MMPC CODE NUMBER

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THIRTY-EIGHTH ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by
The Michigan Section of the Mathematical Association of America

Part II

Wednesday, December 7, 1994

INSTRUCTIONS

(to be read aloud to the students by the supervisor or proctor)

1. Carefully record your six-digit MMPC code number in the upper right-hand corner of this page. This is the only way to identify you with this test booklet. **PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.**

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.

3. You are not expected to solve all the problems completely. Look over all the problems and work first on those that interest you the most.

4. Each problem is on a separate page. If possible, you should show all of your work on that page. If there is not enough room, you may continue your solution on the inside back cover (page 7) or on additional paper inserted into the examination booklet. Be certain to **check the appropriate box** to report where your continuation occurs. On the continuation page, clearly write the **problem number**. If you use additional paper for your answer, **check the appropriate box** and write your **identification number** and the **problem number** in the upper right-hand corner of each additional sheet.

5. If you are unable to solve a particular problem, partial credit will be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate an idea of how to solve the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. The competition rules prohibit your asking questions of anyone during the examination. The use of notes, reference materials, computational aids, or any other aids is likewise prohibited. When the supervisor announces that the 100 minutes are over, please cease work immediately and insert all significant extra paper into the test booklet. It is not necessary to return scratch paper on which routine numerical calculations have been made.

8. You may now open the test booklet.

1. [10 points]
Al usually arrives at the train station on the commuter train at 6:00, where his wife Jane meets him and drives him home. Today Al caught the early train and arrived at 5:00. Rather than waiting for Jane, he decided to jog along the route he knew Jane would take and hail her when he saw her. As a result, Al and Jane arrived home 12 minutes earlier than usual. If Al was jogging at a constant speed of 5 miles per hour, and Jane always drives at the constant speed that would put her at the station at 6:00, what was her speed, in miles per hour?

Check the appropriate box if your solution is continued on inside back cover (page 7) ☐

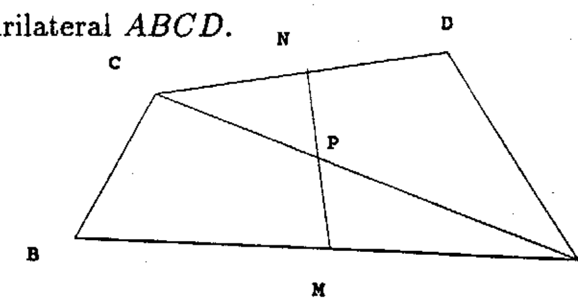
on inserted additional paper ☐

2. In the figure, points M and N are the respective midpoints of the sides AB and CD of quadrilateral $ABCD$. Diagonal AC meets segment MN at P, which is the midpoint of MN, and AP is twice as long as PC. The area of triangle ABC is 6 square feet.

[3 points] (a) Find, with proof, the area of triangle AMP.

[3 points] (b) Find, with proof, the area of triangle CNP.

[4 points] (c) Find, with proof, the area of quadrilateral $ABCD$.



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3.

[4 points] (a) Show that there is a triangle whose angles have measure $\tan^{-1} 1$, $\tan^{-1} 2$ and $\tan^{-1} 3$.

[6 points] (b) Find all values of k for which there is a triangle whose angles have measure $\tan^{-1}(\frac{1}{2})$, $\tan^{-1}(\frac{1}{2} + k)$, and $\tan^{-1}(\frac{1}{2} + 2k)$.

Check the appropriate box if your solution is continued on inside back cover (page 7) ☐

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4.

[3 points] (a) Find 19 consecutive integers whose sum is as close to 1000 as possible.

[7 points] (b) Find the longest possible sequence of consecutive odd integers whose sum is exactly 1000, and prove that your sequence is the longest.

Check the appropriate box if your solution is continued on inside back cover (page 7) ☐

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5. Let AB and CD be chords of a circle which meet at a point X inside the circle.

[3 points] (a) Suppose that $\dfrac{|AX|}{|BX|} = \dfrac{|CX|}{|DX|}$. Prove that $|AB| = |CD|$.

[7 points] (b) Suppose that $\dfrac{|AX|}{|BX|} > \dfrac{|CX|}{|DX|} > 1$. Prove that $|AB| > |CD|$.

($|PQ|$ means the length of the segment PQ.)

Check the appropriate box if your solution is continued on inside back cover (page 7) ☐

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The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America

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ACKNOWLEDGEMENTS

The following individuals, corporations and professional organizations have contributed generously to this competition:

Addison-Wesley Publishing Co.
Ford Motor Company
Jerome J. Kohel
John Wiley & Sons
Kuhlman Corporation
Grand Valley State University

Matilda R. Wilson Fund
Monroe Auto Equipment
Michigan Council of Teachers of Mathematics
The Upjohn Company
Wadsworth, Inc.

The Michigan Association of Secondary School Principals has placed this competition on the Approved List of Michigan Contests and Activities.