1. Solve for n in $4^n + 4^n + 4^n + 4^n = 2^{2010}$.

   A: 1005 B: 2010 C: 1004 D: 1003 E: none of the others

2. If $\sin\alpha \cos\beta = 1$, then $\cos\alpha \sin\beta$ is equal to

   A: 1 B: 0 C: -1 D: $\frac{1}{2}$ E: none of the others

3. Two dice are rolled. Find the probability that the sum is a prime number.

   A: $\frac{5}{12}$ B: $\frac{1}{2}$ C: $\frac{7}{12}$ D: $\frac{1}{3}$ E: none of the others

4. Let $x_0 = \sqrt{3}$, and let $x_n = x_{n-1}^2 - 2$ for $n = 1, 2, 3, \ldots$, what is x_{100}?

   A: 2 B: 1 C: $\sqrt{3}$ D: -1 E: none of the others

5. The diameter of the typical American dinner plate increased by $33\frac{1}{3}\%$ between 1960 and 2010. In the same period, the area of the typical American dinner plate increased by

   A: $33\frac{7}{9}\,\%$ B: $66\frac{2}{3}\,\%$ C: $44\frac{4}{9}\,\%$ D: $77\frac{1}{3}\,\%$ E: none of the others

6. The height of an equilateral triangle is 1. Find its perimeter.

   A: $\frac{6}{\sqrt{3}}$ B: $\frac{3}{\sqrt{3}}$ C: 3 D: $\frac{\sqrt{3}}{6}$ E: none of the others

7. Let k be the smallest positive integer such that $k \cdot 11!$ is a perfect square. Find the sum of the digits of k.

   A: 15 B: 14 C: 16 D: 17 E: none of the others

8. The square below is a *magic square*. All rows, columns, and main diagonals add up to $3a$. What is the value of the center square?

	$a+5$	$a-1$
		$a-3$

   A: $a-2$ B: $a-1$ C: a D: $a+1$ E: none of the others

9. Find the 20102010th place after the decimal point in the decimal representation of $\frac{1}{9}+\frac{2}{99}+\frac{3}{999}$.

   A: 3 B: 1 C: 6 D: 5 E: none of the others

10. How many 3 digit positive integers are there whose digits are distinct?

    A: 648 B: 504 C: 720 D: 560 E : none of the others

11. Eddie, Hugh, and Jennifer have summer jobs painting dorm rooms. If Eddie and Hugh work together, they can paint 5 rooms in 12 hours. If Eddie and Jennifer work together, they can paint 2 rooms in 4 hours. If Hugh and Jennifer work together, they can paint 7 rooms in 12 hours.

If all three of them work together, how many rooms can they paint in 8 hours?

A: 4 B: 6 C: 10 D: 14 E: none of the others

12. Suppose that a_0 and a_1 are real numbers, not both 0, and for $n \geq 2$, $a_n = |a_{n-1}| - a_{n-2}$. Then the sequence a_n is always periodic with the same period. What is the least period?

A: 6 B: 7 C: 12 D: 9 E: none of the others

13. Suppose $y > 1$ and $y^2 + \frac{1}{y^2} = 2010$. Find $y - \frac{1}{y}$.

A: $\sqrt{2009}$ B: $\sqrt{2008}$ C: $\sqrt{2010}$ D: $\sqrt{2007}$ E: none of the others

14. The Celtics and Lakers are playing a series of basketball games. The first team to win four games wins the series. In the first three games, the Celtics had two wins and the Lakers had one win. Assume that the teams are evenly matched, so that the probability that the Celtics will win a given game is $\frac{1}{2}$. What is the probability that the Celtics will win the series?

A: $\frac{11}{16}$ B: $\frac{5}{8}$ C: $\frac{3}{4}$ D: $\frac{67}{128}$ E: none of the others

15. A triangle has vertices A, B, and C, and the respective opposite sides have lengths a, b, and c. If $a = 1, b = 3$, and $C = 60°$, then $\sin^2 B$ is equal to

A: $\frac{27}{28}$ B: $\frac{3}{28}$ C: $\frac{1}{9}$ D: $\frac{1}{3}$ E: none of the others

16. A fair die is tossed 7 times. Suppose that on tosses 2 through 7, the value seen is different from the value seen on the preceding toss. Let S denote the sum of the 7 values seen, the probability that S to be even is

A: $\frac{19}{36}$ B: $\frac{17}{36}$ C: $\frac{1}{2}$ D: $\frac{81}{128}$ E: none of the others

17. Consider the quadratic polynomial $x^2 - 21\,x + 101$, where the coefficients are written in base three. What is the largest real root of this polynomial written in base three?

A: 2 B: 12 C: $\frac{21 + \sqrt{37}}{2}$ D: this polynomial has no real roots E: none of the others

18. If $f(x) = \frac{x}{\sqrt{1 + k\,x^2}}$, where k is a positive integer and $f^{(n)}(x)$ is defined below.

$$\begin{aligned} f^{(2)}(x) &= f(f(x)) \\ f^{(3)}(x) &= f^{(2)}(f(x)) = f(f(f(x))) \\ &\vdots \\ f^{(n)}(x) &= f^{(n-1)}(f(x)). \end{aligned}$$

$f^{(100)}(1)$ is equal to

A: $\frac{1}{\sqrt{k+1}}$ B: $\frac{1}{\sqrt{100\,k+1}}$ C: $\left(\frac{1}{\sqrt{1+k}}\right)^{100}$ D: $\left(\frac{1}{\sqrt{1+k}}\right)^{99}$ E: none of the others

19. Two race cars, cleverly named 'Abby' and 'Betty' are going to race two laps around a $\frac{1}{2}$ mile oval. Abby drives a steady 60 mph. Betty goes 30 mph through her first lap. How fast must she drive during her lap in order to at least tie Abby?

A: 1200 mph B: 90 mph C: 120 mph D: 150 mph E: none of the others

20. Find the sum of the solutions to $2^x + \dfrac{32}{2^x} = 12$.

A: 3 B: 2 C: 4 D: 5 E: none of the others

21. A number is selected at random from the set of all six-digit numbers in which the sum of the digits is equal to 46. What is the probability that this number is divisible by 9?

A: 1 B: $\dfrac{1}{9}$ C: $\dfrac{1}{2}$ D: 0 E: none of the others

22. Consider the following array of letters

```
      S
    Q   Q
  U   U   U
A   A   A   A
  R   R   R
    E   E
      S
```

One can spell the word SQUARES by starting at S and then constructing a path through the array. One is allowed to move only to the nearest letter(s) in each row. How many such paths are there?

A: 15 B: 18 C: 20 D: 25 E: none of the others

23. When $(5x^6 + 10x^5 + 2x^4 + 19x^3 + 2x^2 + 1)^{10000}$ is multiplied out, what is the coefficient of x^4?

A: $2 \cdot 10^4$ B: $2 \cdot 10^8$ C: $2 \cdot 10^8 + 2 \cdot 10^4$ D: 0 E: none of the others

24. Some clocks use a seven-bar digital display where the digits 0–9 are given in the following form:

0 123456789

When the time is 11 : 30, the display reads 1130, and there are 15 illuminated bars. When the time is 1 : 30, the display reads 130, and there are 13 illuminated bars. What is the maximum of illuminated bars that will show on this clock? Assume that the display is in 12-hour mode.

A: 21 B: 20 C: 19 D: 15 E: none of the others

25. Let A, B, C be events in a probability space. Suppose that $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$, $P(A \cap B) = 0.15$, $P(A \cap C) = 0.1$, and $P(B \cap C) = 0.06$. What is the smallest possible value of $P(A^c \cap B^c \cap C^c)$?

A: 0.31 B: 0.25 C: 0.26 D: 0 E: none of the others

26. Suppose x and y are real numbers that satisfy $|x| + y = 3$ and $x + |y| = 7$ simultaneously. Find the product of x and y.

A: -12 B: -8 C: -10 D: -16 E: none of the others

27. What is the largest number of points that can be placed in a unit square, if it is required that the distance between any two points is at least $\frac{1}{\sqrt{2}}$?

A: 8 B: 4 C: 5 D: 6 E: none of the others

28. Let p and q be the length and width of a rectangle where p and q are primes. Suppose the perimeter is 36, find the largest possible area.

A: 86 B: 65 C: 88 D: 77 E: none of the others

29. A triangle inscribed in a unit square has area at most of

A: $\frac{1}{2}$ B: $\frac{1}{3}$ C: $\frac{2}{3}$ D: $\frac{1}{4}$ E: none of the others

30. A 10 foot long ladder is initially standing vertically along a wall. Gradually, the foot of the ladder slides away from the wall along a horizontal floor, until the ladder is horizontal. The midpoint of the ladder describes what kind of curve?

A: a quarter circle of radius 5 B: a parabolic arc C: a circle of radius 5 D: a straight line segment E: none of the others

31. Two cyclists are riding through a 1 mile long railroad tunnel. When they are $\frac{2}{3}$ of the way through, they spot the light of an oncoming train. One of them speeds forward, and the other tries to escape by turning back. The one who went forward escaped from the tunnel just at the moment the train entered, and the other had an equally close call at the other end. The train was traveling at 60 mph. Assuming that the cyclists pedalled at the same speed, how fast did they go?

A: 30 mph B: 20 mph C: 15 mph D: 30 mph E: none of the others

32. Let $\mathcal{C}$ be the circle that circumscribes the triangle that is formed by the lines $y - 2x = 0$, $x + y = 1$, and the x-axis. The area of $\mathcal{C}$ is equal to

A: $\frac{5\pi}{36}$ B: $\frac{2\pi}{9}$ C: π D: $\frac{5\pi}{18}$ E: none of the others

33. Let x and y be real numbers which satisfy $|x-1| + |y+1| < 1$, the range for the expression

$$\frac{x-1}{y-2}$$

is

A: $\left(-\frac{1}{3},\ \frac{1}{3}\right)$ B: $\left(-\frac{1}{2},\ \frac{1}{2}\right)$ C: $\left(-\frac{1}{2},\ \frac{1}{3}\right)$ D: $\left(-\frac{1}{3},\ \frac{1}{2}\right)$ E: none of the others

34. Let $f(u) = 3\,u^2 - u + 4$ and $g(x)$ be a polynomial with integer coefficients. If

$$f\,(g(x)) = 3x^4 + 18x^3 + 50\,x^2 + 69\,x + 48,$$

then the sum of the coefficients of $g(x)$ is equal to

A: 10 B: 9 C: 8 D: 11 E: none of the others

35. The function $h(x) = \dfrac{x}{1-2^x} - \dfrac{x}{2}$ is

A: neither an even nor odd function B: an odd function C: an even function D: both an even and odd function E: none of the others

36. Let x be a real number which satisfies the condition

$$\log_3 x = 1 + \frac{1}{2}\sin\theta,$$

where $\theta,\ 0 \le \theta < 2\,\pi$, is a real constant. Then

$$\left|x - \frac{3}{2}\right| + \left|\frac{11}{2} - x\right|$$

is equal to

A: 7 B: $2x - 7$ C: $7 - 2x$ D: 4 E: none of the others

37. The numbers a, b, c, d, e, f represent $1, 2, 3, 4, 5, 6$, though not necessarily in that order. In other words, $\{a, b, c, d, e, f\} = \{1, 2, 3, 4, 5, 6\}$. If $abcd = 60$, $e + f = 8$, and $e < f$, then

A: $e = 3$ B: $e = 2$ C: $e = 4$ D: the given information is not enough to determine e E: none of the others

38. Let $0 < x < 1$, a, b be positive constants. The minimum value of $\dfrac{a^2}{x} + \dfrac{b^2}{1-x}$ is

A: $(a+b)^2$ B: $a^2 + b^2$ C: $4ab$ D: $(a-b)^2$ E: none of the others

39. The function $f(x)$ is defined below.

$$f(x) = \begin{cases} \left(\dfrac{1}{2}\right)^x, & x \geq 4; \\ f(x+1), & x < 4. \end{cases}$$

Then $f(2 + \log_2(\dfrac{3}{2}))$ is equal to

A: $\dfrac{1}{12}$ B: $\dfrac{1}{48}$ C: $\dfrac{1}{24}$ D: $\dfrac{1}{6}$ E: none of the others

40. A cube with side length 2010 cm is painted on all six sides. The cube is cut into unit cubes. How many cubes have exactly two painted sides?

A: 25000 B: 24090 C: 20100 D: 24096 E: none of the others