STUDENT-INSPIRED RESEARCH FROM A LIBERAL ARTS MATH COURSE

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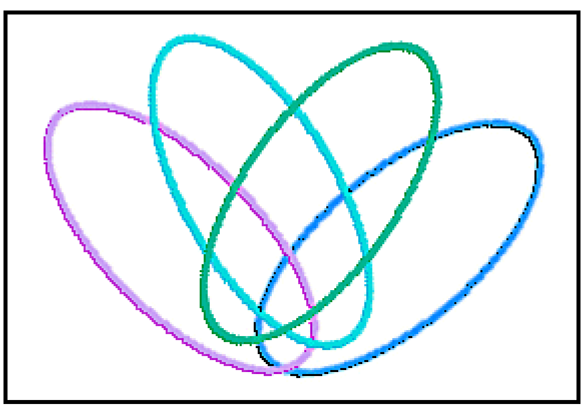
During a lecture for my Math Structures class, I was showing the students how you can use Venn diagrams to solve problems. We had done one or two "two set" problems and then were starting in on three set problems, something like the following:

Sample Problem: When the members of the NSU photography club discussed the types of film they had used during the past month, the following information was gathered: 77 used black and white, 24 used only black and white, 65 used color, 18 used only color, 101 used black and white or color, 27 used infra red film, 9 used all three types of film and 8 did not use any of these types of film.

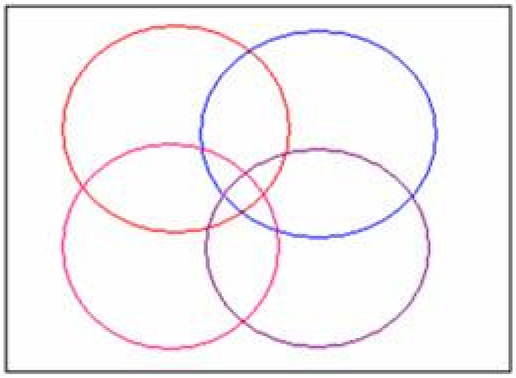
a. What percent of the students used only infrared film?
b. How many of the students used at least two types of film?

Well, in a casual comment I mentioned that you could do four set Venn diagrams (and thus Venn Diagram problems) but that my students did not have to concern themselves with this but just concentrate on two and three set problems. However, I had a student as the question "How would you even draw a four set Venn diagram?" And I replied with, "If you are really interested, I will show you after class."

I never really thought anyone would stick around, but two interested students did stay and want to see what it looked like. So, I drew them the picture I remember seeing for four sets:

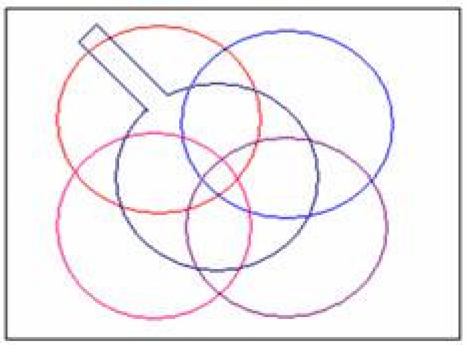


For some reason, the students tried coming up with their own four set diagram (I am not sure if they were "unhappy" with this one, or were just feeling creative!) and the "best one" they came up with looked like:



I asked the student how they had come up with this one, and they commented about being a "tile layer" before coming back to school; this made sense from looking at their design.

I am not sure how the topic got from here to "the number of overlapping regions"… but it did. They had noticed there was 1 (for 2 sets), 4 (for 3 sets) and now with what they had as their "four set diagram" they counted up 9. They thought and thought about how they would draw a five set diagram and finally settled on:



From here they counted the overlapping regions and found 16. It was at this point that they noticed the following patterns:

(2)	(3)	(4)	(5)	(6)	← # of sets
1	4	9	16	?	

(then they looked at the "differences" from (3) to (2), and so on…)

3 5 7 ??

(and then concentrated on the differences of these entries…)

2 2 ???

They noticed this bottom set of differences was always 2, so why shouldn't ??? = 2. Then going up their diagram, they predicted ?? should = 9 and hence, ? = 25!
This was pleasing to me, as they had just finished chapter one in our text, which was on inductive reasoning skills, pattern recognition, etc. This showed me they were really trying to use this stuff, on this new and challenging problem!

Now, their ne t goal was, “let s try to draw a si   set diagram and see if we are right!”   ut neither could think of how this could be done (they had now been there 50 e tra minutes, and both had obligations they were late for!) but agreed we d talk about it later.

However, when I went home for an afternoon run the problem was still on my mind and then the big revelation hit   why not relate this question to the notion of counting subsets of a given set?   Here is an outline of the argument relating the two problems:

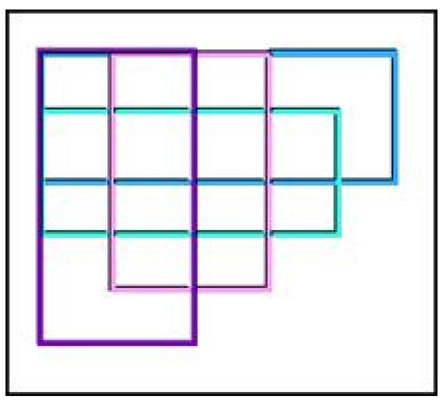
So, consider a set with two elements   a,b   which has 2 2   1   3 non-empty subsets which are analogous to the three regions in the following Venn diagram:

Since we are counting “intersections”/”overlaps” only, we can subtract the number of single sets involved in the diagram, which gives us 3-2 1 overlap.

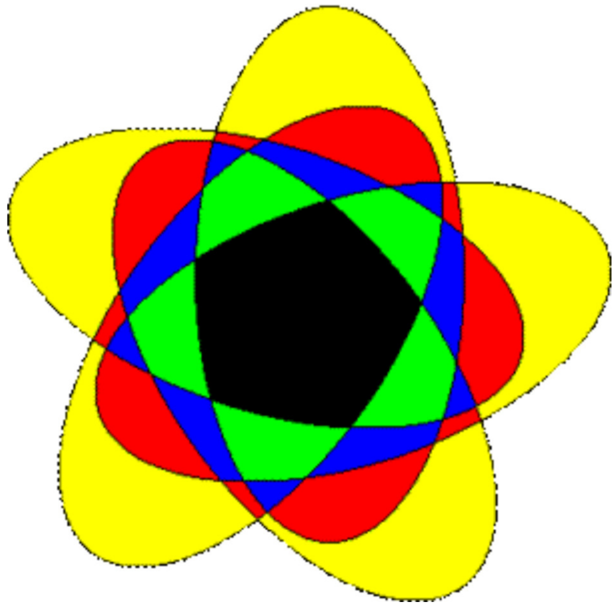
Continuing in this manner, consider a set with three elements   a,b,c   which has 2 3   1   7 non-empty subsets. These are analogous to the seven overlapping regions in the Venn diagram below:

And, since we are again only interested in the number of overlaps we can subtract the number of single sets involved in the diagram, which gives us 7-3 4 overlaps.

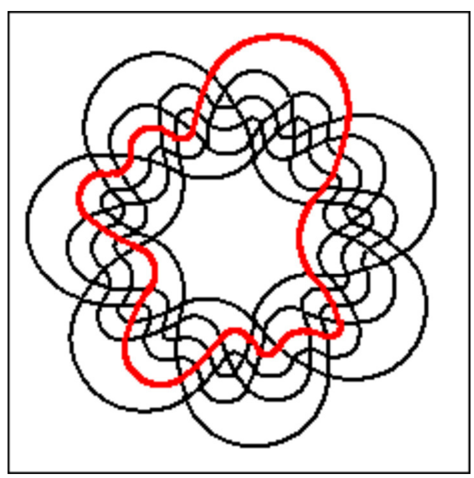
In e actly the same fashion we can look at a set with four elements   a,b,c,d   and find it has 2 4-1 15 non-empty subsets and subtracting the number of sets in a four set diagram we get 15-4 11 overlaps. This can be clearly seen in the following Venn diagram of four sets:



And even for a set with five elements   a,b,c,d,e   and find it has 2 5-1 31 non-empty subsets and subtracting the number of sets in a five set diagram we get 31-5 27 overlaps. These also can be clearly seen in the following n 5 set Venn diagram:



The pattern (i.e., number of overlaps) continues to hold for any si e set… I leave it to the reader to count the overlaps in an n=7 set Venn diagram:



and then with an n=11 set Venn diagram:

ven though the students really proved most of their conjectures falsely, I still consider the entire experience a resounding success! ow often can we get liberal arts math students to be excited enough (even for a brief moment) to stay after class 50 minutes trying to prove a "non trivial" result which they have never seen before? ven with their erred results, I think they got a taste of what mathematicians do on a routine basis; always a good thing!