### Annual Meeting of the Pacific Northwest Section of the Mathematical Association of America

Linfield College McMinnville, Oregon April 13-14, 2007



Annual Meeting of the Pacific Northwest Section of the Mathematical Association of America

Sponsored by The Mathematical Association of America Linfield College Clark College Foundation The Walter P. Dyke Endowment

Co-hosted by Linfield College and Clark College

> Linfield College Mathematics Department McMinnville, Oregon April 13-14, 2007

#### Friday, April 13

7:45	Packet pickup for Project NExT		
	Riley 201		
8:00	Project NExT Meeting		
2:30	Riley 201		
2:45	Minicourse:	<b>Minicourse:</b>	
	Combinatorial Games [1]	<b>Biological Modeling</b> [2]	
5:45	Taylor 101	Taylor 201	
6:00	Project NExT Dinner & Discussion		
	Golden Valley Brewery, 980 NE 4th Street, McMinnville		
	503-472-BREW(2739)		
6:45	Packet Pick-up		
	Melrose		
7:00	Student Reception – All meeting participants welcome!		
	Jonasson, Melrose		
8:00	Walter P. Dyke Lecture: John Conway		
	The Free Will Theorem [3]		
	Ice Auditorium, Melrose		
9:00	Reception		
	Jonasson, Melrose		

### Saturday, April 14

7:30			Execut	tive Committee
8:00				Meeting
		Packet Pickup	Dill	in West Wing
8:30		& Registration		C
9:00	-	Welcome: Barbara Seidman		Barbara Seidman
		Dean of Faculty, Linfield College		ulty, Linfield College
		Murdock	Ice Aud	litorium, Melrose
		Atrium	Invited Talk:	Elwyn Berlekamp [4]
			"Fibonac	ci Plays Billiards"
10:00			Ice Aud	itorium, Melrose
10:15				
	I	Contributed Talks		
10.10	e, T	Murdock, Graf, and Taylor		
12:10	Salo Me		T	
12:15	ok S Dn,	Lunch (provided) Lunch (provided)		ch (provided)
1.00	<b>Boc</b> asso	Murdock Atrium	Jona Dergin ogg	sson, Melrose
1:00		Guring	Business	Community Collogo Donal
	J	Student Problem	Meeting	College Panel Malroca 210
1.45		Solving Session Malrosa 206	Wielfose 202	Wielfose 210
2.00	-	Invite	d Talk• David '	Wolfe [5]
2.00		"In Tribute to Martin Gardner"		
3:00		Ice Auditorium Melrose		
3:15	-			
		<b>Contributed Talks</b>		TI Demo [80]
		Murdock, Graf and Taylor		Graf 205
5:10			-	(3:15-4:45)
5:30		Social Hour		
		Evergreen Aviation Museum		
6:30		Banquet Dinner		
		Evergreen Aviation Museum		
7:15		Section Awards		
7:45		Invited Lecture: John Conway [6]		
		Infinite and Infinitesimal Numbers and Games		
		Evergreen Aviation Museum		

#### **Program of Contributed Papers**

The program of contributed papers appears on the following pages. In some cases, titles or other information are abbreviated for reasons of space; please see the full abstract for more information.

- This year's two Mathematical Contest in Modeling problems were
  - 1. Problem A: *The Gerrymandering Problem*, which "asked teams to develop a model for 'fairly' and 'simply' determining congressional districts for a state." Talks on Problem A are designated with a (G).
  - 2. Problem B: *The Plane Boarding Problem*, which "required teams to devise and compare procedures for boarding and deboarding planes with varying sizes from 85 to 800." Talks on Problem B are designated with a (P).

See the COMAP website, http://www.comap.com, for more information.

• An asterisk (\*) indicates which of the contributors listed is/are making the presentation if not all contributors are.

#### **Session Organizers**

- Student Papers on Modeling and the MCM: Curtis Feist, Southern Oregon University
- Student Papers: Michael Aristidou, DigiPen Institute of Technology
- Alternative Teaching Methods for Math Courses: Stephanie Salomone, University of Portland; Michael Aristidou, DigiPen Institute of Technology
- General Contributed Papers: Colin Starr, Willamette University
- Research Talks: Michael Puls, Eastern Oregon University
- Junior Faculty Research Talks: Jennifer Halfpap, University of Montana
- Interdisciplinary Approaches to Teaching Lower Division Mathematics: Peter Littig, University of Washington, Bothell

Please contact the session organizer with any questions about a session.

Moderators: Please start each talk on time, but **not** early. Folks from other sessions will want to be there when the talks begin.

Contributed Talks – Saturday Morning			
	Students: MCM and Modeling	Students I	Students II
	Graf 109	Graf 101	Graf 209
10:15–10:30	Simple as a Square (G) [17]	Finding Polynomials that Annihilate Algebraic Numbers [76]	The Logarithmic Derivative: What Is It Good For? [14]
	Holly Bochsler, Karen Lange, WOU	Brian Shearson, CWU	Dale Blem, Pacific
10:35–10:50	Get on the Plane!! Minimizing Plane	Evaluating	Derivatives of Natural Numbers Using the
	Boarding Time (P) [53]	$\int_0^\infty \frac{\sin^{2n+1} x}{x} dx$	Logarithmic Derivative [36]
		for $n \in \mathbb{Z}^+$ [59]	
	Erica Jossi, Jared Pierce, Sisi Zhang, Linfield	Mike Leatherman, CWU	Tara Fechter, Pacific
10:55–11:10	Eliminating Gerry- mandering (G) [31]	Sudoku: What Did I Learn From This Project? [78]	Using Markov Chains to Calculate the Probability of Penetration [9]
	Todd Curtis <sup>*</sup> , Ed- ward Smith <sup>*</sup> , J. Jed- idiah Rembold, Lin- field	Eric Smyth, UW	Brian Bauman, Wil- liam Sehorn, Whit- worth
11:15–11:30	The Aircraft Seating Problem (P) [49]	Cosets and Cayley-Sudoku Tables [73]	Rhythmic Oddity: A Counting Problem in Music Theory [47]
	Chinh Hoang, Rachel Kaneta, Ian McCreary, Huy Nguyen, Linfield	Keith Schloeman, WOU	Kyle Haverly, Whitworth
11:35–11:50	Can a Salamander Become a Turtle? Avoiding Gerryman- dering (G) [75]	How is a Minimal Circle Formed Around a Set of Points? [83]	Welcome to the Neighborhood: Seymour's 2nd Neighborhood
	Jason Shaw*, Eddie Strickler*, Clayton Zylstra, Seattle Pac.	Vanessa Wyffels, WOU	Molly Robinson, Whitworth
11:55–12:10	Now Boarding Rows 25 and Higher(P) [77]	A Tale of Two Triangles [15]	What are Matroids? [25]
	Blair Sherman, Me- lissa Thompson, Ali- sha Zimmer, CWU	Holly Bochsler, WOU	Timothy Carstens, Seattle U

Contributed Talks — Saturday Morning			
Alternative Teaching Methods	Research	General	Junior Faculty Research
Murdock 105	Taylor 106	Graf 107	Taylor 101
Projects in Taxicab Geometry [18]	A Multiplicity Problem Involving Schur-like Triples in a Linear Inequality [70]	A Least-time Path for a Sailing Ship [42]	Singular Points of Real Sextic Curves [82]
Ton Boerkoel, DigiPen	Dusty Sabo, SOU	Gary Gislason, Willamette	Nicholas Willis, Whitworth
"You want me to do what?" Teaching Software Engineering "Formal Methods" [62]	Extremal Eigenvalues and Applications [8]	A Quadratic Approach to a Circular Area [44]	An Application of the Slice Theorem for the Reconstruction of X-Ray Data [64]
Bart Massey, PSU	Laurie Battle, Montana Tech	Russ Gordon, Whitworth	Cristina Negoita, OIT
So You Want to Teach Grad Stud- ents to Teach [54]	Non-existence of Limit Cycles in Analytic and Polya Vector Fields [26]	A Lower Bound for a Two-Forbidden Distance Chromatic Number of the Plane [50]	The Trichotomy Character of $x_{n+1} = \frac{\beta_n x_n + \gamma_n x_{n-1}}{A_n + B_n x_n}$ [66]
Gulden Karakok, Aaron Wangberg, OSU	Don Chalice, WWU	Matt Hudelson, WSU	Carol Overdeep, St. Martin's
Guided Discovery in Discrete Math for Majors [38]	How to Slice a Doughnut [35]	Towards a Com- plete Taxicab Geo- metry: A Tale of Two Metrics [19]	Homoclinic Solutions for a Hamiltonian [13]
Mary E. Flahive, John W. Lee, OSU	Isaac Erskine, Robert Thompson, PSU	Ton Boerkoel, DigiPen	James Bisgard, CWU
Example- Generation Tasks to Probe Students' Understanding of Mathematics [21]	Combinatorial Sums via Finite Differences [79]	An ISETL-based System for Polynomial Algebra [12]	Class and Unit Groups in Number Field Extensions [34]
Marianna Bogomolny, SOU	Michael Spivey, UPS	Dave Beuerman, SUNY	Caleb Emmons, Pacific
Upper Division Mathematical Modeling at a Small Four-Year School [51]		Beaucoup de Sudoku [56]	A Combinatorial- Analysis Invariant of Graphs [52]
Chris Hay-Jahans, UAKSE		Mike Krebs, CSLA	Mohammad Java- heri, UO

Contributed Talks — Saturday Afternoon			
	Students: MCM and Modeling	Students I	Students II
	Graf 109	Graf 101	Graf 209
3:15–3:30	American Airlines' Next Top Model (P) [23]	A Review of Poset Products [71]	Symmetric Plane Curves and Their Kin [69]
	Spencer K'Burg, UPS	Kevin Saff, Calgary	Peter Rowell, OSU
3:35–3:50	How to Make Abe Lincoln Get Off the Vacuum Cleaner: Creating Sensible District Lines in New York (G) [72]	Another Solution to the Postage Stamp Problem [30]	The Hopf Map and Its Connection with Division Algebras [41]
	Akbar Saidov, Maria Walters, Pacific	Paige Cudworth, Willamette	Henry Gillow-Wiles, OSU
3:55-4:10	Flights of Inspiration: Taking the Red Eye Through Optimization (P) [37]	Determinants of Matrices given by Pascal-like Recurrence Relations [84]	Free Groups and Gen- erators and Relations [61]
	Tara Fechter, Kerensa Gimre*, and Kevin Shawver, Pacific	Andrew Zimmer, UPS	Martini Machado, SOU
4:15-4:30	A Visualization of Stable and Unstable Motion of a Rigid Body [43] Amber Goodrich.	Classifying Quasiplatonic Surfaces Admitting Certain Common Automorphism Groups [11]	Asymptotic Behavior of Travelling Wave Solutions to Reactio-Diffusion Equations [46]
	Melissa Thompson, CWU	Robert Benim, UP	Brandi Harrison, Linfield
4:35-4:50	Modeling of Suspension Bridges [28]	Rook Polynomials in 3-D [60]	Comparative Population Modeling for Interdependent Species [22]
	Avery Cotton, WOU	Arthur Low, Shalin Mehta, DigiPen	Tyler Bryson, Linfield
4:55–5:10	The Physics of Tossing a Coin [16]	Calculating Volume using IR-Sensors and Riemann Sums [7]	Classifying Trees and Forests by Game Chromatic Number [33]
	Karen Lange, Holly Bochsler, WOU	Michael Anderson, Stephen Niedzielski, DigiPen	Kira Durand, Linfield

Contributed Talks — Saturday Afternoon				
Alternative Teaching Methods	Research	Interdisciplinary Approaches to the Lower Division		
Murdock 105	Taylor 106	Graf 107		
Method of Strategies for Proof Compression in Advanced Calc [10]	On the Dimensions of Bivariate Spline Spaces [57]	Teaching Quantitative Reasoning through Ethnomathematics [55]		
Scott Beaver, WOU	Wai Lau, Seattle Pacific	John Kellermeier, TCC		
The Use of Protocols and Open-Ended Tasks in an Integrated Mathematics Course for K-8 Teachers [39]	Finitely Generated Groups And p-Harmonic Boundaries [67]	Interdisciplinary Lively Application Projects at Central Washington [20]		
Maria Fung, WOU	Mike Puls, EOU	Stuart Boersma, CWU		
Modified Moore Method in Senior-Level Analysis [45]	Geometry of Mutually Stationary Points of Quadratic Forms [65]	Teaching Mathematical Induction as a Rule of Inference [63]		
David Hartenstine, WWU	Yves Nievergelt, EWU	Ken Meerdink, DigiPen		
Getting It Right: WeBWorK at Albertson College of Idaho [29]	Coverings of Triangular Billiards Surfaces [74]	Learning Experiences that Integrate Algebra & Another Discipline [58]		
Robin Cruz, ACI	Jason Schmurr, OSU	Jennifer Laveglia, BCC		
Teaching with Classroom Voting [27]	Identifying Lie Subalgebras Using Root Diagrams [81]	Supporting Students Learning Interdisciplinary Mathematics [48]		
Kelly Cline, Carroll	Aaron Wangberg, OSU	Nicole Hoover, UW Bothell		
Teaching Statistical Literacy: One Approach [32]	Finite Codimension in the Hilbert Cube [40]	Collaboration Between English and Math Pre-service Teachers [24]		
Jill Dumesnil, UAKSE	Kailash Ghimire, OSU	Beth Buyserie, Kimberly Vincent, WSU		

#### Social Events

#### Friday Project NExT Dinner

Golden Valley Brewery: 980 NE 4th Street, McMinnville. Phone: 503-472-BREW(2739) Sponsored by the MAA and Project ACCCESS

> Friday Student Reception Jonasson, Melrose

#### Friday Invited Lecture: John Conway [3]

Ice Auditorium, Melrose

Friday Reception Jonasson, Melrose

Saturday Lunch Murdock Atrium

#### Student Problem-Solving Session Melrose

#### Saturday Evening Social Hour Banquet Dinner and Awards Ceremony

Evergreen Aviation Museum

A few words from Nancy Neudauer (Pacific University), our Section Governor

Recognition of student presenters, Nancy Neudauer, Pacific University

Introduction of new Section Project NExT Fellows, Nancy Neudauer, Pacific University

Presentation of 25- and 50-year MAA membership certificates, Stuart Boersma, Central Washington University

Acknowledgment of Program Committee, Jennifer Nordstrom, Linfield College

2006 PNW MAA Distinguished Teaching Award, presented by Kimberly Vincent, Washington State University.

John Conway [6]

#### Minicourse Descriptions Friday, April 13

#### **1** A Tour of Combinatorial Games Elwyn Berlekamp, UC Berkeley David Wolfe, Gustavus Adolphus College Taylor 101, 2:45 – 5:45 p.m.

Throughout the twentieth century, games have been used effectively to popularize mathematics thanks to the efforts of authors such as Martin Gardner, John Conway and Raymond Smullyan. Combinatorial game theory is a rich and approachable unified theory, bridging recreational and abstract mathematics, bringing fun to fundamentals. Unlike classical game theory, the field of combinatorial game theory analyzes two-player games of complete information where players take turns. A complete information game is one such as Chess, where the entire situation is known to both players. This is in contrast with Poker, where cards in opponents' hands are, presumably, hidden. The complexity of combinatorial games comes from the large number of possible sequences of moves rather than from lack of knowledge of an opponent's decisions.

In the early 1970s, John Conway initiated an axiomatic theory of partisan games, many fruitful ramifications and extensions of which have continued to evolve and develop ever since. The pioneering works in this subject include John Conway's On Numbers and Games, Don Knuth's Surreal Numbers and the playful yet profound treatise, Winning Ways by Elwyn Berlekamp, John Conway and Richard Guy. This latter book uncovered many new games, which they and others have partially or completely analyzed. Albert, Nowakowski, and Wolfe presented many more new games in an introductory textbook called Lessons in Play.

This minicourse is designed as an introduction to combinatorial game theory aimed at faculty from all fields of mathematics.

**2** An Approach to Population and Biological Modeling for Pre- and Post-Calculus Students

#### Sharon Brown, Humboldt State University Chris Dugaw, Humboldt State University Taylor 201, 2:45 – 5:45 p.m.

Mathematical modeling and other quantitative analyses are becoming an ever more important part of biological research, and it is important to provide future biologists and mathematicians a basic understanding of these methods. This minicourse will outline important topics in biological and population modeling that can be covered in calculus or pre-calculus based courses. The material will be presented in such a way that the audience could incorporate topics into an existing standard course or develop a full course on modeling. We will cover the basics of model development, analysis, and interpretation using data, graphical, and symbolic approaches. We also provide a framework for using projects to get students to think more independently and deeply about modeling and to train students to communicate about mathematics.

### Abstracts of Invited Talks

#### (in chronological order)

#### **3** The Free Will Theorem John Conway, Princeton University

This proves that if, as most of us believe, there exist experimenters having a certain amount of free will, then elementary particles must already enjoy their own small share of this valuable commodity. The theorem was proved jointly with Simon Kochen.

#### 4 Fibonacci Plays Billiards Elwyn Berlekamp<sup>\*</sup>, UC Berkeley **Richard Guy, University of Calgary**

The problem of finding a Hamiltonian path or cycle in an arbitrary graph is a special case of the traveling salesman problem. Both are known to be NP Hard. Interesting special cases include the following: Given a set of positive integers, S, and an integer N, find an ordering of the integers from 1 through N such that the sum of every adjacent pair lies in S. If there is a solution, then we say that N can be "chained" by S. This problem seems more tractable. Typical choices for the set S include the squares, the cubes, or the Fibonacci numbers. Many specific choices of N and S provide puzzles which are easily accessible to elementary school students. There are also harder questions, such as whether all sufficiently large values of N can be chained by a specific set S, and if so, what is the largest integer which cannot.

This talk explores some of these questions.

#### **5** In Tribute to Martin Gardner **David Wolfe, Gustavus Adolphus College**

In this talk, I will discuss at least two fun elementary problems:

First, in how many ways can you place stones on an *m* by *n* grid so that each square is adjacent to an odd number of stones? For example, on a  $7 \times 5$  grid there are exactly two ways:



(Joint work with Erick Knight, St. Peter High School)

For the second problem, it's long been observed (by Dr. Matrix, among others) that if you multiply the number 123456789 by 2, 4, 5, 7, or 8 that the result permutes all 9 digits:

 $123456789 \times 2 = 246913578$  $123456789 \times 4 = 493827156$  $123456789 \times 5 = 617283945$  $123456789 \times 7 = 864197523$  $123456789 \times 8 = 987654312$ 

I will address why this is the case by generalizing the problem to base b numbers with b - 1 digits.

#### 6 Infinite and Infinitesimal Numbers and Games John Conway, Princeton University

Cantor's infinite cardinal numbers are well-known to most mathematicians - his infinite ordinal numbers less so. I shall discuss both types before passing to the larger systems of partisan games and the surreal numbers that simultaneously generalize both the ordinal and real numbers.

#### Abstracts of Contributed Talks

(in alphabetical order, by presenter)

#### 7 A Method of Calculating Volume using IR-Sensors and Riemann Sums Michael Anderson, DigiPen Institute of Technology Stephen Niedzielski, DigiPen Institute of Technology

Volume calculations of irregular shapes often require the use of sampling and function analysis to construct approximations. This project investigated the use of infrared range finding sensors and Riemann Sum calculations to approximate the shape and volume of an irregular shape. A device was constructed to sample points on the object, rotating the shape as the sensors took measurement data. This information was transmitted to a computer for further processing. The data points were grouped into cross-section slices of the object. This allowed the slice to be interpreted at vertices of a polygon, which was subdivided into triangles for volume calculations. Summing the combined slices provided a volume approximation of the entire shape. A software interface allows for data analysis, complete with a geometric model representation of the sample data. While filtering difficulties introduced errors in the final shape geometry, the resulting volume calculation retained a high level of accuracy.

### **8** *Extremal Eigenvalues and Applications* **Laurie Battle, Montana Tech**

Extremal eigenvalue problems, which involve maximizing or minimizing the smallest eigenvalue, have physical applications such as constructing the tallest column or maximizing the fundamental frequency of a vibrating elastic system. I consider a Stieltjes Sturm-Liouville problem and establish conditions under which the eigenvalues depend continuously and differentiably on the problem parameters. These results are then applied to solve extremal eigenvalue problems.

# **9** Using Markov Chains to Calculate the Probability of Penetration through a Barrier of Finite Thickness by Guest Particles with a Finite or Infinite Lifespan Brian Bauman, Whitworth College William Sehorn, Whitworth College

Mathematicians can use Markov chains to find the probability of a particle penetrating through a barrier of finite thickness. We explore statistical models with the conditions of a two-dimensional barrier as well as a three-dimensional barrier. We also consider guest particles with a finite and infinite lifespan. Finally, we discuss closed forms that achieve the results given by Markov chains in both the two-dimensional and three-dimensional cases.

### **10** The Method of Strategies for Proof Compression in Advanced Calculus **Scott Beaver, Western Oregon University**

Many proofs in Advanced Calculus are lengthy and complicated. For such problems, students often experience difficulty in simply getting started on a reasonable path toward a solution; further, once a solution is completed, they often have difficulty re-creating it. For problems of more than minimal complexity, I ask that students write a (scored) strategy, containing no computational or algebraic details, above the problem solution. The goal is fourfold: to manifest similarities and differences between problem solutions and hence help students gain a broader perspective, to help students study for exams without necessarily having to read every detail of every problem solution, to make clear to students that the algebraic and computational techniques they employ in proofs are to be internalized as lowlevel tactics, and to eventually train students to develop clear, strategy-based lines of thought.

### **11** Classifying Quasiplatonic Surfaces Admitting Certain Common Automorphism Groups

#### Robert Benim, University of Portland

A quasiplatonic surface is a compact Riemann surface which admits a Galois cover of the Riemann sphere branched over three points. For a given genus, we can determine the groups which act on a quasiplatonic surface of that genus. Though in principal this method can be extended to calculate all quasiplatonic groups which can act on a surface of a fixed genus, in practice this is unrealistic. Another approach is to fix a group and see what genera this group can act upon as a quasiplatonic group. In this talk we demonstrate how to classify certain special groups which can act on a quasiplatonic surface. This talk will display covers for almost all Abelian groups, all groups that are a semi-direct product with  $C_2$ , and provide an algorithm to classify groups further and then illustrates this algorithm by classifying certain low order alternating and symmetric groups.

### **12** An ISETL-based System for Polynomial Algebra **Dave Beuerman, SUNY**

ISETL (Interactive SET Language) has been much used in teaching mathematics, including algebra. Our ISETL-based system for polynomial algebra consists of funcs and procs in files polyfuns.\* which are used for operations on polynomials. Various special polynomials are defined by their generating schemes. One example to be discussed is a study of sums of powers of integers using Bernoulli polynomials and Lagrange polynomial interpolation; the polynomials obtained are factored by the system. Connections with my previous work on relational algebra and linear algebra and linear programming are discussed, as are the pros and cons of using ISETL and some more recent work on abstract algebra.

#### **13** Homoclinic Solutions for a Hamiltonian James Bisgard, Central Washington University

We look for homoclinic solutions of (DE)  $\ddot{q}(t) = V_q(t,q(t))$ , where  $V \in C^2(\mathbb{R} \times \mathbb{R}, \mathbb{R})$  is 1-periodic in time, and the potential V has a local maximum at q = 0, and a global maximum at q = 1. Moreover, we assume that V(t,0) = 0 < V(t,1) for all t. The sought after homoclinic is found as the limit of a sequence of periodic solutions of (DE). The periodic solutions are critical points of

$$I_n(q) := \int_{-n}^n \frac{1}{2} |\dot{q}(t)|^2 - V(t,q(t)) dt.$$

The key is to control the amount of time that the periodic solutions spend close to 1. To do this, we use a very convenient fact about solutions of the following parabolic semi-linear (PDE)  $w_s(s,t) = w_{tt}(s,t) + V_q(t,w(s,t))$ : the number of times that two solutions cross decreases as *s* increases. As a result, we can control the amount of time that the periodic solutions spend close to 1, and so get a subsequence that converges to a solution homoclinic to 1.

### **14** The Logarithmic Derivative: What Is It Good For? **Dale Blem, Pacific University**

The Logarithmic Derivative is the derivative of the natural log of a function:  $\frac{d}{dx}\ln(f(x))$ . This is more commonly expressed as the ratio  $\frac{f'}{f}$ . It is used throughout mathematics in the fields of Differential Equations and Number Theory. We investigate properties of the Logarithmic Derivative, first strictly as an operator, and then as it is applied to Newton's Method. We illustrate a Modified Newton's Method that uses a higher order logarithmic derivative. We show both algebraically and geometrically that this Modified Newton's Method can converge more quickly than the classical Newton's Method.

#### **15** A Tale of Two Triangles **Holly Bochsler, Western Oregon University**

This project will display the connection between Pascal's Triangle and the Fibonacci numbers. More specifically, the powers of Fibonacci numbers will be explored and utilized to create the Fibonomial Triangle. There are striking similarities between the two triangles, which will be shown.

#### **16** The Physics of Tossing a Coin **Holly Bochsler, Western Oregon University Karen Lange, Western Oregon University**

This talk will demonstrate the physical forces acting on the coin when it is flipped. It will also address the three following questions: 1. Why is tossing a coin considered to be random if it is governed by physics? 2. Why is the probability still one half with the previous restraints? 3. How is the probability calculated for a particular toss? The results will be interpreted from the works of Joseph B. Keller.

#### **17** Simple as a Square **Holly Bochsler, Western Oregon University Karen Lange, Western Oregon University**

Our team consisted of three members, all of whom were seniors. The challenge we chose was to find a method of creating voting precincts that would represent all citizens equally. Our group came up with an algorithm that did just this, and if used in real life, all voting citizens could be represented in proportion. Our talk will discuss the challenge, our algorithm, and the implementation of the algorithm.

#### **18** Projects in Taxicab Geometry **Ton Boerkoel, DigiPen Institute of Technology**

Projects in Taxicab Geometry require students to reevaluate the definitions and theorems of Geometry. They discover that the usual definitions, which they have taken for granted, are not at all obvious or don't even make sense and and that some obvious facts and theorems are no longer true in Taxicab Geometry. Although easily accessible to most students Taxicab Geometry is surprisingly different from normal Euclidean Geometry in many ways. A tool like the Geometer's Sketchpad allows students to explore notions in geometry, and deepen their understanding of the role of definitions, theorems and proofs. We'll survey some projects that may range from explorations of distances, shapes (like lines, circles, conic sections, perpendicular bisectors), isometries, similarities, areas, angles and trigonometry, Ceva's Theorem, Pythagoras' Theorem, perpendicular bisectors, circumscribed circles, inscribed circles and constructions. In fact a project could investigate any topic from normal Euclidean Geometry in the setting of Taxicab Geometry.

### **19** Towards a Complete Taxicab Geometry: A Tale of Two Metrics **Ton Boerkoel, DigiPen Institute of Technology**

Most mathematicians have seen the Taxicab metric in their first topology class, but never ventured beyond this introduction. In fact, aside from some scattered results there doesn't even seem to exist a complete Taxicab Geometry: there are more questions posed than answers given. This talk attempts to sketch out a more complete picture of Taxicab Geometry including isometries, similarities, areas, shapes (lines, circles, triangles, conic sections), angles and trigonometry, Ceva's Theorem, Pythagoras' Theorem, perpendicular bisectors, isoscelines, circumscribed circles, inscribed circles and constructions.

### **20** Interdisciplinary Lively Application Projects at Central Washington Stuart Boersma, Central Washington University

Central Washington University has formed several interdisciplinary teams of faculty. Each team is charged with writing an interdisciplinary project for use in a lower division mathematics course. The logistics and rationale for this model will be presented and specific projects will be available for distribution.

### **21** Example-Generation Tasks as a Methodology to Probe Students' Understanding of Mathematics

#### Marianna Bogomolny

This research focuses on example-generation tasks as a methodology to probe students' understanding of mathematics. It is guided by the belief that better understanding of students' difficulties leads to improved instructional methods. The study introduces example-generation tasks as an effective data collection tool to investigate students' learning of mathematical concepts. In particular, this study focuses on students' understanding of the concepts of linear algebra. Simultaneously, it enhances the teaching of mathematics by developing a set of example-generation tasks that are a valuable addition to the undergraduate mathematics education.

### **22** Comparative Population Modeling for Interdependent Species **Tyler Bryson, Linfield College**

Through the use of nonlinear modeling techniques and Mathematica simulations we will compare and contrast different population models. Specifically, we will investigate the conditions for specific nonlinear phenomena such as limit cycles, stable and unstable fixed points, and their relationship to the models' biological meanings. Additionally, the discussion will address the questions raised by 3-dimensional two predator systems exemplified by Neanderthal-Modern man competition in Europe. Did the two ever meet? Did they compete directly, indirectly, or not at all?

#### **23** American Airlines' Next Top Model **Spencer K'Burg, University of Puget Sound**

I address the multi-million dollar question of how to board airplanes faster. My teammates and I designed a computer simulation to model people boarding airplanes of various sizes. One boarding procedure we created cut boarding times by as much as 67%. I will present this procedure as well as the general modeling approach we took on our way to earning an Outstanding Winner designation on the MCM.

#### **24** An Interdisciplinary Collaboration Between English and Mathematics Preservice Teachers

#### Beth Buyserie, Washington State University Kimberly Vincent, Washington State University

Through team teaching, university teachers Kimberly Vincent and Beth Buyserie integrated their mathematics and language arts methods courses for future secondary school teachers. One component of their semester-long collaboration focused on using quantitative literacy to interpret and make inferences about reading rates. The goal of this project was to help students transform their concept of good language arts or mathematics teaching and to figure out how to analyze reading rates and make inferences about comprehension. By answering these questions within the framework of both disciplines, students began to realize that quantitative literacy is not limited to the field of mathematics. Both disciplines were able to recognize how this QL project could transform teaching: language arts teachers can use QL to improve students' literacy; and mathematics teachers can use reading rates to assess if reading comprehension interferes in solving mathematics problems with a real world context posed using language rather than symbols.

### **25** *What are Matroids?* **Timothy Carstens, Seattle University**

A matroid is a combinatorial structure that attempts to capture the essence of "independence." It has equivalent formulations in vector spaces, graph theory, computer science, and many other areas. This session will serve as a tour of the big ideas in matroid theory with an emphasis on the graceful surprises the structure has to offer.

### **26** Non-existence of Limit Cycles in Analytic and Polya Vector Fields **Don Chalice, Western Washington University**

Kevin Broughan shows that "Holomorphic Flows on Simply Connected Regions Have No Limit Cycles" in MECCANICA, vol. 38, 2003, p. 699-709, and then asks whether this is true if the domain is not simply connected. Using Polya fields, we give a simple proof of his result and also answer his question for any domain and for meromorphic functions with a finite number of poles. (The question of existence and the maximum number of limit cycles for a vector field in the plane originates from Hilbert's Sixteenth Problem, which is still unsolved, but for an analytic function describing a vector field on the plane, the situation is very specialized, and we can give an easy solution.) Before giving our simple answer, for the convenience of the reader, we develop some results about Polya fields (to include a Polya field interpretation of the residue theorem). Most of these facts about Polya fields appear to be somewhat neglected in most complex analysis books. Therefore, we hope that by pointing out these results about Polya vector fields, we can help to further enrich the beginning complex analysis course.

### **27** Teaching with Classroom Voting **Kelly Cline, Carroll College**

Classroom voting can be a very powerful tool for teaching mathematics that we have successfully integrated into our calculus classes at Carroll College. This technique involves posing multiple-choice questions to the class and having them discuss the

issue with their peers before voting on the right answer with a set of hand-held "clickers." Their votes are recorded and tabulated with a computer, giving the instructor almost instantaneous feedback on student understanding. After the vote, we then ask individual students to explain and defend their answers, to justify their vote. Sometimes the answer quickly becomes clear, but the best questions result in an extended debate, as students who chose different answers explain their thoughts. Research on this method indicates that the most powerful effect on student learning comes from these classroom discussions, both before and after the vote. With some organization we have found that classroom voting requires no additional class time, allowing us to cover exactly the same syllabus and give similar exams to what we covered before classroom voting. Instead we replace many of the examples that would be done on the board with similar problems that the students do themselves, as well as use voting questions to provoke common misconceptions and bring up issues that would otherwise be dealt with in a lecture format. We have recently received an NSF grant to develop a library of classroom voting questions for differential equations and linear algebra classes and are looking for colleagues who might be interested in testing these materials.

#### **28** Mathematical Modeling of Suspension Bridges **Avery Cotton, Western Oregon University**

To meet transportation needs, bridges have been an efficient way to cross bodies of water. Suspension bridges, in particular, not only meet transportation demands, but they also tend to be aesthetically pleasing. In this project, a mathematical model of a suspension bridge, courtesy of L. Humphreys and P. McKenna, will be explored. Using numerical analysis, we will attempt to model the Tacoma Narrows Bridge of 1940, note the effect of various parameters, and examine some simplifying assumptions within the model.

#### **29** Getting It Right: WeBWorK at Albertson College of Idaho **Robin Cruz, Albertson College of Idaho**

WeBWorK is an internet-based homework delivery system developed at the University of Rochester. The math department at Albertson College has used WeBWorK since 2001 for our Introductory Statistics courses. In this talk I will report my experiences using WeBWorK in a workshop setting for an Intermediate Algebra course and an Applied Mathematics course and demonstrate some of the problem sets I wrote.

**30** Another Solution to the Postage Stamp Problem **Paige Cudworth\*, Willamette University Travis Dailey, Willamette University**  This project explores the classic Frobenius problem, also known as the postage stamp problem. In its basic form, the question is: given two positive, relatively prime integers, what is the largest number that cannot be expressed as a sum of the two given integers? Or, equivalently, given two relatively prime postage stamp amounts, what is the largest postage amount that cannot be made? We explore the problem in its simplest form, the history of the problem, and a variation as a result of our research. Our research focuses on specific recursively defined sets and their respective Frobenius sets.

**31** Eliminating Gerrymandering: An Algorithm for Establishing Fair Congressional Districts

#### Todd Curtis\*, Linfield College Edward Smith\*, Linfield College J. Jedidiah Rembold, Linfield College

The subject of gerrymandering has long been a subject of contention between challenging and incumbent political parties. Although the constitution states exactly how many politicians will represent each state, it mentions nothing about how the voting districts in those states are drawn. Such ambiguity lends itself to the favorable manipulation of district boundaries by the party in control of the state. In an effort to thwart such schemes, a simple and unbiased method of district division is needed. Our group has modeled two different algorithms for district creation and evaluated them based on simplicity and equality of population. One algorithm relied on a halving technique of the state population using rectangles with slight deviations. The other utilized a cellular growth algorithm in which district boundaries "grew" outward from an initial point until the needed population was reached. After testing both methods on a generated fictional state and New York state, it was apparent that the cellular growth algorithm created districts of simple shape and encompassed districts of equal population to a higher degree than the rectangular method. Both methods proved to be much more unbiased than current district drawing practice.

#### **32** *Teaching Statistical Literacy: One Approach* **Jill Dumesnil, University of Alaska Southeast, Juneau**

I teach a course on statistical literacy to liberal arts and social science majors at University of Alaska Southeast (UAS) in Juneau. I will briefly outline my philosophy and discuss my course. I hope to start a dialog with others who may have tried other approaches or texts or assignments to improve the course.

### **33** Classifying Trees and Forests by Game Chromatic Number **Kira Durand, Linfield College**

Two players, Alice and Bob, are coloring the vertices of a graph G with colors from a set X. The goal for Alice is to make sure that all the vertices of G are eventually

colored. If this happens, Alice wins. Otherwise, there will be an uncolored vertex which cannot be colored and Bob wins. They alternate turns with the only rule being that two adjacent vertices cannot be colored the same color. The game chromatic number of G,  $\chi_g(G)$ , is the least number of colors needed such that Alice will win. It has been shown that if G is a tree or forest, Alice can always win if there are at least four colors in X. However, not every tree or forest requires four colors. What differs in a tree or forest G that has  $\chi_g(G) = 3$ ? What about  $\chi_g(G) = 2$ ? We will provide a classification of all trees and forests G with  $\chi_g(G) = 2$ .

### **34** The Delicate Interplay of Class and Unit Groups in Number Field Extensions **Caleb Emmons, Pacific University**

In a Galois extension of number fields K/k, the Galois group G acts on both the unit group and class group of K-but how? Dedekind's formula  $\zeta_K^* - hR/W$  expresses the leading term (at s = 0) of the zeta function attached to K in terms of the class number, unit group regulator, and number of roots of unity in K. This is a tantalizing hint of the deep connections between these groups; we ought to be able to understand not just their cardinalities, but their structure as G-modules. In this talk I will try to open the door to these connections for abelian extensions from the viewpoint of Stark's conjectures.

#### **35** *How to Slice a Doughnut*

#### Isaac Erskine, Portland State University Robert Thompson, Portland State University

A graph G on a closed surface S is called separating if cutting along G will separate S into two or more pieces. For example, it is well known that a circle (simple closed curve) drawn on a sphere must separate the sphere into two regions. This talk will introduce results on graphs which separate a torus into exactly two pieces in such a way that no superfluous cuts are made. Such graphs will be called minimally separating. A classification up to graph isomorphism of minimally separating graphs on the torus will be given with an explanation of some of the techniques used to obtain this result. As time permits, the analogous problem on the genus 2 torus will be discussed as well as some obstructions to a similar classification on higher genera.

#### **36** *Exploring the Derivative of a Natural Number Using the Logarithmic Derivative*

#### Tara Fechter, Pacific University

We begin with an expository overview of the concept of the derivative of a natural number. We examine the concept of the logarithmic derivative in this unfamiliar setting, a function that at first appears to behave without discernable pattern. We then determine the limit of the average values of this logarithmic derivative by bounding its values between two generating functions.

#### **37** Flights of Inspiration: Taking the Red Eye Through Optimization Tara Fechter, Pacific University Kerensa Gimre<sup>\*</sup>, Pacific University Kevin Shawver, Pacific University

Have you ever spent hours waiting to board your flight from Dallas to Portland, only to miss an important connection to Sacramento? If so, then this is for you! This year's International Math Modeling Competition posed the problem of optimizing boarding and de-boarding time for three different airplane sizes. We designed a realistic model for small, medium, and large-sized planes in which boarding time is minimized, increasing profits for airlines. We analyze four models. "It's A Wonderful Model," although likely to never occur, provides us with an upper bound for efficiency. At the other end of the spectrum lies our least efficient method; "I Should Have Stayed Home". We then model the current airline procedure of boarding priority passengers first in the "Shaken, not Stirred" model and finally arrive at our proposal; "Past their Prime". Executing all algorithms in LabVIEW and Microsoft Excel, utilizing random number generators and counting methods to seat passengers, we arrive at a proposed model that performs 15% faster than the current airline boarding procedure and 40% faster than the least efficient method. We specifically address Delta Airlines, and conclude that an implementation of our program would result in an extra 219 flights per day across the nation, increasing profits by 14% and gaining an extra revenue of 2.24 billion dollars annually.

#### **38** Guided Discovery in a Discrete-Mathematics Course for Math Majors Mary E. Flahive, Oregon State University John W. Lee, Oregon State University

In this talk we describe our department's ongoing adaptation of its discrete mathematics course. The current course is based on guided discovery, in which students work through problem sequences to discover and apply principles in combinatorics and graph theory. The only course prerequisites are calculus and matrix algebra. The principal goal of the course is to increase the mathematical sophistication of the students by using problem solving and writing to develop their ability to think mathematically.

### **39** The Use of Protocols and Open-Ended Tasks in an Integrated Mathematics Course for K-8 Teachers

#### Maria Fung, Western Oregon University

This talk will serve as an introduction to pedagogical techniques inspired by OMLI (Oregon Mathematics Leadership Institute) such as protocols, group roles and norms, and discussion facilitation. This talk will also show sample group tasks that promote student mathematical discourse.

### **40** Finite Codimension in the Hilbert Cube **Kailash Ghimire, Oregon State University**

In finite-dimensional settings, the homological and covering dimensional approach to define codimension are equivalent, which lets us define finite codimension in the infinite dimensional settings, here in our case the Hilbert cube. The key result is the technique to calculate codimension of subsets of the Hilbert cube.

# **41** From One Dimension to Another: a Description of the Hopf Map and Its Connection with Division Algebras

#### Henry Gillow-Wiles, Oregon State University

We will use properties of division algebras to derive a relationship, the Hopf map, between spheres of certain dimensions. We will establish a connection between the Hopf map and the division algebras associated with the real, complex, quaternion and octonion numbers. We will use that connection to establish the existence of only four Hopf maps.

### **42** A Model of a Least-time Path for a Sailing Ship **Gary Gislason, Willamette University**

In this paper a model is developed for the path of a ship sailing from point A(-a,b) to point B(a,b) under the action of wind alone. The wind is assumed parallel to a straight shoreline, with intensity proportional to distance from the shore. The relevant Euler-Lagrange differential equation is solved, yielding an explicit solution for the least-time path.

#### **43** A Visualization of Stable and Unstable Motion of a Rigid Body **Amber Goodrich, Central Washington University Melissa Thompson, Central Washington University**

We were interested in investigating the motion of a football as it moves through the air–why does the football spin properly when the quarterback goes untouched, but rotates erratically when the quarterback is sacked? To explain this situation, we will formulate the Euler equations of motion and then show computer simulations of motion of the body under a variety of conditions. Furthermore, we will demonstrate the construction of the inertia tensor of a body and describe the notion of principal axes. After establishing the Euler equations and the inertia tensor, we will relate the simulations to real world situations, such as tossing a book into the air and our initial example of the football.

### **44** *A Quadratic Approach to a Circular Area* **Russ Gordon, Whitworth College**

Suppose a chord of length c in some circle cuts off a segment with height h. It is possible to write down an exact formula for the area of this segment, but the task is nontrivial and the somewhat messy formula requires trigonometric functions. Ancient Chinese and Egyptian records indicate the use of a formula based on a trapezoid, but their estimate was not all that accurate. Several centuries later, Archimedes found an exact formula for the area of a parabolic segment. When this formula is applied to the circle, the approximation is a bit more accurate than the trapezoid method. Both of these estimates are quadratic in nature. In this talk, we seek a formula of the form  $\alpha c^2 + \beta ch + \gamma h^2$  that gives a better estimate for the area of the region. The search for such a formula leads to some interesting elementary mathematics.

#### **45** *Modified Moore Method in a Senior-Level Analysis Sequence* **David Hartenstine, Western Washington University**

I will discuss my use of a modified Moore method in a senior/masters level analysis/advanced calculus sequence at Western Washington University. More specifically, I will discuss my role as the instructor, assessment and coverage issues, observations of student performance and progress, what a good day looks like, and some of the challenges with using this approach.

#### **46** Asymptotic Behavior of Travelling Wave Solutions to Reaction-Diffusion Equations

#### Brandi Harrison, Linfield College

In this talk we will discuss travelling wave solutions to reaction-diffusion equations of the form

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u^p (1 - u^q).$$

which can be used as a mathematical model for various biological phenomena, as well as to model problems in combustion theory.

We identify conditions on the wave speed so that travelling wave solutions exist for the case  $p \ge 1$  and  $q \ge 1$ . Moreover, we estimate the rate of decay of the travelling wave solutions. For the case p > 1 and  $q \ge 1$  this estimate requires center manifold theory because the typical linear methods fail to work.

### **47** *Rhythmic Oddity: A Counting Problem in Music Theory* **Kyle Haverly, Whitworth College**

The field of computational music theory involves interesting combinatorial problems, many of which involve the modeling and analysis of rhythm. This paper focuses on the property of *rhythmic oddity*; in particular it defines the property and attempts to enumerate rhythms that possess a given level of rhythmic oddity. Topics of discussion include a technique for the graphical modeling of a given rhythm as a binary necklace, as well as the mapping of a rhythm to a binary string.

#### **48** Supporting Students Learning Interdisciplinary Mathematics Cinnamon Hillyard, UW Bothell Nicole Hoover, UW Bothell

This presentation will describe a learning support center designed especially to support students in learning mathematics and quantitative reasoning in an interdisciplinary setting. We will give examples of support given inside and outside of the classroom as well as describing the opportunities and challenges that arise when supporting students learning interdisciplinary mathematics.

#### **49** The Aircraft Seating Problem Chinh Hoang, Linfield College Rachel Kaneta, Linfield College Ian McCreary, Linfield College Huy Nguyen, Linfield College

For airline businesses, time is money. As a result, a lot of research has been done to reduce airplane boarding times. Three boarding strategies that have been tested include the outside-in, back-to-front, and rotating zone strategies.

This project involved three different airplane models. The large-sized Airbus A380, medium-sized Boeing 747-400, and small-sized Boeing 737-800. It was assumed that passengers have all been pre-assigned with a seat number and that all of the flights were full. There was not enough time and resources to account for the interferences involved with each boarding strategy.

A Riemann sum was used to calculate the time it takes to board the passengers onto each plane. In our calculation we used several different parameters such as the average walking speed and luggage loading time for a boarding passenger as well as measurements of the cabin interior such as length and pitch.

For all three aircrafts, the outside-in method was found to be the fastest.

### **50** A Lower Bound for a Two-Forbidden Distance Chromatic Number of the Plane **Matt Hudelson, Washington State University**

We show by a counting argument that no matter how the plane is five-colored then there must be two points of the same color either 1 unit apart or the golden ratio units apart.

### **51** Upper Division Mathematical Modeling at a Small Four-Year School: Approaches, Successes and Frustrations

#### Chris Hay-Jahans, University of Alaska Southeast, Juneau

Mathematical modeling was introduced in the UAS mathematics curriculum to provide an opportunity for some breadth to UAS mathematics majors. This course also provides qualified majors in the applied sciences with an opportunity to gain exposure to the philosophy of and practices in mathematical modeling. At the request of the UAS biology program, the prerequisite for this course was set as first semester calculus. Because of the mixed backgrounds of enrolled students, this course needed to be presented in an alternative format so as to achieve upper division quality yet allow less experienced students an opportunity for growth and to enhance their skills effectively. This course remains a work in progress. Some of the successes and frustrations of the first two offerings are discussed. In addition, general information on the evolution of this course, future plans and remaining (unsolved) issues will be discussed.

### **52** A combinatorial-analysis invariant of graphs **Mohammad Javaheri, University of Oregon**

We define two invariants on graphs that represent graph properties in depth. Among our results, we show that if G is a connected multigraph with e edges and no vertex of degree 1, and if  $f: G \to \mathbb{R}$  is integrable with  $\int_G f d\mu_G = 1$ , then for any  $0 < r \le$ 1/e there exists a connected subset U of size r such that  $\int_U f d\mu_G = r$ .

#### **53** Get On The Plane!! Minimizing Plane Boarding Time Erica Jossi, Linfield College Jared Pierce, Linfield College Sisi Zhang, Linfield College

Boarding an airplane is often a laborious and time-consuming endeavor. Minimizing boarding time can save airlines millions of dollars. We will propose boarding methods for three categories of planes: small, seating 85-121 passengers, medium, seating 210-330 passengers, and large, seating 450-800 passengers.

Small Airplanes: Economy-class passengers will be assigned a seat randomly at check-in. When the seats are randomly assigned at check-in, emphasis should be placed first on the window seats, then the middle seats, and finally the aisle seats. Also at check-in, economy-class passengers will be divided into 5 blocks of 20 persons based on their check-in time. The first 20 passengers to check in will be in block 1, the second 20 passengers in block 2, etc. Thus, when block one is boarded, the 20 people who board will be seated mostly in window seats (or adjacent seats within the same row) and are spread evenly across the plane.

Medium/Large Airplanes: The first- and business-class passengers will be boarded from one door at the front of the plane while the economy class passengers will be boarded from one door at the back of the plane. This will automatically cut a significant amount of time from the total boarding time. The first- and businessclass passengers can be boarded in any manner that the airline sees fit since it will take those 60 passengers less time to board than the 154 economy-class passengers. Economy-class passengers can be allowed to choose their seats, or be assigned seats, as the airline wishes. Passengers will be separated into blocks around each of the two aisles and then boarded, beginning with the back of the plane, by alternating aisles. This allows the first block of passengers time to get into their seats while the block in the other aisle is being boarded.

#### **54** So You Want to Teach Graduate Students to Teach... **Gulden Karakok, Aaron Wangberg, Oregon State University**

When asked to describe their perfect class to teach, many beginning graduate students often describe the technical details: A well-thought out syllabus, balanced assignments, a favorite textbook, and pupils who are motivated, mature, and accountable. These same graduate students describe entirely different things - their professor's behaviors - when asked to describe their own favorite undergraduate course. We present here the results of a seminar we taught at Oregon State University which was intended to help mathematics graduate students realize that they could become effective teachers, even when they have no control over the syllabus, assignments, textbook, or assigned students.

### **55** Teaching quantitative reasoning through ethnomathematics **John Kellermeier, Tacoma Community College**

Over the past twenty years the field of ethnomathematics has emerged as the study of mathematics and how mathematics arises in a cultural context in response to problems, struggles, and endeavors of human survival and development. This paper will discuss the author's development and teaching of a course on ethnomathematics at Tacoma Community College. This course is designed as a liberal arts quantitative reasoning course based on the study of ethnomathematics.

#### 56 Beaucoup de Sudoku Mike Krebs, California State, LA

Sudoku is the latest pencil-and-paper craze; it also provides many excellent illustrations of concepts from undergraduate abstract algebra. Students may well find that applying recently-learned methods to a familiar, concrete example brings the theory to life. For instance: Given a completed Sudoku grid, there are some easy ways to create a new Sudoku grid from it, by, say, switching the top two rows, or rotating the grid ninety degrees. The set of all such actions forms a group G. Can we find some familiar group that it is isomorphic to? The action of G on the set of all Sudokus gives us a notion of "equivalent" Sudokus. Are all Sudokus equivalent?

### **57** On the Dimensions of Bivariate Spline Spaces **Wai Lau, Seattle Pacific University**

We consider bivariate  $C^r$  spline spaces of degree d defined on arbitrary triangulations. Schumaker (1979) established a lower bound for the dimensions of such spaces. It has been conjectured that the lower bound is the exact dimension for  $d \ge 2r + 1$ . We give an equivalent condition for which the dimension is given by the lower bound. The equivalent condition is in terms of certain linear maps between the triangulation and its subsets.

### **58** Learning Experiences that Integrate Algebra and Another Discipline **Jennifer Laveglia, Bellevue Community College**

This presentation outlines a coordinated studies course that integrates a college algebra course for business majors with microeconomics. After a brief description of the development of the course and a general sense of the course's structure, she will share some examples of the integrated content. Additionally, she will provide an example of integrating algebra in a biology assignment (or vice versa) and using self-assessments from a learning strategies course in an intermediate math course.

**59** Evaluating the Improper Integrals  $\int_0^\infty \frac{\sin^{2n+1} x}{x} dx$  for Positive Integers n **Mike Leatherman, Central Washington University** 

It is well known that  $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ . One way to prove this is to introduce a parameter, differentiate the parameter under the integral sign and then use the Fundamental Theorem of Calculus twice to evaluate the given integral. This method will be outlined for the integral given here as well as for the integral  $\int_0^\infty \frac{\sin^3 x}{x} dx$ . An identity of Euler will then be used to find a formula for values of the integrals  $\int_0^\infty \frac{\sin^2 x}{x} dx$ .

#### 60 Rook Polynomials in 3-D Arthur Low, DigiPen Institute of Technology Shalin Mehta, DigiPen Institute of Technology

We present patterns found for Rook Polynomials with respect to an  $m \times m$  chessboard in 2-D, and look for similarities in 3-D. We form conjectures regarding the following: the maximum number k of non-attacking rooks that could be placed in an  $m \times m \times m$  chessboard in 3-D, the number of placeable combinations of k non-attacking rooks in an  $m \times m \times m$  chessboard in 3-D, and a generator for Rook Polynomials with respect to an  $m \times m \times m$  chessboard in 3-D.

#### **61** Free Groups and Generators and Relations **Martini Machado, Southern Oregon University**

I will begin with the development of free groups using the common alphabet analogy. From there, I will discuss the Universal Property of Free groups and show how it leads to the generators and relations notation that is often used in group theory.

### **62** "You want me to do what?" – Teaching Software Engineering "Formal Methods"

#### Bart Massey, Portland State University

One of the key ideas of modern software engineering is the use of "formal methods": mathematical / logical modeling of software systems with proofs of key properties. Over the last 5 years, my colleagues and I have taught SE formal methods to a number of non-mathematician software engineers and computer scientists. Lessons we have learned include the importance of starting at the beginning, the importance of software tools, and the difficulty of choosing a formalism that is sufficiently general yet accessible to the students.

#### **63** Teaching Mathematical Induction as a Rule of Inference **Ken Meerdink, DigiPen Institute of Technology**

Students get a better grasp of what the Principle of Mathematical Induction is and how it works when it is taught as a rule of inference in logic. Interdisciplinary approaches to propositional and predicate logic for mathematics and computer science majors can help students better understand math concepts, applications of logic and programming methods. The Principle of Mathematical Induction is a good example. After learning simple rules of inference and applying them to concrete problems, students easily mastered the Principle of Mathematical Induction.

### **64** An Application of the Slice Theorem for the Reconstruction of X-Ray Data **Cristina Negoita, Oregon Institute of Technology**

An introduction to the problem of inverting the Radon transform is given, along with some examples. The Slice Theorem is introduced and applied to invert the Radon transform. To address discrete sampled data, we need to consider the Slice Theorem along with the Sampling Theorem. Both will thus be discussed from a computational point of view, and applied to the reconstruction of images of sinograms of the human brain using Positron Emission Tomography.

#### **65** Geometry of Mutually Stationary Points of Quadratic Forms **Yves Nievergelt, Eastern Washington University**

The problem of locating the constrained minimum of an objective defined by a quadratic polynomial with several variables on a curve or surface also defined by such a polynomial occurs in various applications. For instance, if the objective is the squared Euclidean distance from a fixed point, then the problem amounts to computing the distance from the fixed point to the curve or surface. Moreover, if the constraining surface is an ellipsoid, then the problem arises in the computation of the geodetic altitude or depth of a point above or below the surface of such a body as Mars or its satellites. Similarly, if the constraining surface is paraboloid or a one-sheeted hyperboloid, or a two-sheeted hyperboloid, then the problem arises in the quality control of microwave antennae or cooling towers, for manufacturing or safety purposes. The talk will show algebraically and geometrically that the constrained minimum corresponds to the Lagrange multiplier with the smallest magnitude.

# **66** The Trichotomy Character of $x_{n+1} = \frac{\beta_n x_n + \gamma_n x_{n-1}}{A_n + B_n x_n}$ with Period-two Coefficients **Carol Overdeep, Saint Martin's University**

We extend the known results of the non-autonomous difference equation in the title to the situation where (i) the parameters  $\beta_n$  and  $\gamma_n$  are period-two sequences of nonnegative real numbers with  $\gamma_n$  not identically zero; (ii) the parameters  $A_n$  and  $B_n$ are period-two sequences of positive real numbers; and (iii) the initial conditions  $x_{-1}$  and  $x_0$  are such that  $x_{-1}, x_0 \in [0, \infty)$  and  $x_{-1} + x_0 \in (0, \infty)$ .

### **67** Finitely Generated Groups And p-Harmonic Boundaries **Mike Puls, Eastern Oregon University**

In this talk we will define the *p*-harmonic boundary of a finitely generated group *G*. We will then state some of the properties of the *p*-harmonic boundary and give a characterization of the vanishing of the first reduced  $\ell^p$ -cohomology space of *G* in terms of the cardinality of its *p*-harmonic boundary.

### **68** Welcome to the Neighborhood: Seymour's 2nd Neighborhood Conjecture **Molly Robinson, Whitworth College**

Seymour's  $2^{nd}$  Neighborhood Conjecture states that if *D* is a simple digraph without loops or digons, then *D* contains a vertex such that  $|N^{++}(v)| \ge |N^{+}(v)|$ . Paul Seymour of Princeton University formulated this conjecture in the 1980's. Since that time, Kaneko and Locke made substantial contributions towards the proof of the conjecture by proving it for digraphs with a vertex of out-degree at most 6. Godbole, Cohn and Wright proved it for almost all digraphs. Related to this conjecture,

Chen, Shen and Yuster proved that every simple digraph without loops or digons has a vertex v such that  $|N^{++}(v)| \ge \gamma |N^+(v)|$ , where  $\gamma = 0.657298...$  is the unique, real solution to  $2x^3 + x^2 + 1 = 0$ . This paper looks at the types of graphs that satisfy Seymour's  $2^{nd}$  Neighborhood Conjecture.

### **69** Symmetric Plane Curves and Their Kin **Peter Rowell, Oregon State University**

The figure-8 plane curve has 180° rotational symmetry, but, if you draw the figure-8, it will probably turn out slightly crooked. However, it is easy to see that an imperfect figure-8 is diffeomorphic to a perfectly symmetric figure-8. Determining whether or not an asymmetric curve is diffeomorphic to a symmetric curve becomes more difficult when the curve is more complicated.

In his work on the classification and enumeration of closed plane curves, V.I. Arnol'd characterized four types of symmetry exhibited by plane curves. Arnol'd's four symmetry types are defined in terms of immersions of the circle into the plane, so identifying the Arnol'd symmetry type of a curve depends on having knowledge of the mapping  $S^1 \rightarrow R^2$  and cannot be determined from its image alone.

In this talk, we show how all symmetries of a curve on the plane or a curve on the sphere can be identified by looking at the word formed from the sequence of self-intersections of the curve-the "Gauss word." Since the Gauss word depends only on the image of the curve and is invariant under diffeomorphism, this method enables us to define a more extensive classification of symmetric plane curve diffeomorphism classes and a classification of symmetric spherical curve diffeomorphism classes both of which are independent of parametrization, so members of each class can be identified by their images. We also describe how the Gauss word method applies to the open problem of the enumeration of plane curves.

### **70** A Multiplicity Problem Involving Schur-like Triples in a Linear Inequality **Dusty Sabo, Southern Oregon University**

In 1916, Issai Schur proved the following theorem: For every integer *t* greater than or equal to 2, there exists a least integer n = S(t) such that for every coloring of the integers in the set  $\{1, 2, ..., n\}$  with *t* colors there exists a monochromatic solution to x + y = z. The integers S(t) are called **Schur** numbers and are known only for t = 2, t = 3, and t = 4. During a combinatorics conference in June of 1996 at Tianjin, China, Ron Graham posed the following problem. For a given integer *n*, what is the least number of monochromatic solutions to x + y = z that are guaranteed to occur in an arbitrary coloring of the set  $\{1, 2, ..., n\}$  with two colors? A problem such as this one is referred to as a multiplicity problem. This problem was solved in 1998 by Aaron Robinson and Doran Zeilberger and also independently by Tomasz Schoen. In this paper we solve the following similar problem. For a given integer *n*, what is the least number of monochromatic solutions to x + y < z that are guaranteed to occur in an arbitrary coloring of the set  $\{1, 2, ..., n\}$  with two colors?

#### **71** A Review of Poset Products **Kevin Saff, University of Calgary**

The general concept of a *partially-ordered set*, or *poset*, is introduced. The standard finite operations on these will be defined. While it is clear how to define an infinite sum of posets, many different definitions of infinite products have been suggested by Sierpinski, Fraissé, and others. I will present a brief history and present a new definition which generalizes many of these.

#### **72** How to Make Abe Lincoln Get off the Vacuum Cleaner: Creating Sensible District Lines in New York

#### Akbar Saidov, Pacific University Maria Walters, Pacific University

Isn't it strange that a state's congressional district lines gradually transform into abstract art? The practice of gerrymandering, allowing political incumbents to change the boundaries of districts in their states in order to give their party an unfair advantage, has long been a problem in populous states such as New York. The recent gerrymandering in the Empire state has resulted in odd district shapes, including one labeled "Abraham Lincoln riding a vacuum cleaner" by political blogger Blair Horner. The 2007 Mathematical Contest for Modeling challenged teams to tackle this problem by finding an algorithm for creating geographically simple districts in a given state. The only requirement: each district must contain the samepopulation. In creating ideal districts, our team strove for compactness, which we could measure by finding the sum of the interior edges between the districts. The smaller the sum, the more compact the districts. Our solution turned out to be almost twice as compact as the current lines. We also labeled and dealt with the two major obstacles of making a simple congressional district: irregularly shaped states and the uneven distribution of residents within those states. In defending our approach to residents, we deal with political issues including ethnic and economic divisions.

#### 73 Cosets and Cayley-Sudoku Tables Keith Schloeman, Western Oregon University

Cayley tables are similar to Sudoku puzzles in that every element of the group appears exactly once in each row and once in each column. There are many ways of arranging the Cayley table of a group, using cosets, so that it also satisfies the additional requirements to be a Sudoku table.

### **74** Coverings of Triangular Billiards Surfaces **Jason Schmurr, Oregon State University**

Consider a polygonal region and a single particle moving in a straight-line trajectory within it. Suppose that when the particle collides with a wall of the region it reflects

off the wall, with the angle of incidence equal to the angle of reflection. This seemingly simple model (sometimes called polygonal billiards) quickly leads to difficult questions, such as: what are the periodic trajectories in this system? If a trajectory is not periodic, will the particle eventually visit every open subset of the polygon? There is a highly successful and visual technique for studying such questions which involves "gluing" several copies of the initial polygon together to form a topological surface. Such surfaces often have high degrees of symmetry. It is of interest to know how different billiards surfaces are related: can different billiards tables lead to the same surface? Which of these surfaces are simply composed of several copies of other billiards surfaces? How do the symmetries of different billiards surfaces compare? The presentation will consist of a brief introduction to this topic and a presentation of preliminary research results regarding triangular billiards tables.

#### 75 Can a Salamander Become a Turtle? Avoiding congressional gerrymandering Jason Shaw, Seattle Pacific University Eddie Strickler, Seattle Pacific University Clayton Zylstra, Seattle Pacific University

"Gerrymandering" is the name given to politicians redrawing congressional districts to preserve the power of their own party. Besides calling into question the integrity of congressional elections and creating bizarrely-shaped districts, the process can be a drain on congressional productivity, as minority parties filibuster votes on proposed redistricting plans. We propose a method to objectively define congressional districts, preventing this unpopular practice. Currently, districts are required only to be connected and have approximately equal populations. Our method achieves this; it also does not needlessly divide metropolitan centers between districts, and creates districts of much simpler shapes. The algorithm is appropriate for all states, and includes parameters which may be adjusted in special situations. We construct districts one ZIP code at a time according to a few simple rules, allowing for precise population control and associating each postal code with exactly one district. We also introduce a scoring function to choose the initial ZIP codes for each district.

### **76** Finding Polynomials that Annihilate Algebraic Numbers **Brian Shearson, Central Washington University**

An algebraic number is a number that is the root of some non-trivial polynomial with rational coefficients. For example,  $\sqrt{2} + \sqrt{3}$  is algebraic since this number is a root of  $x^4 - 10x^2 + 1$ . Using techniques from linear algebra, I will show how, given an algebraic number  $\alpha$ , to find a polynomial p(x) over the rationals that annihilates  $\alpha$ ; that is,  $p(\alpha) = 0$ .

#### 77 Now Boarding Rows 25 and Higher... Blair Sherman, Central Washington University Melissa Thompson, Central Washington University Alisha Zimmer, Central Washington University

In the age of technocapitalism, travel by air is a vital form of transportation, and airlines depend on quick flight turnaround times to maximize profits. While the so-called "back-to-front" procedure is most widely employed, this is not necessarily the best way to minimize boarding time.

Our tasks were to determine the boarding process that minimizes boarding time and to persuade airlines that it will be monetarily advantageous to employ this method of boarding. In order to develop our recommendations for the airlines, we created a model to simulate the boarding of passengers. Based on both research and personal experience, we hypothesized that minimizing aisle delay is the most important factor in minimizing overall boarding time.

We developed a robust computer simulation to emulate the boarding process. This software allows us to customize the internal layout of the plane, the positions of the entryways, and the manner in which passengers will form lines to board the aircraft. It then models the actions of the passengers during boarding. This includes actions such as passenger movement, luggage storage, and taking their seats. Our goal was to determine the most efficient passenger lineup methods for various situations.

### **78** Sudoku: What Did I Learn from This Project? **Eric Smyth, University of Washington**

Sudoku is an addictive game. In this project, we implemented creation and solving Sudoku puzzles. The emphasis was using CS and mathematical structures to implement it. Our first solver was a stack, which translates to a very simple strategy to solve a puzzle. Our second approach was a list coloring of a graph. We used a dynamic coloring scheme to find solutions. Finally, we compared both approaches for efficiency and also in an attempt to rank puzzles' difficulties.

### **79** Combinatorial Sums via Finite Differences **Michael Z. Spivey, UPS**

We present a new approach to evaluating combinatorial sums by using finite differences. Let  $\{a_k\}_{k=0}^{\infty}$  and  $\{b_k\}_{k=0}^{\infty}$  be sequences with the property that  $\Delta b_k = a_k$  for  $k \ge 0$ . Let  $g_n = \sum_{k=0}^n {n \choose k} a_k$ , and let  $h_n = \sum_{k=0}^n {n \choose k} b_k$ . We derive an expression for  $g_n$  in terms of  $h_n$  and for  $h_n$  in terms of  $g_n$ . These expressions allow certain kinds of binomial sums to be evaluated fairly easily. We then extend our approach to handle binomial sums of the form  $\sum_{k=0}^n {n \choose k} (-1)^k a_k$ ,  $\sum_k {n \choose 2k} a_k$ , and  $\sum_k {n \choose 2k+1} a_k$ , as well as sums involving unsigned and signed Stirling numbers of the first kind,  $\sum_{k=0}^n S_n^k a_k$  and  $\sum_{k=0}^n s(n,k)a_k$ .

# **80** The TI-Nspire Handheld and Software: A New Level of Communications for Multiple Representations Delivered through a Document Model **Paul Riopel, Texas Instruments**

The TI-Nspire from Texas Instruments employs a significantly higher level of communications for students and teachers through a document model of information delivery and interaction. This new product is available with or without a Computer Algebra System (CAS) and has a changeable faceplate that enables compatibility with the TI-84 Plus and TI-83 Plus. The TI-Nspire greatly enhances the use of multiple representations for concept development and problem solving. The combination of these capabilities helps teachers increase learning opportunities for more students.

### **81** *Identifying Lie Subalgebras Using Root Diagrams* **Aaron Wangberg, Oregon State University**

Although Lie algebras are usually classified using Dynkin diagrams, it is their root and weight diagrams which are most useful in applications to quantum mechanics when describing the properties of fundamental particles. In this talk, I will demonstrate two methods that use root and weight diagrams to visually identify a given algebra's subalgebras. In particular, these methods will be applied to algebras whose root and weight diagrams have dimension greater than 3, including the exceptional Lie algebras  $F_4$  and  $E_6$ .

### **82** Singular Points of Real Sextic Curves Nicholas Willis, Whitworth College

Complete classifications of the individual types of singular points are given for irreducible real quartic curves, reducible real quartic curves, irreducible real quintic curves, reducible real sextic curves, and reducible real sextic curves. These classifications are derived by using the computer algebra system Maple. We clarify that the classification is based on computing just enough of the Puiseux expansion to separate the branches. Thus, the proof consists of a sequence of large symbolic computations that can be done nicely using Maple.

### **83** How is a Minimal Circle Formed Around a Set of Points? **Vanessa Wyffels, Western Oregon University**

This paper will explore a new technique due to Felix Friedman finding a minimal enclosing circle for a set of n points. Partitioning of the plane will be examined as it is the basis for extending the circle when needed (when more points are added). Also, examples will be presented in the actual process of adding points and extending the circle.

### **84** Determinants of Matrices defined by Pascal-like Recurrence Relations **Andrew Zimmer, University of Puget Sound**

It is a surprising fact that matrices with the recurrence relation  $M_{i,j} = M_{i-1,j} + M_{i,j-1}$  and the initial condition  $M_{1,1} = 1$  have determinant of one. Such matrices are often referred to as Pascal matrices. In this presentation I will discuss the determinant of matrices having the more general recurrence relation of  $M_{i,j} = aM_{i-1,j} + bM_{i,j-1} + cM_{i-1,j-1}$  on their entries. I will derive the value of the determinant using a proof that relies on the invariance of the determinant under column and row additions. Although the proof presented is original several people including Krattenthaler, Zakrajšek and Petkovšek, and Neuwirth have found the determinant using different methods. Their methods will be briefly discussed and I will also mention how the row and column addition proof provides insight into the *LDU* decomposition of *M*.

#### **Campus Information**

Wireless internet access is available in Murdock Hall (2nd floor), Melrose (East end of the top floor), and Graf Hall (East end classrooms). Use network CatNet-Public.

Linfield Safety Department Phone Number: (503) 883-SAFE (7233) From campus phones: SAFE (7233)

McMinnville Police Department Emergency Number: 9-911 (from campus phones) Non-Emergency Number: (503) 434-7307

#### LINFIELD COLLEGE

#### 1. Melrose Hall

Administration, Classrooms, College Relations, English, Philosophy, Religious Studies, Ice Auditorium Jonasson Hall International Programs

2. Riley Campus Center

O'Rileys Deli, Bookstore, Student Activities, ASLC, Fred Meyer Lounge, Meeting Rooms, Employee Lounge, Game Room, Women's Resource Center

3. Walker Hall

Academic Advising Office, Student Health Center, Counseling Center, Career Center, Learning Support Services, Anthropology Museum, Sociology-Anthropology, Archaeology Lab, Modern Languages, Classrooms

- 4. Mac Hall Integrated Technology Services
- Graf Hall 5 Physics, Mathematics
- 6. Murdock Hall Biology, Chemistry
- 7. Taylor Hall Business
- 8. Northup Hall
- 9 Malthus Hall Division of Continuing Education, Economics Department
- 10. Michelbook House Office of Admission
- 11. Upward Bound
- 12. Emmaus House
- 13. President's Home
- 14. Pioneer Hall Classrooms, History, Political Science, Psychology, Residence Hall, KSLC Radio Station
- 15. Newby Hall Student Apartments
- 16. Greenhouse
- 17 Storage
- 18 Cozine Hall Auxiliary Services Department
- 19. Delta Psi Delta Fraternity
- 20. Potter Hall Pre-school, Residence Hall, Early Childhood, Elementary/Secondary Education
- 21. Theta Chi Fraternity
- 22. Pi Kappa Alpha Fraternity
- 23. Kappa Sigma Fraternity
- 24 Whitman Hall Residence Hall



46. Withnell Commons Post Office

Residence Hall

Residence Hall

Residence Hall

Residence Hall

Residence Hall

Residence Hall

38. Anderson Hall

39 Terrell Hall

40 Hewitt Hall

41 Frerichs Hall

37. Larsell Hall

- 47. Hewlett-Packard Park Apartments
- 48. Health, Human Performance and Athletics Building Ted Wilson Gymnasium
- 49. Aquatics Building
- 50 Rutschman Field House
- 51 Memorial Hall Residence Hall

- Art. Gallerv
- 57. Vivian A. Bull Music Center
- 58a. Jereld R. Nicholson Library
- 58b. Kenneth W. Ford Hall Marshall Theatre, Theatre and Communication Arts
- 59. Facilities Services and Campus Safety Departments
- 60-1 & 2. Warehouse Storage
- 61. Sports Practice Fields

#### Acknowledgments

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Jennifer Firkins Nordstrom

Local Arrangements Chair