Undergraduate Contributed Paper Sessions

EPaDel/MAA-NJ Joint Section Meeting
11-November-2023

2:20 pm – 3:20 pm

Undergraduate Session 1: Driscoll 221
Speakers: Aaron Kolaric, Joseph Aulenbacher, Peter de Bruin

Undergraduate Session 2: Driscoll 227
Speakers: Abdullah Alshamrani, Jerome Grant, Derek Wescoe, Rylee Barnhart, Shannon Travers, Danielle Konnick, Emily Medwid, Farhanaz Asskaryer

Undergraduate Session 3: Driscoll 240
Speakers: Eilis Casey, Juliana Pitts, Juliana Abruzzi, Alexa Fisher, Jayna Penn, Michael Fiore

Undergraduate Session 4: Driscoll 244
Speakers: Amine Boukardagha, Hung Nguyen, Iris Horng

Undergraduate Session 5: Driscoll 246
Speakers: Arwen Hertzler, Santure Chen, Xinxin Fang, Hillary Kim

Undergraduate Session 6: Driscoll 248
Speakers: John Seibert, Emily Shambaugh, Victor Greene, Christopher Heitmann

Undergraduate Session 1: Driscoll 221

2:20 pm, Aaron Kolaric (Shippensburg University)

Text Analysis of Alternative Protein Discourse

Abstract: Over the past several years, there has been an increase in discussion around genetically modified agriculture, especially on social media. X has been a prominent outlet for these discussions among users on both sides of the dialogue. In search for what discussions have been occurring, obtained were posts from 2016 to 2021 that contain certain key words that are common in the alternative protein discourse. Using the posts, can network analysis be used to see communication that is happening between users?

2:35 pm, Joseph Aulenbacher (Shippensburg University of PA)

Counting Sums of Subsets mod m

Abstract: This project was inspired by a YouTube video by 3Blue1Brown which details the solution to the question 'How many subsets of the set {1, 2, 3, …, 2000} have a sum that is divisible by 5?' That question appears very difficult at first, but the video presents an elegant solution involving a generating function and complex roots of unity. For our research, we discovered a formula for the more general case: 'How many subsets of the set {1, 2, 3, …, n} have a sum that is divisible by m where m|n?' Beyond that, an even more general case was solved, that being 'How many subsets of the set {1, 2, 3, …, n} have a sum that is congruent to r modulo m, where m|n?' Overall, this was an interesting project that combines a discrete combinatorial question with generating functions, modular arithmetic, and complex roots of unity.

2:50 pm, Peter de Bruin (Shippensburg University of PA)

9 and g-Collatz Functions

Abstract: Since the late 1930's Collatz conjecture has continued to baffle and intrigue mathematicians of all skill levels despite its apparent simplicity. I will be introducing the 9-Collatz variant of the original conjecture and describing some characteristics of the function and the sequences generated. We will also look at how these characteristics generalize across similar functions.

Undergraduate Session 2: Driscoll 227

2:20 pm, Abdullah Alshamrani, Jerome Grant (Saint Joseph's University)

Al-khwarizmi Presentation

Abstract: Discover the remarkable contributions of Al-Khwarizmi, the Persian scholar of the Islamic Golden Age. This presentation explores his enduring impact on mathematics, science, and education, as well as his legacy in shaping modern algebra and introducing the term "algorithm."

2:35 pm, Derek Wescoe, Rylee Barnhart (Saint Joseph's University)

Euler

Abstract: Euler was a Swiss mathematician and Physicist who was born in Basel, Switzerland and died in St. Petersburg Russia due to a brain hemorrhage. He contributed to the subjects of geometry, calculus, mechanics, and number theory, which is still used today in modern methods of mathematics. Euler developed the theory of trigonometric and logarithmic functions, reduced analytical operations to simpler form, and contributed to many parts of pure mathematics.

2:50 pm, Shannon Travers, Danielle Konnick (Saint Joseph's University)

Sir Isaac Newton

Abstract: We will be presenting about Isaac Newton. This includes his life and his contributions to mathematics. Our presentation will highlight Newton’s early life, the impacts this had on how he presented himself and how he worked to achieve his goals, and several of his contributions.

3:05 pm, Emily Medwid, Farhanaz Asskaryer (Saint Josephs University)

The Life and Legacy of Pythagoras

Abstract: Pythagoras was an ancient Greek mathematician and philosopher who left an enduring legacy. This presentation will delve into his remarkable journey, the development of the Pythagorean theorem, and learn how his contributions continue to shape mathematics, philosophy, and science today.

Undergraduate Session 3: Driscoll 240

2:20 pm, Eilis Casey (Villanova University)

Arithmetical Structures on Graphs

Abstract: This project focuses on the study of arithmetical structures on graphs. Given a graph, an arithmetical structure on it is an integer labeling of its vertices that satisfies some divisibility requirements. For each structure, there is an associated matrix that contains information about labels and adjacent vertices. In this talk, we will discuss two questions: 1) how many arithmetical structures exist for specific types of graphs, and 2) which structures produce matrices with the largest eigenvalues.

2:35 pm, Juliana Pitts, Juliana Abruzzi (Saint Joseph's University)

Koningsberg Bridge Problem

Abstract: There are many challenges in the world that can be left unsolved if they are not looked at in a different way. The town of Konigsberg, Russia had an issue with crossing over each of their towns bridges only one time, visiting each of the islands and coming back to your starting point. Euler studies this problem and crafts a theorem that it is impossible to cross seven bridges only passing over the same bridge once due to an odd number of bridges. During this presentation, we will discuss the troubles of the townspeople of Konigsberg, Euler's dedication to help solve the problem, and the importance of the birth of graph theory since it is a helpful tool used in the world today.

2:50 pm, Alexa Fisher, Jayna Penn (Kutztown University)

Girth of Algebraically Defined Bipartite Graphs

Abstract: This research pertains to the girth of algebraically defined bipartite graphs, which are graphs made through equations. Each node is assigned (x,y)-coordinates, and edges are only drawn if the coordinates from the two nodes, one from each partite set, satisfy a specific equation. Others had studied these types of graphs before using various fields, but not $\mathbb{Z}_n$. We investigated how changes to the modulus and changes to the equation impacted the girth of the graph.

3:05 pm, Michael Fiore (St. Joseph's University)

Chromatic Polynomials: An Algebraic Method in Graph Theory

Abstract: Graph theory has become one of the most important fields of study in discrete mathematics, giving rise to some of the most famous and elusive problems in recent history. Specifically, one very well-researched facet of graph theory is graph coloring, which has seen great use in both theoretical research and practical applications. In this talk, we will study some general properties of the polynomial, such as its relation to the chromatic number of a graph as well as an important recurrence property which finds use in this subject as well as others in graph theory. We conclude with some remarks on the computation of the chromatic polynomial by algorithm.

Undergraduate Session 4: Driscoll 244

2:20 pm, Amine Boukardagha (Swarthmore College)

Closed-Form Solution for the Heston PDE for European Put Options with Dividends

Abstract: We study Heston's PDE for European put options with dividends and prove the existence and uniqueness of solutions when asset prices and volatility are non-zero. We use a diffeomorphism to transform the PDE and boundary conditions in a simpler formulation, namely, the Laplace transform methods to transform the result. Finally, we derive a closed-form solution for the problem.

2:35 pm, Hung Nguyen (University of Pennsylvania)

Dynamics of a Vibro-Impact Energy Harvester Under Non-smooth Forcing

Abstract: We study the dynamics of a ball-and-capsule vibro-impact energy harvester (VI-EH) under triangle wave forcing to contrast with ideal models based on harmonic forcing. We obtain a comprehensive bifurcation structure of our model via simulations, a semi-analytical approach based on nonlinear maps, and linear stability analysis of the system under both the triangle wave forcing and its smooth Fourier approximations. Across a range of relevant parameters, we observe and characterize general shifts of periodic solutions and the bifurcations to smaller capsule lengths. Further analysis of these bifurcation structures also reveals novel phenomena not seen under harmonic forcing. Energy-harvesting analysis shows that low-order Fourier approximations provide an accurate estimate of the energy harvested under the non-smooth triangle wave. We find that the VI-EH is more efficient under the harmonic forcing than the triangle wave forcing in the same regime of motion, while the energy harvester generally remains in the more efficient regime of alternating periodic motion for a larger range of capsule lengths under the triangle wave forcing. This bridges the gap between previous work and experimental conditions.

2:50 pm, Iris Horng (University of Pennsylvania)

Algorithmic Generation of DNA Self-Assembly Graphs

Abstract: With recent advancements in the field of nanotechnology, there has been increasing interest in self-assembling nanostructures. These are constructed through the process of branched junction DNA molecules bonding with each other without external guidance. Using a flexible tile-based model, we represent molecules as vertices of a graph and cohesive ends of DNA strands as complementary half-edges allowing the molecules to bond with each other. Due to the unpredictability of DNA self-assembly in a laboratory setting and the risk of undesirable products being incidentally constructed, predicting what structures can be produced from a given list of components, referred to as a "pot of tiles" is useful but has been proven NP-hard. This project introduces an algorithm to computationally generate and visualize at least one valid graph and for smaller cases, all non-isomorphic graphs, given a pot of tiles. By adjusting a number of construction parameters, we can produce graphs of various orders and proportions of tiles.

Undergraduate Session 5: Driscoll 246

2:20 pm, Arwen Hertzler (Franklin & Marshall College)

Projective Geometry in Perspective Art

Abstract: A branch of theoretical mathematics, called projective geometry, provides many tools that can help artists. Projective geometry is the study of projections, or "images" and these projections are particularly applicable to perspective art. For example, though a cube is known to have six sides of equal area, when drawing a picture of a cube, only two (or possibly none) of the sides are actually shaped like a square. Projective geometry also demonstrates that math isn't all about calculations or algebra; rather, it can be a visual discipline and a unique manner in which we might approach problems. In this talk, we will present several art puzzles that we will solve with projective geometry. (No artistic experience is necessary!)

2:35 pm, Santure Chen (Franklin & Marshall College)

Same Base, Same "Parallels": Triangles on a Sphere

Abstract: An elementary fact from Euclidean geometry is that two triangles sharing the same base, say on a line *l*, will have the same area if and only if their third points lie on a line m that is parallel to *l*. However, the analogy of this relationship on the sphere seems to have been unknown until 1976, when it was asserted but not proved by David Huffman. This talk will give a proof for the following theorem using spherical trigonometry, and suggest a possible application of it in rigid origami: for spherical triangle ΔABC and spherical triangle ΔDBC with non-zero overlapping area, they have the same area if and only if the midpoints of line segments AB, AC, DB, and DC are collinear, say on great circle *l*, and A and D, B and C are on two symmetric latitudes on either side of *l*, respectively.

2:50 pm, Xinxin Fang (The Episcopal Academy)

A Graphic Approach to the Frobenius Number Problem in Three Variables

Abstract: The Frobenius number for a set of relatively prime positive integers, where the smallest integer in the set is at least 2, is the largest integer that cannot be expressed as a nonnegative linear combination of those integers. We analyze the Frobenius number for three variables by analyzing the lattice points associated with the line ax+by=ab and its downward parallel translations, along with each lattice point's corresponding value ax+by. As an application of our graphic approach, we recover that key theorems such as results from Sylvester and Selmer can be reduced to a graphic problem of locating, followed by an evaluation of the linear form ax+by at, a lattice point. Therefore, an arithmetic problem of finding the Frobenius number is reduced to a visual/graphic problem of locating the lattice point, followed by an evaluation, both of which are simple tasks. We then extend our shortened proof of Selmer's explicit formula to derive relatively simple formulae for the general Frobenius number of three variables. Compared to previous works, our results are not algorithmic in nature nor complex, a significant improvement.

3:05 pm, Hillary Kim (Swarthmore College)

A Partial Resolution of Hedden's Conjecture

Abstract: Our research investigates the properties of the self-maps induced by pattern knots in solid tori on the smooth knot concordance group. We began by testing two-component links, one by one, to see which of them, if any, described patterns which induce homomorphisms on the knot concordance group. The initial goal of our research was to add small pieces of evidence to support Matthew Hedden's conjecture that the only patterns which induce homomorphism are those which induce the zero map, the identity map, or reversal. However, our goal changed somewhat as we started to recognize patterns in our findings, and we were eventually able to prove a stronger result, which resolves Hedden's conjecture for patterns of certain winding numbers. We prove that if the winding number of a pattern is even but not divisible by 8, then the corresponding map is not a homomorphism, thus partially establishing Hedden's conjecture. This is the first result to obstruct all patterns of a given winding number from inducing homomorphisms.

Undergraduate Session 6: Driscoll 248

2:20 pm, John Seibert (Bloomsburg University of Pennsylvania)

Evaluating the Cost of an Integer using Primitive Recursive Functions

Abstract: In 2020, Max Norfolk defined the cost of an integer relative to S, denoted C_S, a finite set of binary operations. Here, we will present an approach to constructing C_S by expanding the scope of S to sets of primitive recursive functions. $C_S(n)$ is defined as the minimum of n or the sum of the cost of a primitive recursive function, f: $N^k \to N$, in S and the cost of the argument, x, where f(x) = n. We will discuss the definition of C_S, the computability of C_S , and several theorems and open questions.

2:35 pm, Emily Shambaugh (Dickinson College)

Ghost series and a motivated proof of the Bressoud-Göllnitz-Gordon identities

Abstract: We present what we call a "motivated proof" of the Bressoud-Göllnitz-Gordon partition identities. Similar "motivated proofs" have been given by Andrews and Baxter for the Rogers-Ramanujan identities and by Lepowsky and Zhu for Gordon's identities. Additionally, "motivated proofs" have also been given for the Andrews-Bressoud partition identities by Kanade, Lepowsky, Russell, and Sills and for the Göllnitz-Gordon-Andrews identities by Coulson, Kanade, Lepowsky, McRae, Qi, Russell, and the third author. Our proof borrows both the use of "ghost series" from the "motivated proof" of the Andrews-Bressoud identities and uses recursions similar to those found in the "motivated proof" of the Göllnitz-Gordon-Andrews identities. We anticipate that this "motivated proof" of the Bressoud-Göllnitz-Gordon identities will illuminate certain twisted vertex-algebraic constructions.

2:50 pm, Victor Greene (Gettysburg College)

An Investigation of Exact h-Spanning Sets of Cyclic Groups

Abstract: Let $n \in \mathbb{N}$, and consider the Abelian group $G = \mathbb{N}_n$. For a subset $\{a_1, a_2\} \subseteq G$, we define the h-fold signed sumset of $\{a_1, a_2\}$ to be

$$h_{\pm}\{a_1, a_2\} = \{\lambda_1 a_1 + \lambda_2 a_2 \mid |\lambda_1| + |\lambda_2| = h\}$$

We say that $\{a_1, a_2\}$ h-spans G if $h_{\pm}\{a_1, a_2\} = G$. We are interested in finding groups G and positive integers h for which $\{a_1, a_2\}$ h-spans G. We'll present a result for when G is a group of odd order and h is even.

3:05 pm, Christopher Heitmann (Temple University)

New Examples of Fixed Point Free and Contractive Maps

In my talk, I first prove a general theorem that once a map is fixed-point-free (fpf) and contractive on an appropriate domain, previously established series technique will always produce a fpf and contractive map. I also use a known fixed point free and non-expansive map TΔ, inspired by Alspach's famous example, to construct a new fixed-point-free and contractive map.