Dirichlet's Theorem and the Rise of Analytic Number Theory

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Abstract: In 1837, Peter G. L. Dirichlet proved the following theorem: If a and d are relatively prime integers, then the arithmetic progression a, a+d, a+2d, ... contains infinitely many prime numbers. His proof ushered in a revolution in number theory because it relied in a critical way on complex analysis. The use of analytic methods to solve problems in number theory was a tremendous innovation at the time. We shall consider some of the details of Dirichlet's proof, focusing on understanding why there is a deep connection between these seemingly unrelated branches of mathematics.

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