation with the instance of lung cancers. Field burning and wood burning for heat combined with the unique atmospheric conditions in Burns appear to be contributing to these high levels. In this talk we discuss the methods used in establishing a regression model to predict PM<sub>2.5</sub> for Burns. In particular, we worked in collaboration with The National Oceanic and Atmospheric Administration (NOAA) as well as The Oregon Department of Environmental Quality (ODEQ), who had established a preliminary regression equation. Improvements we made to this model were adding interaction terms and including data gathered from the previous day. The resulting equation improved the predictions of PM<sub>2.5</sub> by a 10% higher R-squared adjusted and displayed a narrower range of errors. Our model is now implemented as part of a pilot air quality alert system for the City of Burns sent out by ODEQ in collaboration with NOAA.

**60** *Examining Matroids with Unique Addresses*

**Roger MadPlume, University of Montana**<sup>\*\*†</sup>

(*Session*: General Papers II)

Moved from 15:05 to 14:45

In the September 2013 issue of Math Horizons Gary Gordon posed the following problem: For a finite set of points in the plane, write down the following data: For each point P, record the number of 3-point lines through P, the number of 4-point lines through P, and so on. Is there a finite set of points in the plane where each point has a unique nonempty address? Stan Wagon posted this same problem in the Macalester Problem of the Week forum. We use geometric representations of matroids to find the minimal solution to this problem. Then through construction identify a class of matroids with the property of unique addresses.

**61** *Looking for a (Super)resolution to an Image Processing Problem.*

**Alletha Maier, Linfield College**<sup>\*†</sup>

(*Session*: NUMS III—Computational/Statistical)

In recent years sparse coding has been employed to efficiently process images. Since recovering sharp images from images corrupted with noise is a well-known ill-posed problem, small perturbations in the image lead to large deviations in the reconstructed image. We look to combine research in superresolution with that of sparse coding for elucidation.

**62** *Commuting Pairs in Finite Nonabelian Groups*

**Tyler McAfee, Western Oregon University**<sup>\*†</sup>

(*Session: NUMS VI—Algebra/Geometry*)

The study of the probability that two group elements commute dates back to 1968 with the work of Paul Erdos and Paul Turan. Since then, much has been deduced about these probabilities, including its bound of  $5/8$  for nonabelian groups. During this talk, we will look at the associated probabilities of finite nonabelian groups and how to calculate such probabilities using several methods. Furthermore, reports will be made on known probabilities associated with dihedral groups and how to calculate probabilities with specified denominators as well as specified numerators. Finally, we will wrap up with looking at the group of  $GL(2, \mathbb{Z}_p)$  matrices and deducing the probability that two of these matrices commute.

**63 Decomposable Cyclically Presented Groups and Shift Dynamics**  
**Kirk McDermott, Oregon State University\*\*†**

(*Session: Algebra and Number Theory I*)

A group with a cyclically symmetric presentation admits an automorphism of finite order called the shift. In this talk we look at cyclically presented groups which admit a certain decomposition, and relate the shift dynamics for the group to the components of the decomposition. Topological methods are used to identify fixed points for powers of the shift.

**64 Geometric Invariants of Knots**  
**Christian Millichap, Linfield College†**

(*Session: Junior Faculty Research*)

Given two arbitrary knots (tangled up strings with their ends tied together), how can we (easily) tell if they are different or not? In general, this problem is extremely difficult to answer, and has led to the development of a variety of knot invariants. In this talk, we will examine geometric invariants of knots. These are knot invariants that arise from examining the (often hyperbolic) geometry of the space surrounding a knot. Historically, these invariants have been extremely useful in helping classify knots. However, it is possible to construct large sets of knots that are geometrically similar, that is, all the knots in such a set are different, yet these knots have a number of geometric invariants in common. We will give two examples of geometrically similar sets of knots and raise some interesting questions about geometric invariants.

**65 Infinity (Still) Blows My Mind**  
**Aaron Montgomery, Central Washington University†**  
**Doug DePrekel, Central Washington University**

(*Session: General Papers I*)

As filler in an undergraduate abstract algebra class, I tossed out a question that I had encountered in a high school math puzzler involving the final state of an infinite process. In the process, chips are added and removed from a bag and the question asks what remains at the end of all additions and removals. As the class discussed the solution, more questions arose than we could answer. The question sparked enough interest that an undergraduate and I decided to spend some time exploring the questions raised. Some of the early exploration suggested basic computational techniques, but soon we found that the more interesting questions led through an introduction to cardinality theory and the point-set topology of the real line. In the talk, I will present the original question and discuss some of our results. Hopefully, you will leave the talk with some new questions for your students to ponder.

**66 Overpartition Statistics****Thomas Morrill, Oregon State University\*\*†***(Session: Algebra and Number Theory II)*

Many partition results and  $q$ -series identities are classically derived through analytical techniques, though the results beg for a combinatoric interpretation. – the standard examples being the Ramanujan congruences. Historically, the combinatorics were filled in by studying integer valued functions on the set of partitions, namely the rank and the crank functions. We show how the ordinary partition statistics can be extended into the more general overpartition case.

**67 Mathday at University of Washington****James Morrow, University of Washington†***(Session: Thinking Outside the Circle: Alternate Outreach)*

Mathday is a day in which 1600 high school students and teachers are on the UW campus to attend lectures and activities. We have grants to support the attendance from remote schools and schools with under-represented populations. This is its twenty-sixth year.

**68 Kummer subspaces of Central Simple Algebras****Nhan Nguyen, University of Montana\*\*†***(Session: NUMS VI—Algebra/Geometry)*

Let  $F$  be a field containing a primitive  $d^{\text{th}}$  root of unity  $\rho$  and  $\text{char}(F) \nmid d$ . Let  $A$  be the tensor product of  $n$  cyclic  $F$ -algebras. An element  $v \in A$  is Kummer if  $v^d \in F$ . A subspace of  $A$  is Kummer if every element is Kummer. Kummer spaces have been used to bound the symbol length of a central simple algebras. In this talk we will discuss some open problems concerning Kummer subspaces

**69 Starved? Let's Solve That Math Hunger!****TJ Norton, Klamath Community College\****(Session: NUMS I—Mathematics Applications)*

We all enjoy that time of day when we get to sit down and enjoy our favorite meal. We typically eat three or more times per day. However, being a hard-working college student and living on a college budget often severely restricts the amount of money we can spend. Essentials like lunch, dinner, and second dinner could be much cheaper than what we currently spend. Everyone knows that coupons save us money on our everyday needs, but how many people know how to use mathematical logic to solve these expensive cravings? It is possible to reduce the size of your food budget by using equations to calculate the savings from coupons. Let's apply math so we can buy more late-night grub and keep up with the massive amounts of math homework!

**70** *Math and Sudoku: Exploring Sudoku boards through graph theory, group theory, and combinatorics*

**Kyle Oddson, Portland State University**<sup>\*†</sup>

(*Session: NUMS II—Combinatorics*)

Encoding Sudoku puzzles as partially colored graphs, we state and prove Akman's theorem regarding the associated partial chromatic polynomial; we count the 4x4 sudoku boards, in total and fundamentally distinct; we count the diagonally distinct 4x4 sudoku boards; and we classify and enumerate the different structure types of 4x4 boards.

**71** *Klein links versus torus links, part II*

**Hayley Olson, Gonzaga University**<sup>\*†</sup>

**Bryan Strub, Gonzaga University**<sup>\*†</sup>

**Ryan Lattanzi, Gonzaga University**<sup>\*</sup>

(*Session: NUMS IV—Topology*)

We will examine the relationship between Klein links and torus links, using both diagrammatic techniques and link invariants. We determine the types of components in a Klein link, and use this result to look at which Klein links are torus links and which are not.

**72** *Hamiltonian Properties of Toroidal Zero Divisor Graphs*

**Shannon Overbay, Gonzaga University**<sup>†</sup>

(*Session: Algebra and Number Theory I*)

The zero divisor graph of a commutative ring  $R$  is formed by taking the nonzero zero divisors of  $R$  as the vertices and connecting two vertices exactly when the corresponding product of the two elements is zero. We will show that all 44 planar zero divisor graphs are subgraphs of planar graphs with a Hamiltonian cycle and that all 46 genus one zero divisor graphs are subgraphs of toroidal graphs with a Hamiltonian cycle.

**73** *Algorithmic Variants of QR*

**Sally Peck, Western Oregon University**<sup>\*†</sup>

(*Session: NUMS III—Computational/Statistical*)

One of the fundamental computations in numerical linear algebra is the QR factorization. A QR factorization decomposes a matrix into the product of an orthogonal matrix and an upper triangular matrix. The algorithms that compute these decompositions can often be costly, and at times, do not perform well for particular matrices. We investigate different methods of computing a QR factorization on a tall and skinny matrix, that is a matrix with more rows than columns. We discuss algorithmic variants and the move to a new family of algorithms based on tiles.

**74** *G-Sets and Sequences Associated with  $n$ th Order Linear Recurrences Modulo Primes*

**Robert Ray, Gonzaga University**<sup>†</sup>

(*Session: Algebra and Number Theory I*)

We consider  $n$ th order linear recurrence relations of the form  $S_k = a_{k-1}S_{k-1} + a_{k-2}S_{k-2} + \cdots + a_{k-n}S_{k-n}$  over the finite field  $Z_p$ , where  $p$  is a prime not equal to 2. The results regarding the distribution of elements in the sequence  $\{S_0, S_1, \dots\}$  are well known for second order linear recurrence relations, however, we expand some results using matrix groups, linear algebra and  $G$ -sets in the finite vector space  $(Z_p)^k$ . It is our hope that this alternate approach may provide a set of material or examples that could be utilized in undergraduate mathematics courses.

**75 *Girls' Day or Mdchen-Zukunftstag***

**Annie Raymond, University of Washington<sup>†</sup>**

(*Session: Thinking Outside the Circle: Alternate Outreach*)

We discuss what happened when a whole country decided to reach out to its teenage girls to get them to be more involved in math, science and technology.

**76 *Student Generalizations of Distance and the Cauchy Property***

**Zackery Reed, Oregon State University<sup>\*\*†</sup>**

(*Session: Research in Undergraduate Mathematics Education I*)

The Cauchy Property is an important characterization of convergent sequences in complete metric spaces. Students were observed reflecting on the nature of Cauchy Sequences on  $\mathbb{R}$ , and were then prompted to generalize the definition of a Cauchy sequence into more abstract settings. Their generalizations were implemented into various metrics to eventually be leveraged for the re-invention of the abstract metric definition. This talk will explore some of the students' initial generalizing activity and will describe some of their cognitive schemes developed through interactions with various notions of distance.

**77 *The Difficulty of Classifying Decomposable Torsion-Free Abelian Groups***

**Kyle Riggs, Eastern Washington University<sup>†</sup>**

(*Session: Junior Faculty Research*)

An abelian group is decomposable if it can be written as a direct sum of two (or more) nontrivial subgroups. Otherwise it is indecomposable. The only indecomposable torsion groups are cyclic groups of the form  $\mathbb{Z}(p^n)$ , where  $p$  is prime (as well as Prüfer groups,  $\mathbb{Z}(p^\infty)$ ). Mathematicians have been unable to describe the class of decomposable torsion-free groups. As it turns out, this problem is analytic complete (it cannot be characterized by a first-order formula). We show this by comparing it to the problem of determining whether an infinitely branching tree in  $\omega^{<\omega}$  has an infinite path.

**78 *Infinitesimal-Based Calculus***

**Eric Rogers, Gonzaga University<sup>\*†</sup>**

**Ethan Snyder, Gonzaga University<sup>\*†</sup>**

(*Session: NUMS VII—History/Math Ed*)

When Newton and Leibniz first developed calculus, they did so by using infinitesimals (really really small numbers). Infinitesimals were used until calculus was made more rigorous by

Weierstass. The calculus that we are taught today is based on Weierstass's - definition of the limit. However, people have been arguing that we go back to an infinitesimal-based calculus, not only for its historical roots, but because many proofs and concepts seem to be much cleaner when using infinitesimals. Using Keisler's *Elementary Calculus: An Infinitesimal Approach*, our group set out to relearn calculus using infinitesimals. First we will define the hyperreal number line (an extension of the real line that contains the infinitesimals). Then we will walk through the familiar ideas and concepts of single variable calculus, such as limits, derivatives, and integrals, reformulated in terms of hyperreals.

### 79 *An Active STEM-Prep Curriculum*

**Francisco Savina, University of Texas at Austin**

(*Session: General Papers I*)

The STEM-Prep Pathway is designed as two one-semester courses created by the New Mathways Project that prepare students beginning at the elementary algebra-level to succeed in college-level calculus. All lessons are designed to be contextual and meaningful, with guided student inquiry at the core. Each lesson is 25 minutes long and is typically preceded by a short preview assignment, which students complete before class. The preview assignment reviews skills required for the upcoming lesson, and asks readiness questions that are designed to help the student determine if they need additional support prior to class. Each lesson begins with an easily accessible opening question which includes the experiences and opinions of all students. Students complete practice assignments to cement their learning. All lessons include detailed instructor notes suggesting pedagogical approaches, facilitating questions, and the lesson's constructive persistence (CP) level. Early in the course, lessons are designated as CP 1 or 2 as students build their ability to work independently. A CP 3 level promotes productive struggle with engaging problems that are more open-ended. For this talk I will share examples from the curriculum that exemplify these design principles.

### 80 *Numerical Inversions of the Broken Ray Transform with fixed initial and terminal directions*

**Brian Sherson, Oregon State University\*\*†**

(*Session: General Papers II*)

Moved from 14:25 to 14:05

The Broken Ray transform is a transform used in single-scattering tomography, and was introduced by Lucia Florescu, Vadim A. Markel, and John C. Schotland, in 2009, and shown to be invertible in the case of fixed initial and terminal directions. An inversion formula for this case, involving a second-order derivative and an integration was presented previously in 2012, along with some numerical results. In this talk, we will explore improvements to the numerical inversion, as well as view a numerical inversion from two sets of data that requires only first-order differentiation and no integration.

### 81 *Systems of parameters and the Cohen-Macaulay property*

**Katharine Shultis, Gonzaga University†**

(*Session: Algebra and Number Theory II*)

Cohen-Macaulay rings play a central role in commutative algebra and there are many connections between systems of parameters and the Cohen-Macaulay property. In a Cohen-Macaulay

ring, every system of parameters is also a regular sequence (roughly speaking it behaves like a set of polynomial variables). A classical result due to Rees says that when working in a Cohen-Macaulay ring, a certain class of modules of homomorphisms defined by systems of parameters is always isomorphic to a certain free module of rank one. Recently, K. Bahmanpour and R. Naghipour showed that in a non-Cohen-Macaulay ring, the same class of modules of homomorphisms is sometimes decomposable as a direct sum, and therefore is not a free module of rank one. In this talk, we will present stronger theorems in the non-Cohen-Macaulay case, and present illustrative examples about the decompositions obtained.

### **82** *A Basis for the Space of Order 5 Chord Diagrams*

**Allison Stacey, Oregon State University\*\*†**

(Session: NUMS IV—Topology)

In the study of Vassiliev Knot Invariants, the algebra of chord diagrams plays a key role. A chord diagram of order  $n$  is a circle with  $2n$  vertices around it with chords through the circle connecting the vertices pairwise. The algebra of such diagrams is isomorphic to closed Jacobi diagrams which are trivalent graphs with  $2n$  vertices and a circle around the edge but here the vertices are inside the circle. I used the relationship between these two algebras to find a basis for each in order 5. I write up my results as art pieces so this talk will include many pieces of my art.

### **83** *Perturbing Equilibria of the Three-Body Problem*

**Joseph Stauss, Gonzaga University\*†**

(Session: NUMS III—Computational/Statistical)

The Lagrange-Relative Equilibrium and the Figure-8 Equilibrium are the only known periodic solutions of the Three-Body problem in the case of equal masses. These equilibria were analyzed by using a finite-difference method to approximate their perturbation-response for various quantities. The resultant behavior is discussed, explained, and classified.

### **84** *Specific Examples, Generic Elements and Size Tuning - Overcoming Student Roadblocks in Linear Algebra*

**Jeffrey Stuart, Pacific Lutheran University†**

(Session: Teaching Tricks, Techniques and Discoveries – Share what works for you)

Linear algebra is often the first math course in which sets play an explicit and fundamental role. Consequently students typically struggle with writing proofs for set-based results. In this talk, I focus on four key strategies to improve student success. 1. Emphasize the role of specific (fully specified) examples as examples to highlight definitions, and, more importantly, as counterexamples to universal statements. 2. Emphasize what a generic element from a set is, how to write one, and what role it plays in proofs about sets. 3. Emphasize the different and noninterchangeable roles of specific examples and generic elements. 4. Thoughtfully tune the sizes of vectors and matrices in problems to focus students on the primary idea at hand. Specifically, use small shapes to encourage students to populate objects and to free students from dealing with the technical complications of large shapes. In contrast, use large shapes to discourage students from employing an entrywise approach, and from populating specific entries in an object. These key strategies will be illuminated by a discussion about spans.

**85 *Pathological Continuity: A Zoo of Nowhere-Differentiable Functions (Cancelled)*****Nathan Taylor, University of Montana, Missoula\*\*†***(Session: General Papers II)*

Poincare said of pathological functions, "logic sometimes makes monsters." We will investigate some classical examples of the monsters which are continuous but nowhere-differentiable real functions. We will take a historical perspective, with emphasis on visualizing the various examples of these functions.

**86 *Rearrangement in an Infinite Series*****Lane Thomason, Southern Oregon University\*†***(Session: NUMS VII—History/Math Ed)*

Typically we are free to use associative and commutative properties without problems with real numbers. However, under certain conditions, these can break in infinite series. This talk will look into what happens when these conditions are met.

**87 *Breaking Bad Symmetries*****Gerald Todd, University Of Montana\*\*†***(Session: General Papers II)*

Moved from 14:45 to 14:25

Point-line configurations in the plane can have many types of symmetries. We will investigate bijections of point-line configurations that preserve a certain structure (automorphisms). Of course, to 'break' these symmetries, we can simply fix all points, but we are interested in the fewest number of points that 'break' all non-trivial automorphisms, called the fixing number. We can view these point-line configurations as matroids and get some more general results.

**88 *Hypothesis Testing with Person-Time Data*****Terri Torres, Oregon Institute of Technology†***(Session: General Papers I)*

In the area of epidemiology person-time data is a common measurement. I will address the inference associated with this measurement.

**89 *The process of representing food webs as interval competition graphs*****Kayla Vincent, Western Oregon University\*†***(Session: NUMS I—Mathematics Applications)*

A food web is defined as an acyclic graph where vertices represent different species and there is a directed edge from species  $x$  to species  $y$  if species  $x$  preys on species  $y$ . Food webs are important in Biology because they model the flow of energy in an ecosystem. A competition graph has the same vertex set as a food web, but now two vertices are adjacent if they prey on a common species in the food web. Most of these competition graphs are interval graphs. Interval graphs are graphs where vertices can be represented as intervals of the real line such that vertices are adjacent if and only if their intervals overlap. In this talk, we will explain these structures and their relationships with real examples from Biology.



**90 Student Conceptions of Isomorphism****Kristen Vroom, Portland State University\*\*†****Kate Melhuish, Teachers Development Group***(Session: Research in Undergraduate Mathematics Education II)*

During creation of the Group Concept Inventory (GCI), we discovered that the initial question related to isomorphism adapted from Weber & Alcock (2004) (Are  $Q$  and  $Z$  isomorphic?) contained a number of hidden complexities related to student understanding of isomorphism. We developed two new questions to attempt to better parse apart student conceptions around the topic. One question targeted cardinality and the second structural sameness. The questions were piloted with group theory students across the country. We analyzed the data using a thematic analysis approach. Additionally, we conducted five follow-up interviews to further make sense of student reasoning related to isomorphism. In this presentation, we focus on student use of formal and informal definitions, their ability to explain the relationship between structural sameness and the formal definition, and their flexibility addressing non-familiar properties.

**91 A Game of 1s and 2s: Constructing level 2 Demazure Filtrations of level 1 Demazure modules****Jeffrey Wand, Gonzaga University†***(Session: Algebra and Number Theory I)*

In this talk we dive into the representation theory of Lie algebras. Representation theory is an area of math that studies algebraic structures and the objects they act on. In addition, it is also a great tool that takes problems in abstract algebra and turns them into linear algebra problems. More specifically, we will be looking at what is called the current algebra  $sl_2 \otimes \mathbb{C}[t]$ , where  $sl_2$  is the space of complex  $2 \times 2$  matrices whose trace is zero, and  $\mathbb{C}[t]$  is the space of polynomials with complex coefficients. The combinatorics involved in this talk are motivated by giving an explicit filtration (chain of submodules) of a well-studied family of modules, the level  $\ell$  Demazure modules, which are indexed by a natural number  $\ell$  and a vector  $\lambda$ . We know such a construction must exist by Naoi who proved the existence for every  $\ell \geq 1$ . In this talk we will construct maps that determines our explicit filtration. In defining these maps we have created a game that amounts to turning 1s into 2s. This game can also be played on a directed graph, turning the filtration question into one that investigates certain ways you can traverse this graph. We will investigate this graph and talk about some generalizations.

**92 An Investigation of Subtraction Algorithms Utilized in the US during the 18th and 19th Centuries****Nicole Wessman-Enzinger, George Fox University†***(Session: General Papers II)*

Over 30 arithmetic texts and 280 cyphering books utilized in the United States during the 18th and 19th centuries were examined for subtraction algorithms. A framework for different types of subtraction algorithms utilized at this time will be presented. The investigation revealed that same algorithms were not utilized in arithmetic texts and cyphering books providing evidence of support that printed books as solitary source of historical investigations are an insufficient representation of this time period in the United States.

**93 *Prospective Teachers Reasoning about Integers and Temperature***  
**Nicole Wessman-Enzinger, George Fox University<sup>†</sup>**

(*Session: Research in Undergraduate Mathematics Education II*)

Ninety-eight elementary and middle school prospective teachers participated in a study focusing on integer addition and subtraction while enrolled in an introductory mathematics content course emphasizing number concepts and operations. Across two academic semesters, the prospective teachers posed 784 stories for integer addition and subtraction number sentences. Of these, 108 of the stories included the context of temperature. The stories about temperature were analyzed for mathematical correctness, consistency, realism, and problem types. A framework for different problem types for integers was modified. Other complexities for posing these types of stories, such as realism of the temperature stories, will be discussed.

**94 *The Diversity of the national Math Circle movement***  
**Brandy Wieggers, Central Washington University, National Association of Math Circles<sup>†</sup>**

(*Session: Thinking Outside the Circle: Alternate Outreach*)

Originating in Eastern Europe, Math Circles spread to the United States in the 1990s. They emerged approximately at the same time on both the east and west coast, and have spread to almost every state, numbering around 200 today. While the first wave of Math Circles in the United States started with a focus on preparing mathematically talented high school age youth for advanced mathematical competitions, their focus has now broadened. Math Circles now serve a range of ages (preschool through high school) as well as teachers. Some still focus on preparing students for local or national competitions; others provide non-competitive, mathematical enrichment experiences for all interested students. In this presentation, I will share my work with the San Francisco and Kittitas Valley Math Circles, both of which take a unique approach to outreach. I'll also share resources of the National Association of Math Circles ([mathcircles.org](http://mathcircles.org)) to help you start your own Math Circle.

**95 *Reading  $\pi$ : Helping children move from symbol to meaning***  
**Emma Winkel, Pacific University<sup>\*†</sup>**

(*Session: NUMS VII—History/Math Ed*)

Students currently learn of  $\pi$  in a formulaic context; as a value needed for the effective calculation of the circumference or area of a circle. In this talk, we present an activity that uses manipulatives to help middle grade students develop an understanding of the geometrical meaning of  $\pi$ .

**96 *The subregular part of Lusztig's asymptotic Hecke algebra***  
**Tianyuan Xu, University of Oregon<sup>\*\*†</sup>**

(*Session: Algebra and Number Theory I*)

Given an arbitrary Coxeter system  $(W, S)$ , Lusztig defined its asymptotic Hecke algebra  $J$ , an associative algebra closely related to the usual Hecke algebra and the category of Soergel bimodules for  $(W, S)$ . The algebra  $J$  decomposes as a direct sum of subalgebras indexed by the 2-sided Kazhdan-Lusztig cells of  $W$ , and we present some results on the subalgebra  $J_c$

corresponding to a particular cell  $c$  known as the subregular cell. We show that products in  $J_c$  can be computed by repeated use of (variations of) the Clebsch-Gordan formula arising from the representation theory of  $sl_2$ , and we use this multiplication rule to obtain alternative descriptions of  $J_c$  for Coxeter systems of certain types (such as all simply-laced ones).

**97** *Improving algebra skills of university students through participation in academic service-learning*

**Ekaterina (Katya) Yurasovskaya, Seattle University<sup>†</sup>**

(*Session: Research in Undergraduate Mathematics Education I*)

Seattle University has a long history and a solid institutional structure for implementing academic service-learning in its courses. For the present study, we developed a Precalculus course with a service-learning component, allowing university students to work in the tutoring labs at a local middle school, an immigrant assistance center, and a community college, and to tutor algebra prerequisites to middle-school students and to adults returning to complete their GED diploma. One of the primary goals of the project was to improve basic algebra skills of the student tutors by explaining foundational material to others 2-3 hours per week over the course of the quarter. Through weekly pedagogical diaries, the student tutors analyzed the source and nature of mathematical successes or misconceptions of their own students. Review of the final exams via a special rubric revealed a significant reduction in the number of fundamental mistakes between the Precalculus section with the service-learning component and the control section of the same course. In our talk, we discuss the collaboration with community partners, the course structure and the key components that enhanced student learning, and academic and non-academic benefits to the participants.