VECTOR ALGEBRAIC THEORY OF ARITHMETIC: Part 1

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ABSTRACT: In the subject of vector algebra, the scalars traditionally constitute a field. But when it is allowed that the scalars constitute merely a **system of whole numbers**, the so modified space yields a bona fide **vector theory of arithmetic calculations** – suitable as one mathematical cornerstone for re-forming the (k-calculus) core curriculum in mathematics. Its vector spaces are the familiar **numerals-vocabularies** – for wholes, integers, fractions, rationals, reals, and complexes. The algebraic theory opens the way (1) for improving the effectiveness of instruction in arithmetic, and (2) for giving all students a comfortable, but thorough grounding in selected aspects of linear algebra, by the time they fully achieve basic literacy in arithmetic. That is because the theory of **whole-scalars vector spaces** integrates all of the usual, but traditionally separated numerals-vocabularies – while fully clarifying most (if not all) of the traditionally hazy or mathematically absurd arithmetic notions within the core-curriculum – including "place values", "borrowing", "carrying", "bring down", "proper", and "reduced", "ratios", "proportions", "imaginary", and more.

[Part 1 of this survey, sketches the vector algebra of calculations up through the Arabic numerals. Its sequel, Part 2, extends that vector theory to cover the arithmetic of whole-scalars (vector) numerals for the other kinds of number systems. For brevity, this two-part paper is written to professional mathematicians who already are well versed in vector spaces. For other educators and students, more casual presentations of the same mathematical theory are intended for publication in **The Mathsense Library** at http://www.mathsense.org.]

THE ARABIC CONSTRUCTION OF WHOLE-NUMBERS LINES

Development of the whole-scalars vector theory requires immediate access to systems of whole numbers – and to their standard numerals-vocabulary, the (Euro-Hindu-) **Arabic Numerals**. For curricular purposes in early childhood education, systems of whole numbers cannot be merely postulated – they must be constructed in the most natural, simplest and most expedient way. So, the first step in developing the algebra of arithmetic is the **Arabic construction of the whole numbers**.

The Arabic numerals are **finite strings** of **Euro-Arabic digits**, 0-9 – with the **simple Arabic numerals** being 0 and all digit-strings that start with non-0 digits. (The non-simple ones use more than one digit and start with 0). As a string, each Arabic numeral is **pronounced, digit-by-digit,** from left to right.

The family of all simple Arabic digit-strings is **ordinally alphabetized** by the following rule: (a) shorter strings are before longer strings, and (b) same-length strings are **alphabetically ordered**. That kind of alphabetical ordering is a **linear** ordering, and the resulting line is of the **Peano kind**. Every Peano line is discretely ordered, has a minimum point, and has no maximum point. [That concept of a Peano line is a common-sensible equivalent to the three classical versions of "finite mathematical induction".]

Every system of whole numbers uses a Peano line of points. Since all whole-numbers systems are isomorphic, any one Peano line can be used as a representation for each of the others. The **Arabic-numerals vocabulary** is the ten-digit case of an infinitude of **ordinal-alphabetic vocabularies** for systems of whole numbers – constituting an infinitude of systems of whole numbers. Included, of course, are the familiar binary, octal, duodecimal, and hexadecimal systems.

Alphabetics is a mathematical topic that properly belongs in the early-school mathematics curriculum – if only as a context for comprehending the Arabic and decimal-point numerals. Moreover, the alphabetic constructions of Peano lines reveal that the fundamental meaning of an Arabic numeral is simply its position as a line-point – a meaning that **in no way depends on the elusive curricular notion of "place values"**. Actually, place-values and polynomial expansions (as "expanded notations") are relatively advanced vector-theoretic concepts. So, **PREMATURE encounters with those notions overly complicate the work of teaching or learning arithmetic**.

VECTOR DEFINITION OF WHOLE NUMBER OPERATIONS

Every Peano line can be upgraded, to become **a system of whole numbers** – by imposing two classes of operations: "additions" and "multiplications". In particular, the line of simple Arabic numerals thus becomes **an (Arabic) system of whole numbers** – in which the **whole-numbers are digit-strings**. That happens as soon as the learner achieves, and owns, appropriate definitions for those two numeric operations – which might be long before he or she knows how to calculate the results of such combinations.

A vector-looking "slide rule" representation of the needed operations already is being used the schools – where a length-n arrow is drawn upward from a number-line point, m, to specify the value of m+n, also as a line-point. But that is not actually a construction of the whole-numbers addition functions – since it presumes a *continuous* "number line" with *continuous* arrows. However, the usual geometric model for vectors – wherein, arrows are vector-added to each other, or scalar-multiplied by numbers – can be adapted for use with Peano lines. That yields a "measuring tape" definition of the operations – by **counting spaces**.

Along Peano lines, each non-empty interval is **finite** if it has both a **lower endpoint** and an **upper endpoint**. A **space** for that line is an interval that has two distinct endpoints, but nothing in between them. The corresponding **Peano scale** is the alternation of line-points (called the **marks** on that scale) and line-spaces (or "places" on that scale): **0**,[0,1],**1**,[1,2],**2**,…

Along such a Peano scale, an **arrow** consists of a finite interval (of marks and spaces) that has two **endmarks** – together with one of the two inverse directions: "upward" (or "plus"), and "downward" (or "minus"). Depending on the arrow's direction, one endmark is its **notch** and one is its **tip**. An arrow's **length** is the (cardinal/ordinal) number of spaces that it comprises.

Those "space-counting" arrows yield the needed definitions for whole-number additions and multiplications. Following the usual mode for developing geometric vectors, the arrows are organized into equivalence classes, called **vectors**. Each (upward) **plus-class** has one **original arrow** whose notch is at 0, but also has an up-arrow from every other mark. The usual "notch to tip" composition of arrows onto original arrows – to get original arrows – constitutes a space-counting model for whole-number additions. The geometric vectors present each whole-number multiplication as a repeated application of an addition.

Those (geometric) vector operations thus upgrade the Peano line of Arabic numerals into a line of whole numbers. In the process, that "Peano vectors" model also integrates the cardinal, ordinal, and arrow developments of the whole-number translations. The values of the combinations can initially be got by using measuring tapes and/or calculators – long before the learner has access to "the tables" or to Arabic arithmetic.

INVENTORY SPACES

Vector algebra is intuitively used by all young children (and by all adults) – albeit a very informal use of the most primitive kind of vector algebra. The basic ingredient is the (whole amount) **quantity** – as with "2shoes" or "4hearts". Those are collected into **inventories** – as with {2shoes, 2boots, 1hat, 1coat} or {2S, 2B, 1H, 1C} or even 2S+2B+1H+1C. Since such combinations can be vector-added, and multiplied by scalars, they constitute a whole-scalar vector space. Despite their primitive nature, such spaces present many important ingredients of the algebra of vectors (which covers a very substantial part of introductory algebra). They also provide many vector elements that are basic to the arithmetic of numerals.

Inventory algebra begins as a "kindergarten algebra" that relies on naming and counting various *kinds* of things. But the greater conceptual importance of the inventory theory is that it **allows quantities whose denominations do not have numeric values**.

Kinds: Any "thing" whose nature is of much mathematical interest can be abstractly described in terms of one or more of its **properties**. A property is **an abstract**, in the sense that it speaks only about some aspect of an object – and that property might be shared also by other objects. A property thus identifies the collection of all things to which it refers. Accordingly, **a list of properties (all of which are shared by some things) defines a kind of things** – and the listed properties serve as the **defining postulates** for things of that kind. Through use of such **postulational definitions**, the objects of a universe of discourse become classified into various kinds-of-things – imposing a taxonomic structure onto the semantics for that universe. All of that intuitively begins to happen whenever a learner begins to a acquire *names* for things.

Denominations: " Nomen" means "name", and a "**denomination**" is a name for things of a kind – as with "cow". (In contrast, a **denominator** is something that identifies what denomination is being considered – as might "Bossie".) In accord with convention, we reserve the term, "**poly-nomial**", for multi-denomination combinations that admit also to polynomial multiplication.

An inventory space normally invokes several denominations (kinds-of-things). Built into the meaning of "inventory" is that those denominations must be **independent** of each other, in the sense that an object that is covered by one of the so-used denominations is not also covered by another denomination. Any initial selection of independent denominations defines a whole-scalars vector space – by constituting its **basis.** The basis-denominations also classify the objects of the discourse-universe into "equivalence classes". Although each inventory-denomination is "a variable" in that it refers to EACH object within its "domain" of things (rather than to a particular example, thereof) – inventory denominations are not *numeric* variables, because their domains are not sets of numbers.

Quantities: A **quantity** is the "of" coupling of a number with a denominated kind-of-things, as with "3Cows" meaning 3 of the cow things. The denomination bespeaks **the kind** of that quantity – and its numerical **coefficient** is **the amount** or **numerator** of that quantity. The quantity, 3cows, illustrates why we are reluctant to regard the coefficients as always being "multipliers" of the denominations. For, "3 cows" speaks of the cardinal class, 3, from the power-set of "cows" – which would be a rather remote meaning for "multiplying" the cows-class, by 3.

Evaluations: Arithmetic is largely about operating with combinations of quantities in various denominations. The inventory theory does not presume any numeric values for its denominations. But neither does it exclude the possibility of assigning numeric values (e.g. "prices") to things of various kinds. Any denomination-values are assigned to each its constituents – resulting in numeric values for the quantities and for the inventories – just as routinely is done in grocery stores and in introductory algebra. But when numeric values are assigned to the denominations, vector equations naturally emerge. So, even the primitive inventory algebra can lead into some algebraic operations with equations.

"Dependence" of basis vectors: To constitute a basis, the denominations must be independent of each other. But arithmetic often requires that some basis-vectors are scalar multiples of other basis-vectors. That need is nicely met by **partitioning the vector space into equivalence classes** – wherein some basis vectors become equivalent to combinations of other basis vectors. However, that kind of extension goes beyond the scope of inventory theory. Instead, one nicety of the inventory theory is that its pursuit does not necessarily entail equivalence classification of vectors – so allowing for the vector-algebraic operations to be learned within the simplest possible context.

Vector descriptions: As a **combination of quantities**, a vector may be described in alternative forms – including the (non-ordered) **quantity-set form** for the vector, {4B, 5C, 3A} – and the (commutable) **additive form** for the same vector 3A+4B+5C. The sequential forms – as with (3A, 4B, 5C) – tacitly assign a **place** to each quantity. Although their permutations present the same combinations, they hint at the possibility also of locking each denomination into one sequential place.

So arises a **matrix format** format for the vectors – which is nicely seen through the context of inventories. Therein, it is common practice (e.g. in MSExcel) to post the (basis) denominations as **headings** for rows or columns. In the resulting "inventory matrix", each inventory appears only as a (non-permutable) **string of whole-number amounts**, one coefficient (or "numerator") for each of the specified denominations. [Even within the primitive context of whole-scalar inventories, vector operations with the rows of coefficients lead toward the development of matrix algebra. But that would be a digression from developing the arithmetics of numerals.]

For reasons that stem from English rhetoric, curricular preference is for writing inventories as **rows** of coefficients – under a basis-row of column-heading denominations. For reasons that stem from current financial practices, preference is for expressing those **tuples** in box-strings of the kind often seen in daily life – [__|__|__|__].

Thus, relative to an [A | B | C | D] basis of column-heading denominations, the {2B, 3C, 4D} vector is matrix-presented as the row, [0 | 2 | 3 | 4]. In **coordinate form**, the same row-vector is (0,2,3,4). Either way, the total matrix of all such number-strings also is a whole-scalar vector space – and the algebra of such spaces yields an arithmetic of calculations.

Arithmetic of inventories: In matrix format, inventory-vectors are represented by number-strings. Those tuples usually are not used as "numerals", but in very specific cases, they are. Either way, the vector operations with the rows of coefficients do constitute a rudimentary form of arithmetic – one that paves the way for the arithmetics of numerals. Those row operations include: (1) place-wise scalar addition; (2) where possible, place-wise scalar subtraction; (3) distributive multiplications of number-strings, by scalars (4) where possible, factor-division of number-strings, by scalars; and always (5) remainder-division of number-strings, by scalars.

The conceptual importance of whole-scalar inventory arithmetic lies in the **logic for processing the number-strings** – rather than in immediate mental fluency with calculations. The educational goal for very young children is fully met if they can perform those row operations, through some mode of counting or through assistance from some kinds of calculators.

The axes: Within any vector space, a **unit-vector** has 1 in exactly one place, and 0 in all other places. Within the inventory theory, their primary importance is that the family of all whole-multiples of a unit vector is **an axis** or **dimension** of the space. On ancient Arabian counter-boards, the axes were parallel vertical grooves into which **calculi** (i.e. stones) could be placed. Thereby, a whole-scalar inventory appeared as a vertical bar-graph. Today, such **Arabian calculator** counting-boards can enable children to digest the inventory algebra of number-strings.

Proportional inventories: Among the subspaces of an inventory space are its **proportions**, each of whose vectors are the **ratios** in that proportion. A **reduced whole-scalar vector** is one for which the GCF of its coefficients is 1. The set of all whole multiples of a reduced vector constitute a proportion (subspace) – for instance, each of the spaces axes.

Every vector thus belongs to exactly one proportion – except that the 0-vector belongs to every proportion. But the space's vectors become ratios only when they are classified into that space's proportions. That shift in context is often indicated by expressing the vectors in "ratio mode" – as with 2B: 3C: 4D, or with [0 : 2 : 3 : 4]. As with any vector space, an inventory space's proportions constitute a starburst of "straight lines" that radiate from the origin. Each non-proportion line in the space is a vector-offset of a proportion. [The essence of coordinate "linearity" is proportionality – whence the descriptor, "linear combinations".]

So, even the lowly, "early childhood" **vector algebra of (multi-dimensional) inventories** fully reveals that **a ratio is NOT merely a single-number quotient – and that a proportion is NOT merely an equation of quotient-formulas**. [The curricular tradition of focusing on only 2−dimensional ratios long has led to widespread confusion of binary ratios with quotients, fractions, and rationals.]

MEASUREMENT SPACES

While all kindergartners intuitively venture into inventory spaces, all primary students intuitively venture also into the more advanced arena of **measurement-algebra** – well before they are mathematically ready to perceive the common-sensibility of arithmetic calculations with Arabic numerals. The **vector algebra of measurements** is informally used throughout modern family life, and invokes such denominations as: nickels, cups, pounds, minutes, (temp-)degrees, miles, watts, and now, bytes.

The conceptual importance of measurement algebra is that it is a natural extension of inventory algebra, in the direction of Arabic arithmetic – so serving as an algebraic bridge between those arenas. Most of the conceptual difficulties with Arabic arithmetic are with elements that are not intrinsic to inventory-algebra – but whose common-sensibility becomes obvious within the algebra of measurements. However, that common-sensibility is obscured by the slickness of traditional "shortcuts" of Arabic arithmetic.

As with inventories, measurement-denominations speak of kinds of things. But unlike with more rudimentary inventory spaces, **measurement-spaces** are partitioned into equivalence classes – and that difference is the algebraic bridge from inventory-arithmetic, to Arabic arithmetic.

Consider a set of "rainbow" poker chips in 7 colors – RΩY G BIV. (The "colored chips" concretely provide an abstract model that can be adjusted to fit almost any commonplace system of measurement.) In that context, every inventory of chips is a 7-dimensional **rainbow-vector** – a combination of 7 quantities, one amount in each color. That rainbow-inventory space can be converted into **measurement spaces** of numerous kinds. For brevity, we consider only the simplest kind.

Now, suppose that "a sandbox-containers game" uses fluid measurement containers: gal., qt., pt., c., oz. tbs., tsp. – represented on the tabletop by the RΩY G BIV game chips, and also by number-rows. From the sandbox come the **quantity-conversion equations**, as follows: 1R=4Ω; 1Ω =2Y; 1Y=2G; 1G=8B; 1B=2I; 1I=3V. Those equations align the RΩY G BIV rainbow denominations, by establishing the **adjacent pairs** in that line – with R as the "highest" denomination and V as the "lowest". Each of those **adjacency equations** also presents a whole-number **conversion factor** for the lower denomination in that equation.

The adjacency equations also imply conversion equations/factors between **every pair** of unit vectors. In particular, each of the rainbow units **equates with a corresponding V(iolet) quantity**. That comes very close to assigning numerical values to each denomination. The difference is that those V-quantities are not numbers. However, assigning a numeric value to the V-unit (e.g. 5 per tsp.) would immediately yield a value for each of the vectors.

Moreover, the adjacency equations also imply equations among some "rainbow" *quantities* – as with 6pints = 3quarts (the lower coefficient, for the higher denomination). As a result, those equations also distinguish between **reducible quantities** and **irreducible quantities** – the 6pints quantity is reducible to the 3quarts quantity, but in the specified rainbow system, the 3quarts cannot be reduced to another single quantity. Notable here is that ***reducing*** a quantity converts it to an equivalent quantity that has a **smaller numerator**. That criterion for "reducing" generalizes for use throughout the arithmetic of numerals.

Vector Conversion Classes: We here persist with the simplistic "sand-measurement" space of rainbow vectors. Because every quantity is equivalent to one and only one V-quantity, every *rainbow-vector* also is equivalent to one and only one *V-vector* – corresponding to one V-quantity. That V(iolet)-quantity is the **magnitude** of that rainbow vector – relative to the basis for that space.

It means that the preceding adjacency equations partition the inventory-space of rainbow vectors into equivalence (or equi-magnitude) classes. Two rainbow vectors are equivalent iff both have the same magnitude. Within each class, each vector can be "converted" to any equivalent vector – whence the label, **"conversion classes"**

Since there is only one Violet-vector in each class, the ("violet") Peano line of V-quantities induces a Peano alignment also of the equivalence classes. Because that line of **measurement classes** (of rainbow vectors) is Peano-ordered, it could be made into a line of whole numbers – if the needed "arrow addition" and "multiplication" were imposed. The vector additions among the rainbow-vectors naturally abstracts into additions among the measurement-classes. But no such abstraction yields also a "multiplication" among the measurement-classes. So it is not always useful to regard the measurement classes as being "numbers".

"Making" and "Breaking" quantities: The quantity-conversion equations also lead to two special kinds of conversions that are essential for executing the arithmetics of measurements and of numerals, "properly". In whole-scalar measurement spaces, a **proper quantity** is one whose numerator is less than its conversion factor (to the next higher denomination). All other quantities are **improper**. It is notable that every proper rainbow quantity is reduced, but not conversely – e.g. 3quarts is proper and reduced; 7pints is reduced, but improper.

A **proper vector** is one in which each of its quantities is proper. Every other rainbow vector is **improper** – and at least one of its quantities is improper. Each measurement class contains exactly one proper vector. It is the **reduced** vector in that class, in the sense that the sum of its coefficients is the lowest such sum from that class.

The reduction of an improper vector to its proper equivalent may be done through repeatedly applying, to its quantities, remainder-division by their respective conversion factors – as with 7pints = 2·3pints + 1pint. → 3qt.+ 1pt. – thereby **making** quantities of higher denominations. In reference to "stone age" calculator-boards, educators classically called such quantity-conversions **carrying**. In the "reformed" curricular vernacular, that concept has been lost within a more inclusive (and even more elusive) formalistic notion of "regrouping".

Inverse to the making of quantities is **breaking** quantities – as with 7pints → 6pt + 2c. → 5pt. + 4c., etc. Also in reference to "stone age calculators", educators classically called such conversions **borrowing**. In the contemporary scholastic vernacular, that concept, too, has been buried under the formalistic "regrouping".

The corresponding conversion of vectors by "un-reduction" is used for "expanding" measurement-vectors by breaking some of their quantities. Judicious vector-conversion by quantity-breaking is what makes it possible to perform vector subtractions and remainder-division by scalars, "properly".

The Proper Arithmetic of Measurements: The "proper" arithmetic of measurements can be attended even while the vectors are represented by quantity-set or additive-forms. [That often is an appropriate medium for beginners.] But the matrix format of number-string row-vectors is where the "proper" arithmetic of measurements best foretells the proper arithmetic of the Arabic numerals. (The conversion factors may be posted between adjacent headings.)

The arithmetic of measurements is done, "properly", by starting and ending with proper vectors. But the vector-addition of proper measurements often yields an improper vector, which can be carried into a proper one. Scalar multiplication of proper measurements often proceeds, likewise. When subtraction of proper measurements is even possible, it often requires borrowing the minuend vector, into an improper one that admits to vector subtraction. Likewise, scalar division of a proper measurement often requires borrowing of the dividend.

THE ARABIC AND ILLION DENOMINATIONS

The vector-algebraic bridge from (grade 1) inventory spaces, over (grade 2) measurement spaces, to the (grade 3) arithmetic of Arabic numerals is completed when initial intervals of Arabic numerals are recognized as being row-vectors in the matrix format of a **decimal-measurement space** – subject to all of the vector-conversions and operations that apply throughout measurement spaces, in general. For instance, set the RΩY G BIV measurement space so that all adjacency conversion factors are ten. Such measurement spaces include some "metric" systems, and also several decimal systems of coins or currency. The rainbow game-chips can so serve as **decimal play-money** – occurring in both (decimally) proper and improper combinations.

For every whole-scalars decimal-measurements system, the matrix format's proper row vectors are strings of Arabic digits – if the space is infinite dimensional. In the finite dimensional cases, only Arabic digits occur in all places except the highest place – and sometimes there, as well. That might seem to suggest that the natural place for the first recognition of Arabic numerals as vectors would be within the matrix format for decimal-measurements spaces. But the vector-structure of Arabic numerals actually is seen in kindergarten, if not before.

Almost as soon as children begin to pronounce Arabic numerals, they are instructionally led to depart from the phonics of a digit-by-digit reading of the digit strings, in favor of **pronouncing Arabic denominations**. In English, the string, “3945”, is properly pronounced, “3 thousand, 9 hundred, 4T, 5” – which might be written as 3M+9C+4X+5I or as 3K+9H+4T+5S.. In effect, the phonics **analyze** that digit-string Arabic numeral into a combination of four quantities – whose coefficients are digits (3, 9, 4, and 5), and whose respective denominations mostly are spoken. The very young normally learn to **speak in Arabic vectors** long before they seriously tackle Arabic arithmetic. Unfortunately, the English phonics for the first ten-thousand whole numbers are not totally vector-structured: the Saxon "elev's" and "teens" induce unfortunate wrinkles into the English phonics vector space. Phonic-vectors are used also for much larger whole-numbers. The English phonics for 123,456,789,876,543 uses a **base-thousand formal vocabulary with one-thousand digits** – 123T,456B,789M,876t,543. The phonic “illions” vectors use the spoken denominations, “thousand”, “million”, “billion”, etc.; and the one-thousand coefficients are spoken in Arabic phonics.

Within the context of measurements, the phonics for the first four Arabic denominations speak about *kinds-of-things* – NOT about the *numbers*. It means that development of **the algebra of “Arabic phonics”** can be integrated first into the (grade-1) algebra of inventories, and then into the (grade-2) algebra of measurements – even into the measurement-arithmetic of number-strings, including the make/break conversions. That includes using Arabic denominations even with **improper decimal** vectors.

ALGEBRA OF ARABIC ARITHMETIC

The **Arabic vector space** is the infinite-dimensional decimal-matrix of numerical row-vectors (improper and proper – organized into decimal conversion classes). Therein, the (proper) Arabic numerals constitute an infinite-dimensional decimal abacus that has nine “beads” at each place. The rest of the Arabic counting-board consists of **improper base-ten numerals**, each of which at least one coefficient that is higher than 9.

The logic of Arabic arithmetic is largely the logic of the Arabic vector space –including conversions between proper decimal forms (the Arabic numerals) and their improper equivalents. Of course, **proper additions** and **proper multiplications** by numbers usually entail vector reduction (carrying). **Proper subtraction** often requires expanding proper minuends to improper ones. **Proper division** by scalars normally requires converting proper dividends to improper ones. [“Long” division is division of an Arabic vector by a whole-number scalar – and “bring down” is borrowing. In the classical format, the improper, divisible vector occurs along the diagonal of digit-multiples of the divisor.]

Missing from the basic vector theory, however, is the “long” multiplication of vectors by vectors – an operation that, in many kinds of inventory and measurement spaces, is senseless – but one that is an essential ingredient in the arithmetics of alphabetic-numerals vocabularies. So extending a vector space is normally done by defining a multiplication among the unit-vectors – from which follows a multiplication among the axial vectors – which then allows for a cross-distributive multiplication among the vectors. A special case is the place-shift **polynomial multiplication** among unit-vectors. Within the alphabetic- numerals vocabularies, the polynomial multiplication inherits from scalar multiplication by **the base-number** – the number of digits in the alphabet. Proper Arabic multiplication is the polynomial multiplication of Arabic numerals, with reduction (carrying) to proper vector-products.

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