# MATHEMATICAL-SYLLABUS METHODS FOR OPTIMIZING INSTRUCTIONAL EFFECTIVENESS: THE MATHEMATICAL INTEGRITY CRITERION

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**ABSTRACT:** For instructional guidance of the learner's personal mathematical development, the most basic tool is perhaps the instructor's selection of a point-by-point flow of mathematical information that the learner is to encounter — herein called the **mathematics syllabus**. Despite decades of recurrent "reforms", core-curricular (K-calculus) instruction in mathematics continues to rely on mathematics syllabi whose instructional effectiveness is woefully inadequate. The primary cause of such misjudgments has been the non-availability of a scientifically reliable technology (1) that discloses that, how, and why the "normal" syllabi are so ineffective, and (2) that also reveals how to achieve mathematics syllabi that are maximally effective.

Accordingly, **the science of syllabus methods** is being developed as a foundation for improving the instructional effectiveness of mathematics syllabi. Thereby, science can provide invaluable guidance for future reforms of curricular instruction in mathematics. The methods are adapted from the **critical path methods** that long have been used in **the managerial science of operations analysis**. The adaptation is made possible by the (constructivistic) **state-transition theory of psycho-mathematical development**. The empirical aspect of the methods consists of superposing **clinical research and development** activities onto an ongoing program of clinical instructional services in mathematics.

[In accord with the conventions of axiomatic mathematics, this paper invokes many phrases — including "learner" and "instructor" and "progression" and "mathematical points" — in relatively abstract senses. Those notions can be applied with alternative specific meanings, within various particular contexts, according to the users' respective needs and purposes.] .

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## INDIVIDUAL, MEAN, AND NATIONAL MATHEMATICS SYLLABI

The scientific methods pivot around a very specialized concept of "mathematics syllabi" — which must not to be confused with other meanings of "syllabus". Throughout the practices of curricular education, various kinds of syllabi often are used for diverse purposes: a table of contents, a list of topics or of textbook sections, a “scope and sequence chart” or even a good catalog description of a course. Even of such global syllabi, critical examinations often shed some light on how to improve effectiveness of the instructional guidance of learning. But for purposes of **discerning where and why multitudes of students have difficulty with curricular mathematics** — and of perceiving how to improve instructional effectiveness — focus must be on the intricate details of precisely what mathematical information the learner is to encounter, in what order.

For preliminary illustration, simplistically consider the case of an oriental immigrant who will be newly learning the (geometric) capital letters of the English alphabet. The minimal **instructional intent** thus **targets** 26 **mathematical points** for the learner to encounter, ingest, digest, assimilate, and accommodate. It is conceivable that the instructor-selected syllabus might begin by introducing any randomly selected letter, and then randomly proceed to introduce any second one, and so on, until all are learned. There are 26! progressions that introduce each of those letters.

To invoke mathematics syllabus methods would be to research the alternative syllabi that cover the targeted mathematical points with regard for what progressions might be **optimal choices**. It is conceivable that some of those paths might be (geometrically) easier for the learner — as with O before C before G … or K before R before P before B … or I before L and T ... or F before P, and E before B. A “best choice” progression might be made strictly on the basis of the instructor's own theory (or deliberations) about the geometric constructions of those letters. Moreover, that choice might also be clinically scrutinized (e.g. if there were to be a continuing flow of such learners).

The preceding considerations also might be expanded to include possible use of some non-targeted ‘auxiliary’ mathematical points such as circle, arc, vertical, and slant. Still other, potentially useful mathematical points might be of a less formal nature — such as awareness that, for the letter, K, it is more reliable to start the sloped lines on the vertical line, than to start them at their outer ends (its a statistical theorem!). However, a learner with severe dyslexia might not be able to follow a syllabus that the instructor had judged to be quite adequate.

That "ALPHABET" scenario roughly outlines what mathematics syllabus methods are about. It also illustrates how this concept of a mathematics syllabus (as a progression of mathematical points) invokes a highly technical meaning — one that is substantially different from most commonplace meanings of “syllabus”. More specifically, **a mathematics syllabus** is a progression of **mathematical points** that are specific ingredients of a mathematic theory or of a mathematical process. So, the methods are most concretely, effectively, and beneficially used point-wise with a very specific mathematics syllabus — such as the one that a particular instructor is currently using for a specific class-course over a specific mathematical subject.

The label suggests that the methods are for use with progressions of mathematical points that are intended for a learner to internalize. But the same methods can be used also on progressions that already have been presented to a learner — or are encountered by a learner — or are actually acquired or assimilated by a learner — or any combinations or variations of such occurrences. For example, the mathematics syllabus that an instructor actually uses in an interactive setting (especially a clinical setting) is best detected through post-facto review of a videotape of the instructional proceedings.

It is even much easier to perceive and dissect a point-wise syllabus when it is woven through a particular mathematics textbook. In a sense, text-embedded syllabi also are more important, in that they normally are intended for use by a large number of instructors, with an even much larger number of learners. For brevity, our primary concern, herein, is with text-embedded mathematics syllabi.

Every mathematics textbook uses, as its mathematical skeleton, its own progression of mathematical points — its own intrinsic mathematical syllabus. A first approximation of that syllabus can be gleaned by critically examining the textbook's development, page by page, phrase by phrase, and word by word — to generate a report on the point-by-point progression that is used in that text. Almost any teacher capable of teaching from that book can generate at least gross reports of that kind. Thereby, mathematics-syllabus methods can be used, even very roughly, for selecting a "best choice" textbook, or for deciding how best to use or supplement a pre-determined textbook.

However, such reports would vary in accuracy and in detail, depending on the reporters' respective expertise in mathematics, in psycho-mathematics, in the semantics and grammar of formal mathematical rhetoric, in the philosophy of mathematics, and in meta-mathematics. In fact, the mathematical points that occur within a text-embedded mathematics syllabus often are invoked so subtly, tacitly or obscurely, that those points are non-discernable even to most who teach from that book — often being unrecognized even by the textbook's authors. So, the most reliable identifications of the intrinsic mathematics syllabus would come fromindividuals or teams having expertise in all of the fields cited, above.

Of course, the above method for examining a single textbook can be used also for comparing and contrasting two or more textbooks on the same mathematical subject. Similar methods can be used on a specified collection of particular syllabi, for surmising **the mean syllabus** for that collection — and the nature of each sample's deviation from that mean. Especially when comparing several textbook syllabi, it becomes quite evident that a realistic and reliable "mean" syllabus must be of the "best fit" kind — having minimal variance from the various samples in that collection.

The extreme in that direction would be to discern such a (best fit) mean-syllabus for a lengthy curriculum in mathematics — by compiling the syllabi from a very large selection of textbooks and comparable instructional media. A viable model of that kind is the U.S.A.'s core-curriculum in mathematics — grade K through calculus, including elements of probability, statistics, finite mathematics, and other material that is required in popular college degree programs. The U.S.A. does not have a nationally prescribed mathematics syllabus for its core-curriculum in mathematics. But most American core-curriculum teachers of mathematics follow commercially published textbooks — and in any one core-course, the nation's curricular instruction is dominated by only a relatively small number of textbooks. So a realistic portrait of the nation's de facto mean-syllabus can be estimated by examining the prevailing textbooks.

In the above manner, even **the national-mean mathematics syllabus for the core-curriculum** becomes subject to the use of syllabus methods, for purposes of improving instructional effectiveness, nationwide. By improving national professional knowledge about curricular mathematics syllabi, the methods can lead to elevation of national standards for the instructional quality of prevailing textbooks. Of course, identifying a mean-syllabus for such a global educational complex can be so costly that it is justified only by its promise of yielding major, lasting, beneficial improvements in the instructional effectiveness of the curricular program.

Any scientifically rigorous mode of so discerning the de facto national-mean syllabus would require very powerful (and expensive) resources. But even very gross estimates of the nation's mean-syllabus (by highly qualified experts) immediately disclose that the nation's mean-syllabus has **many mathematical flaws that seriously suppress the instructional effectiveness of the national core-curriculum in mathematics**. Such flaws, and the ease of mathematically correcting those flaws, prompted a MALEI Institute initiative that is represented by The Mathsense Library website at http://www.mathsense.org.

## STATE-TRANSITION OF THE LEARNER

The scientific heart of syllabus methods is the psychological **state-transition theory of mathematical growth**. Mathematical aspects of that theory are perceived through various models. The simplest version is a topological model in which **a learner** occurs within **a domain (a set) of "learnable points"**. (For simplicity and generality, no particular properties are assigned to the "learnable points" or "domain" until the topological model is applied for specific purposes.)

Within that domain, the learner is a point-set — one that continually (perhaps even continuously) changes through combinations of expansions and contractions. More precisely, the learner is **a time-indexed family of point-sets**, called **developmental states**, which are contained within the learning-domain — with the "growth" proviso that the closer the learner's states are to each other, with regard for time, the closer they are with regard for their learnable-point contents. The state-changes over time thus constitute **the learner's state-transition**. [Syllabi describe what state-changes did, do, or are intended to occur.]

Within a learning-domain, the learner's state-transition looks much like how an expanding layer of water or rocks can progressively cover a geographic surface, when fed by an inflowing stream of water (the continuous-case) or of rocks (the discrete-case). [Syllabus methods are used for managing the inflow of information.]

However, that same growth also can be viewed within **the learning-domain's power set** — which serves as **the learning-lattice** for that domain. Along that lattice, the learner's state-transition is seen to constitute **an evolutionary path of time-indexed (lattice) points**. Evolutionary changes that are strictly expanding over time are upward along that lattice, and changes that are strictly contracting over time are downward. It is important to note that, within the learning-domain, the state-transition appears as a progressive warping of a point-set — while on that domain's learning-lattice, the same state-transition appears as a progressive movement of a lattice-point. [Syllabus methods are concerned with how to "steer" such movements, by managing the inflow of information.]

The learner's present **progress-track** is the time-ordered (so linearly ordered) progression of instantaneous developmental states that were previously achieved — a bit like the visible trail of an aircraft or watercraft. That past-progression converges to, and includes, the learner's present developmental state. But from that state onward, into its future, the learner's **present potentials** fan out (like the headlight beam of a moving vessel) as the myriad of all viable state-transition paths leading from the present state, into the future.

Along such a learner-evolution, any two of its states, together with all of the learner's interim states, constitute the learner's **path of progress during that time-interval**. Each pair of distinct developmental states along a progress-path also identifies a **differential** — the coupling of (1) the symmetric difference of the two point-sets, and (2) the associated time-interval. Best seen on the lattice is that, by fixing one instant of time (and the corresponding state), and attending all later times as a variable, the associated convergent nest of ("right-side" or "forward looking") time-intervals defines a corresponding convergence of (the symmetric-difference) point-sets. That convergence describes, for time-forward motion along the progress-path, the learner's **instantaneous thrust** (for continuous lattice-paths) or **incremental thrust** (for the discrete case), at that (fixed) point in time — giving such a (thrust) point-set for each instant of time. Over the full time-span, the progress-path's forward-differentials thus converge to its **"outside" derivative** — as a time-entries function that gives, for each point in time, a point-set that describes "what was learned next".

Of course, the analogous construction yields an **"inside derivative"** of the progress-path — representing "what was just learned". The **"two-sided" derivative** represents the current boundary along which changes in the evolving point-set define the current state-transition. Along that boundary is where the "Pacman™" learner progressively "eats" or "spits out" learnable points from the domain. (Of course, all of the above warrants very precise formal mathematical exposition — but the purpose of this paper precludes such a formalistic digression.)

Thus, **the progress-path's derivative**, too, is a time-indexed family of point-sets, each contained in the underlying domain of learnable points. On the domain, the derivative portrays how the "stages" of the learner's boundary edge across the terrain. But on the learning-lattice, the derivative appears (like discrete stepping stones in a garden, or like a continuous walkway) as **the "boundary-path"** of lattice-points (the boundary-sets) that the learner progressively acquires within the learning domain — while the learner's state-transition concurrently progresses, elsewhere along the learning lattice. [Syllabus methods focus on such boundary-paths. When the state-transition's outside derivative consists only of single domain-points, the boundary-path on the lattice consists only of single-point sets — and the derivative/boundary-path (of lattice-points) is essentially synonymous with a point-succession within the domain. The methods are applied to such point-progressions within the domain.]

Every domain-point that occurs in at least one of the learner's developmental states occurs also in at least one "stage" of the progress-path's derivative — as an incoming boundary point, or an outgoing one (perhaps even as an "in-again, out-again" point). In fact, the progress-path, itself, is the progressive accumulation of such boundary changes. In effect, "from that (beginning) time forward" (or backward), the learner's evolution is the result of how it adsorbs, absorbs, internalizes, retains, discards, etc., the progression of learning-domain "items" that it encounters. Accordingly, from any developmental state that is selected as a beginning state for a path of progress, the **progress path (of the evolving state) is the integral, over time, of its own derivative** (also over time).

## REALIZATIONS: ALEKS AND MACS

Except for the vocabulary, the proceeding topological model abstractly describes the "evolution" of any "complex" whose instantaneous states can be characterized as point-sets. So, any effort to apply that model within a particular context begs for evidence that such application is realistic, perhaps even useful. The state-transition theory's applicability to the context of education in mathematics is demonstrated by the two realizations aired below — the latter of which led to **mathematics-syllabus methods**.

**ALEKS**: At present, the most visible practical application of the state-transition theory is the widely used ALEKS System of internet-tutoring. Its learning-domain is a battery of skills that commonly are curriculum-prescribed for students to achieve. The learner's current state of know-how consists of whatever of those skills the learner already owns. The learner's evolution along the learning-lattice is through progressive acquisition of additional skills. Via the internet, computers are used for assessing the learner's present (such) state, and for providing performance-training activities for learning additional skills.

That tutorial system clearly is intended for use with students who are expected, by various curricula, to gain the know-how for performing those tasks. Within current realities of curricular education, there is a growing market for improved technologies for performance testing and for performance training. Were it not for the marketability of some ALEKS-like practical service, scientific advancement of the academic state-transition theory of learning probably would be very lethargic. Fortunately for mankind, the financial viability of ALEKS is an important stimulus for the associated scientific progress.

Underlying the ALEKS tutorial system is a corresponding particularization of the above topological model. Doignon and Falmagne describe their mathematical model in detail in their (1999) book, Knowledge Spaces — an elaborate extension of their original paper (1985) by the same title. Their learning-domains are hierarchies of "questions", and their "states" are of know-how for producing "correct answers". Although their mathematical theory is the foundation for ALEKS, it also has broad potentials for other applications.

**MACS**: A less visible realization of the topological model has been used, since 1978, as a theoretical backdrop for operations of **The (MACS) Project for Teaching and Learning Mathematics As Common Sense** (to the learners, themselves) — sponsored, since 1980, by the MALEI Mathematics Institute. The MACS-use of the state-transition theory invokes the above topological model within the **psychological science of psycho-mathematics** — i.e. of the development of functional personal mathematical intelligence within the human mind. The learning-domain is any **mathematical arena** of **mathematical content** and **mathematical power**. The content "points" constitute any identifiable mathematical theory within which the learner is growing. The power "points" are mental processes that enable learner-growth within that theory. Each developmental state includes a content-component within the mathematical theory, and also includes a power-component of currently functional processes. In that sense, each is the learner's **current state of functional mathematical intelligence** within that mathematical arena.

The MACS model comes from this author's (1963) document, The Physiology of Mathematical Concept Development, composed at Illinois Institute of Technology, as an unpublished graduate thesis. As an initial mathematical venture into **the psychology of theoristic learning of mathematics** — more simply called **mathematical learning** — it presented a partial topological description of how mathematical intelligence grows, through use of mental "operators" that are well known and routinely used by all professional mathematicians. Soon thereafter, that description of mathematical growth was further developed into a lattice-theoretic mathematical model of mathematical learning. Its preliminary description was presented at a 1968 national conference on the psychology of mathematics cognition, and soon after was published in The Journal of Structural Learning, as A State-Transition Model for the Mathematical Study of Learning and Instruction.

In those days, the state-transition theory of theoristic learning of mathematics already was being used at IIT, for purposes of teacher education — both in a formal seminar in mathematics curriculum, and within an experimental "mathematics remediation clinic" conducted as a teacher-internship program. (That experiment is described in Innovative Practices in Teacher Education, an ERIC-published report of proceedings of the NCTM's first national conference for mathematics teacher-educators.) However, it still was a state-transition theory only of mathematical learning. It did not yet extend to include **a mathematical model of how mathematics instruction works** — i.e. of how instruction does, could, or "should" guide development of the learner's functional personal mathematical intelligence. Nonetheless, that teacher-education experiment set the stage for subsequent clinical development of the MACS model — and its use of syllabus methods.

During the next decade, the state-transition model was used as a backdrop for innovations in the areas of mathematics curriculum and program management — which eventually revealed the essence of a **mathematical state-transition theory of learning-guidance**, and how it could lead to the advent of a bona fide **science of mathematics instructology**. That revelation led to the author's 1978 creation of the MALEI Mathematical-Learning Clinic, where the state-transition model still serves as a learning-theoretic basis for clinical instruction — and serves also for clinical R&D, including the model's extension into a theory of learning-guidance. Early findings were profound and compelled prompt creation (1980) of the MALEI Mathematics Institute, for continued pursuits along those lines. Ever since, MALEI has sponsored the MACS Project's clinical use and development of its psycho-mathematics version of the state-transition model of learning-guidance. Throughout, syllabus methods have been used as one of the major tools — for clinical instruction, as well as for R&D.

## INSTRUCTIONAL GUIDANCE OF LEARNER TRANSITION

In the initial topological model, above, the learner's state-transition is viewed as though it already were past history — which is why the learner's progress-path is the integral of its own derivative. But instruction normally focuses more on the future, than on the past. The learner's track and present state are pre-established — but branching out from the present state are numerous present-potential progress-paths, and the learner will follow only one of those. The function of instruction is to exert managerial influences for which of those future paths will be realized.

In some cases, the instructor actually **targets** some mathematical points, with the **instructional intent** that the learners encounter, adsorb, absorb, assimilate, and accommodate, each point in the **target set**. Within the domain, where the learner's present developmental state is a point-set, the instructional intent is for that **state-set** to be progressively re-shaped so that it eventually will encompass the target-set. That scenario looks much like any dynamic geographic map of growth or of conquest.

On that domain's learning-lattice, however, the target-set appears as a single **target-point** — and there may be many other lattice-points that **cover** the target-point. Each of those target-covering lattice-points is, in the domain, a point-set that includes the target-set. So, achievement of any one of the target-covering lattice-points suffices for achievement of the target-point on the lattice. Those target-covering lattice-points constitute a (**target-zone**) sub-lattice. The instructional goal is met if the learner enters that zone. In that context, the instructional intent is to "shepherd" the learners so as to "herd" them into that target zone.

In effect, the instructor strives to **navigate the learner's transition** along the lattice — so that its evolving progress-path eventually will reach the targeted "harbor" zone. One means of doing so is reminiscent of "Pacman™" — wherein the learner encounters a linear progression of mathematical points (within the domain), and absorbs and assimilates those points, in that order. Thereby, within the domain, the state-set progressively expands (hopefully to encompass the target-set) — while the state-point climbs upward on the learning-lattice (hopefully into the target zone). In navigational perspective, the point-progression within the domain is an **itinerary** of points for the learner to acquire — and the point-progression along the learning-lattice is a **course** for the learner's state-transition.

When such an itinerary is instructor-prescribed for purposes of (lattice) steering the learner-transition into a target zone, that progression of mathematical points (in the domain) is **a mathematics syllabus**. So, the instructor-prescription of a mathematics syllabus (within the domain) tacitly prescribes also a flow of **mathematics growth of the learner**, both as an expansion of a state-set within the domain, and as a movement of a state-point along the learning-lattice. Typically, a mathematics syllabus covers all points of an instructor's target-set — but also typically, it also covers many auxiliary mathematics points. However, instructor-prescribed or not, any such progression of mathematical points likewise defines a possible progressive expansion of the state-set, and a corresponding movement of the state-point along the learning lattice. So arises **the syllabus problem** of which of those progressions is an "adequate;" or "optimal" or "best;" choice for a syllabus.

In that context, instruction is seen to have three distinct functions. One is **instructional decision-making** — about what progression of mathematical points to use as a syllabus, and what actions are needed in order to implement that syllabus. One is **research/analysis** — for purposes of attaining a reliable basis for such decision-making. One is **execution** of those decisions. Those same functions also are the three components of any managerial effort. The "head" makes the decisions; the "executive arm" carries out the head's directions; and the "intelligence arm" provides the head with the information it needs for making good decisions. Instruction, then, is a specialized form of learning-management — and **the science of learning-guidance is an aspect of the managerial sciences**.

Syllabus methods are research/analysis tools of the intelligence arm of (learning-management) instruction. Locally, mathematics-syllabus methods can be used by the individual instructor, for improving personal intelligence about instruction in a specific topic or subject. That is how The MACS Project's MALEI Mathematical Learning Clinic has progressively developed the nation's most effective program of clinical mathematics instruction. More globally, mathematics-syllabus methods can be used by agencies or organizations, for improving national professional intelligence about instruction in specific topics or subjects — which is a major purpose of the MACS Project.

Regardless of who uses them for what purposes, mathematics-syllabus methods always are used within some particular context of whatever **dimensions of instructional concern** constitute the **orientation** of the particular syllabus-research effort. Identifying those dimensions can greatly facilitate analysis of whatever syllabi are being considered. It also can enable selection of **criteria** for the acceptability of any syllabus. Of course, substantial differences in orientation can call for widely differing criteria — and even for notable differences in what syllabus methods are appropriate for use.

## THE MATHEMATICAL INTEGRITY CRITERION

Perhaps the most stringent use of mathematics-syllabus methods occurs among professionals in mathematical research. Their **mathematical integrity** criterion has profound, though yet unrecognized implications for future reforms of the core curriculum in mathematics. Among mathematical researchers, **mathematical comprehension** of anything amounts to knowing a mathematical theory that describes (or “models”) that thing — both through an axiomatic description of all such things, and through the theoretic concepts and theorems that derive from that axiomatic definition. Research mathematicians are professional learners — and they learn partly by engaging in the creation or extension of mathematical theories which they personally hold. In fact, **the mathematical arts** consist of the processes and tools that are used in that kind of learning. Much of their research is done through learning-processes that are loose, illogical, informal or subconscious. But the intended end result is absolutely reliable knowledge — about anything that fits the axiomatic definition.

Their use of syllabus methods is woven into the research effort. They rarely do such research merely to entertain themselves. More often, they intend to **provide reports** of their results to other learners — often to other mathematical researchers who will be critically scrutinizing the reports. In the final stages of the process, **the reporting researcher so functions as an instructor** — and his/her report includes a progression of mathematical points for the learners to follow — **the mathematics syllabus** for that report. **The criterion of mathematical integrity** is applied to that syllabus. But the researcher's awareness that the integrity criterion must ultimately be met has a permeating effect throughout the research effort.

To be professionally acceptable, the research findings must eventually be structured for mathematical integrity — i.e. in accord with currently accepted **rational standards for mathematical sensibility**. Necessarily, the reporting researcher presumes that the learner has a basis of a priori knowledge (and in that regard, professional mathematicians routinely exercise demanding license of authorship). But beyond that, each mathematical point that is newly appended to the evolving mathematical theory is **mathematically derived** from within previous states of that theory. Accordingly, **mathematical learning** is theoristic learning in which mathematical points are newly learned by deriving them from previous states of the evolving mathematical theory.

Moreover, the mathematical sensibility standards must be met also by the manner in which the report is presented. The conveying rhetoric need only be effective. But ***the mathematics syllabus of that report must meet the mathematical sensibility criterion of mathematical integrity***. The reporter's mathematics syllabus is instructor-intended to guide the state-transition of the learner's own mathematical theory until the latter, too, incorporates the research findings. In order for that to happen, **the syllabus must mathematically derive each of its newly introduced syllabus-points, from within previous theory-states**. In reality, many researchers are relatively insensitive to their readers' needs for comfortable digestion of the material — as when asserting a "definition" before providing the justifying existence theorem, or when asserting the theorems before presenting their derivations. The nicer ones try to use syllabi that are **developmentally continuous** — wherein the syllabus, itself, derives each of its newly injected theory-points, from theory-states achieved prior to that injection. From the viewpoint of psycho-mathematics, only a developmentally continuous mathematics syllabus fully qualifies as being a bona fide **mathematical** syllabus — because it defines a learner-progress path whose state-transition is through mathematical learning.

The curricular importance of the mathematical integrity criterion for mathematics syllabi — and especially of developmental continuity — stems from the fundamental psycho-mathematics principle on which the MACS Project is based. What mathematicians know as "mathematical integrity" is but a highly refined version of what laymen know as "common sense". The primary cause for student difficulties with curricular mathematics is that the curricular mathematics syllabi often fail to make common sense to the students, themselves — typically because **the mean-national syllabus is wrought with developmental discontinuities**. For a quarter-century, the MACS Project has consistently confirmed that mathematical repairs of curricular syllabi — to make them mathematically sensible to the students — is the most critically needed kind of reform for the core curriculum in mathematics. That is why MALEI initiated The Mathsense Library website at [http://www.mathsense.org](http://www.mathsense.org).

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