Matroids, Positroids, and Beyond!

Anastasia Chavez
(she/her/hers)

Saint Mary's College of California

MAA Metro NYC Virtual Section Meeting
28-April-2024

My True Goal: "With a hammer, everything is a nail"

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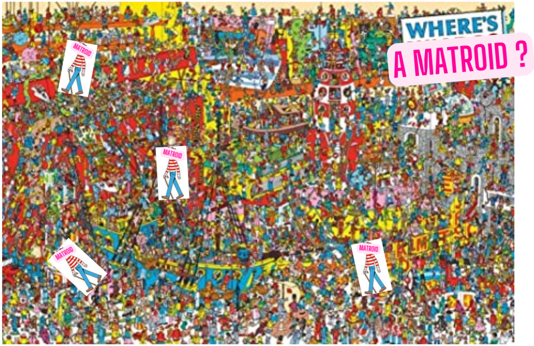

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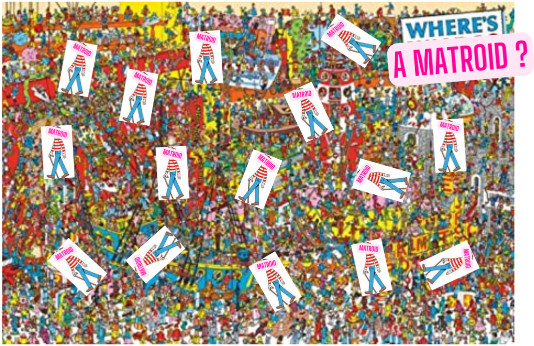

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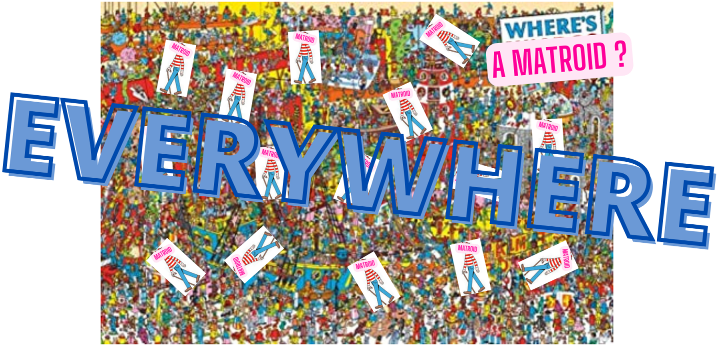
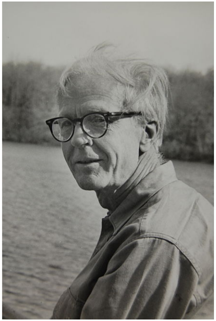


Everywhere? YES!

Gian-Carlo Rota (1932 - 1999)

“It is as if one were to condense all trends of present day mathematics onto a single finite structure, a feat that anyone would a priori deem impossible, were it not for the mere fact that matroids exist.” (circa 1986)

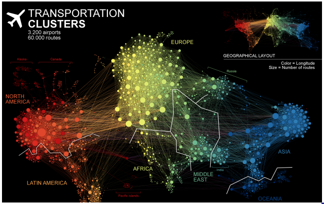
Matroids - the early years

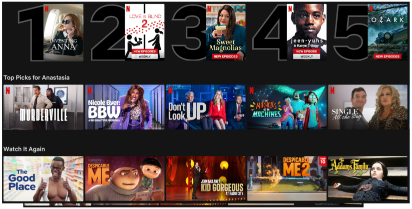


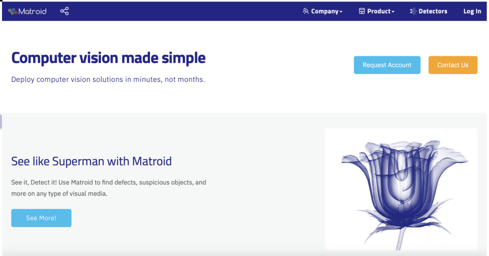
Hassler Whitney
(1907 - 1989)

Takeo Nakasawa
(1913 - 1946)

Really, there are matroids here?







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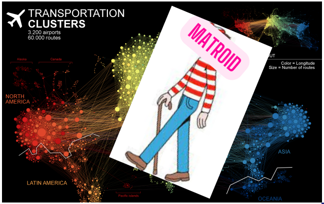
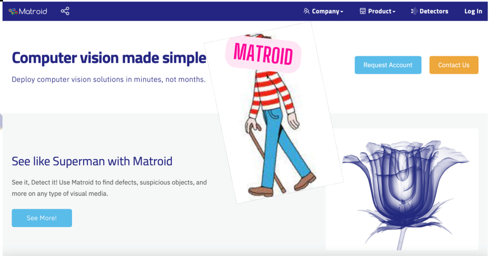
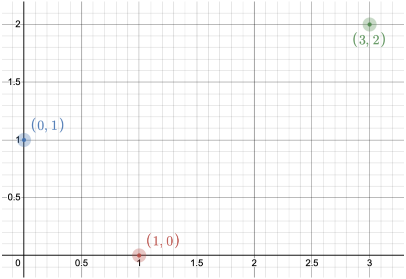




Matroids generalize dependence ... how is that?

Linear Spaces



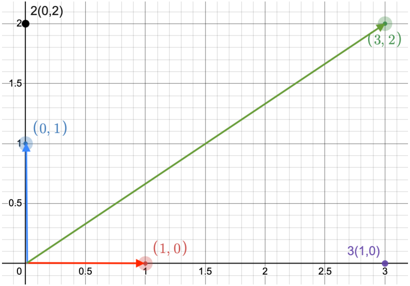
$$a = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \text{and } c = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Graphs

Matroids generalize dependence ... how is that?

Linear Spaces

Graphs



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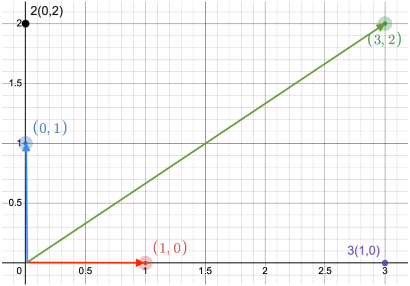

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and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *are linearly independent*

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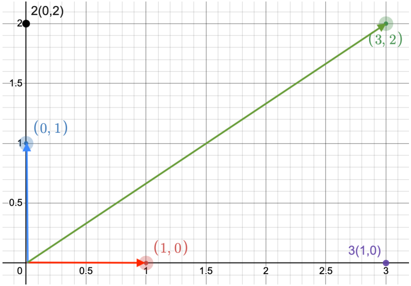

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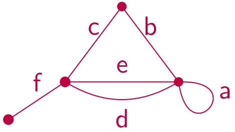

Graph = (Edges, Vertices)

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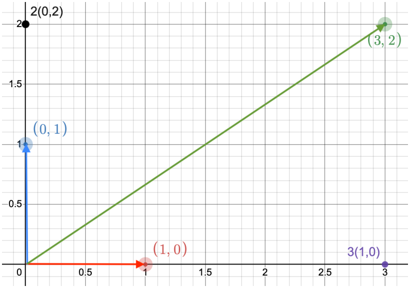

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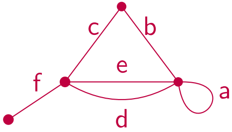

Graph = (Edges, Vertices)

Dependent = Closed paths (CP)

Min. Dependent = Cycles

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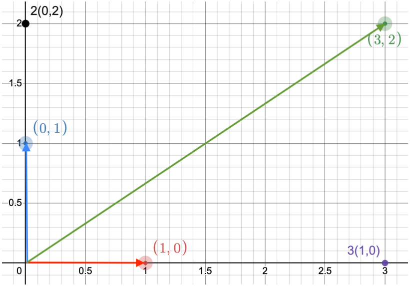

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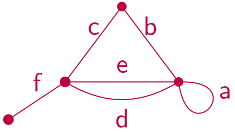

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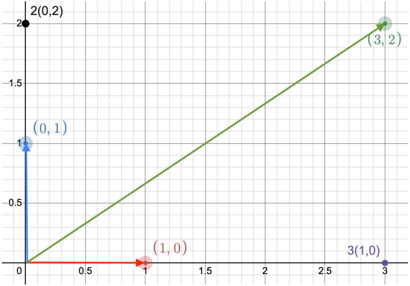

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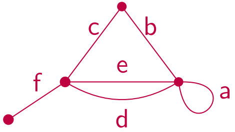

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= {spanning trees}

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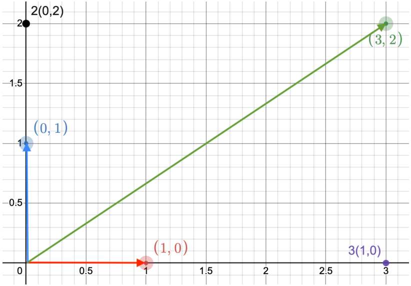

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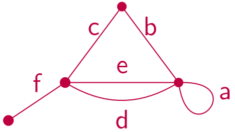

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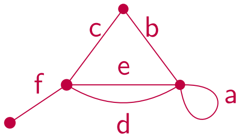
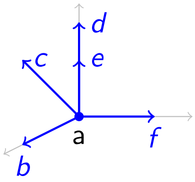
$= \{fcb, fce, fcd, fbd, fbe\}$

The search for (in)dependence: Which subsets are independent? dependent?





a	b	c	d	e	f
0	1	1	0	0	0
0	0	1	2	1	0
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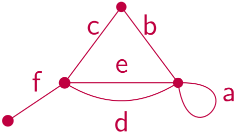

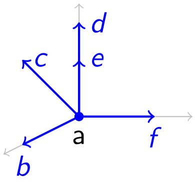

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trees:
bcf
bdf
bef
cdf
cef

bases:
bcf
bdf
bef
cdf
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max. lin. ind. cols:
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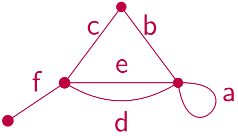

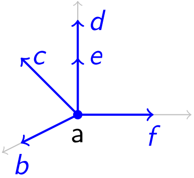

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trees:	bases:	max. lin. ind. cols:
bcf	*bcf*	*bcf*
bdf	*bdf*	*bdf*
bef	*bef*	*bef*
cdf	*cdf*	*cdf*
cef	*cef*	*cef*

Let $A = bdf, B = cef$. Then $A - d + e = bef$ is also a basis.

Matroids: basis description

Theorem (Nakasawa, Whitney 1935)

A matroid is a pair $M = ([n], \mathcal{B})$ *such that*

- $\mathcal{B} \subset 2^{[n]}, \mathcal{B} \neq \emptyset$,
- *For all* $A, B \in \mathcal{B}$, $a \in A \backslash B \Rightarrow b \in B \backslash A$ *s.t.* $A - a + b \in \mathcal{B}$.

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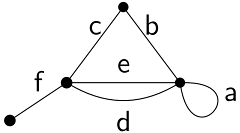

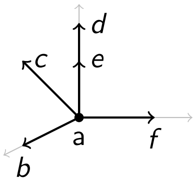

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0	1	1	0	0	0
0	0	1	2	1	0
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trees: bases: max. lin. ind. cols:

$\{bcf, bdf, bef, cdf, cef\}$

Matroids: independent set description

Definition

A matroid over $[n]$ has independent sets $\mathcal{I} \subset [n]$ such that

(I1) $\emptyset \in \mathcal{I}$

(I2) If $J \subset I$ and $I \in \mathcal{I}$, then $J \in \mathcal{I}$

(I3) If $I, J \in \mathcal{I}$ and $|I| > |J|$, then there exists $i \in I - J$ such that $J \cup \{i\} \in \mathcal{I}$.

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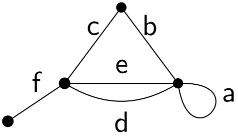

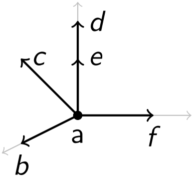

a	b	c	d	e	f
0	1	1	0	0	0
0	0	1	2	1	0
0	0	0	0	0	1

cycle-free edges: independent sets: indep. column sets:

$$\{\emptyset, b, c, d, e, f, bc, be, bf, cd, ce, cf, df, ef, bcf, bdf, bef, cdf, cef\}$$

Matroids: a circuit description

Theorem

A matroid over $[n]$ can be characterized by its set of circuits $\mathcal{C} \subset [n]$, i.e. minimally dependent sets.

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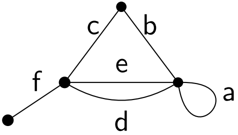
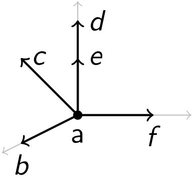




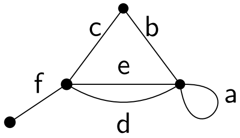
a	b	c	d	e	f
0	1	1	0	0	0
0	0	1	2	1	0
0	0	0	0	0	1

cycles: circuits: min. lin. dep. cols:

$$\{a, ed, bcd, bce\}$$

What matroids have we seen so far?

1. Graphical Matroids
   $E = \{\text{edges of connected graph } G\}$ and $\mathcal{B} = \text{trees of } G$.



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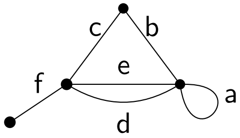
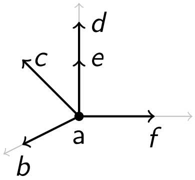
   $E = \{$edges of connected graph $G\}$ and $\mathcal{B} =$ trees of G.
2. Linear Matroids
   $E = \{$set of vectors spanning $\mathbb{R}^d\}$ and $\mathcal{B} = \{$bases of $\mathbb{R}^d$ in $E\}$.





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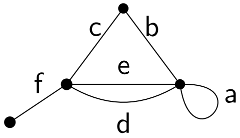
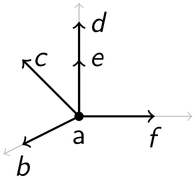
3. Representable Matroids
   $E = \{\text{labeled columns of a matrix } A \text{ over a field } F\}$ and
   $\mathcal{B} = \{\text{bases spanning the column space of } A\}$.

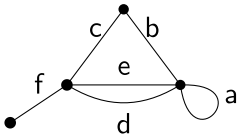




a	b	c	d	e	f
0	1	1	0	0	0
0	0	1	1	2	0
0	0	0	0	0	1

Graphical $\subset$ Representable

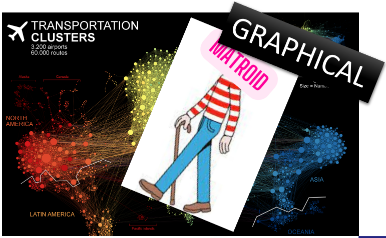
Given a graph G with both vertices and edges labeled, one can describe a representable matroid over F_2 via the vertex-edge matrix!



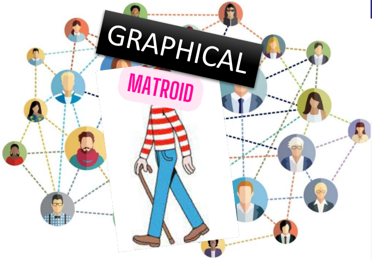
$$A = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 & 1 & 1 \\ 3 & 0 & 1 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}$$

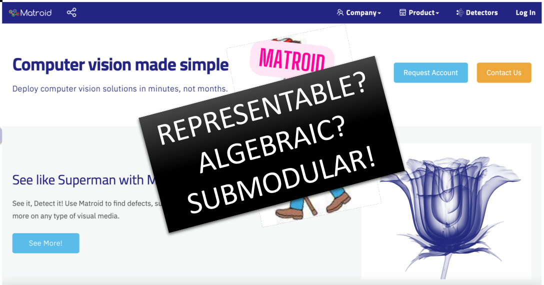
Cycles of G are the sets of minimally dependent spanning columns of A.

Which Means ...


TRANSPORTATION CLUSTERS
3,200 airports
60,000 routes
NORTH AMERICA
LATIN AMERICA
ASIA
OCEANIA
GRAPHICAL
MATROID


Top Picks for Anastasia
Watch It Again
MATROID
REPRESENTABLE?
ALGEBRAIC


GRAPHICAL
MATROID


Matroid
Company
Product
Detectors
Log In
Computer vision made simple
Deploy computer vision solutions in minutes, not months.
Request Account
Contact Us
See like Superman with M
See it, Detect it! Use Matroid to find defects, su
more on any type of visual media.
See More!
MATROID
REPRESENTABLE?
ALGEBRAIC?
SUBMODULAR!

Most matroids are NOT this special

[Nelson 2018] Almost all matroids are non-representable. (as $|E| \to \infty$)

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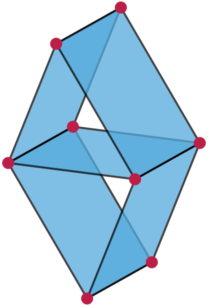

Vámos matroid

Facts:

- A non-representable rank 4 matroid (over any field) of 8 elements.
- All subsets of size 3 or less are independent.
- All subsets of size 4, except for those shown in the diagram below, are independent.
- A classic matroid to know!

All you need is ... linear algebra!

- A great entry point for undergraduate students with linear algebra experience!
- Let Δ_{ij} denote the determinant of the submatrix of columns i and j.

Example

Does $M = ([4], \{12, 23, 34, 14\})$ define a representable matroid?

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Find entries of A such that these pairs are the only bases of the column space,

$$A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \end{pmatrix}.$$

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$$A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \end{pmatrix}.$$

That is, find values for A such that Δ_{12}, Δ_{23}, Δ_{34}, Δ_{14} are all nonzero

AND

that $\Delta_{13} = \Delta_{24} = 0$.

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What if we also wanted that Δ_{12}, Δ_{23}, Δ_{34}, Δ_{14} all nonzero and positive?

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By now you're asking: What about this extra condition?

Positroids

Definition (Postnikov 2005)

A representable (over $\mathbb{R}$) matroid M on $[n]$ of rank k is a *positroid* if there exists a $k \times n$ matrix A such that all maximal minors are nonnegative and A represents M. *Said differently: all* $\Delta_B \geq 0$ *for* $B \in \binom{[n]}{k}$.

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Example

The matroid $M = ([5], \mathcal{B})$ where $\mathcal{B} = \{13, 14, 15, 34, 35, 45\}$ is a positroid:

$$A_M = \begin{pmatrix} 1 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}.$$

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Which means we now know what we need to check to see why $M = ([4], \{12, 23, 34, 14\})$ does not describe a positroid! EXERCISE

Are positroids graphical?

Definition

The uniform matroid $U_{k,n}$ is the rank k matroid over $[n]$ such that the set of bases is $\mathcal{B} = \binom{[n]}{k}$.

Example

Consider $U_{2,4}$ with $\mathcal{B} = \{12, 13, 14, 23, 24, 34\}$.

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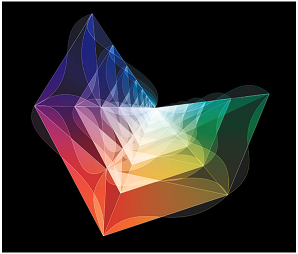
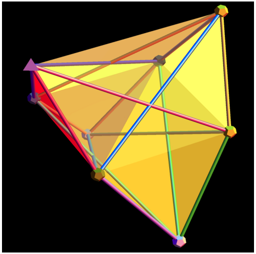
- Positroid? Yes! Check the determinants above!
- Graphical? No! EXERCISE ... what breaks down in the graph?

Positroids and physics





Artists rendition and notional visualization of an Amplituhedron

In 2013, Arkani-Hamed et. al. found a monumental link between particle physics and matroids, in particular positroids, that was described in *Quantum Magazine* in this way:

"Physicists have discovered a jewel-like geometric object that dramatically simplifies calculations of particle interactions and challenges the notion that space and time are fundamental components of reality."
https://www.quantamagazine.org/physicists-discover-geometry-underlying-particle-physics-20130917/

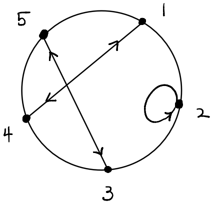
MATROIDS IN SPACE

Combinatorics of positroids [Postnikov 2006]

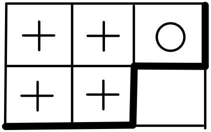
Consider the positroid $M(A)$ with $\mathcal{B} = \{13, 14, 15, 34, 35, 45\}$. Then it can be indexed by the following unique objects:

$$\mathcal{I}_M : (13, 34, 34, 45, 51)$$

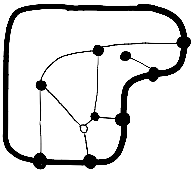
Grassmann Necklace



Decorated Permutation



Le Diagram



Plabic Graph

Positroids in the "wild"

1. **Polytopes:** Combinatorially characterize f-vectors of simplicial polytopes. [C - Yamzon '17]

2. **Posets:** Combinatorially characterize the poset of Unit Intervals as a family of positroids. [C - Gotti '17]

3. **Flag Matroids:** Combinatorially describe a "quotient of positroids" in terms of "decorated permutations". [Benedetti - C - Tamayo '22]

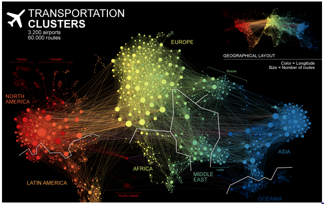

C. Benedetti

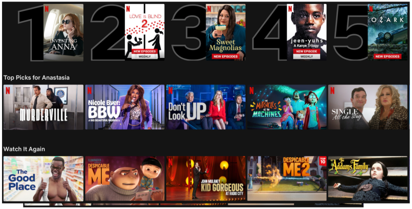

F. Gotti

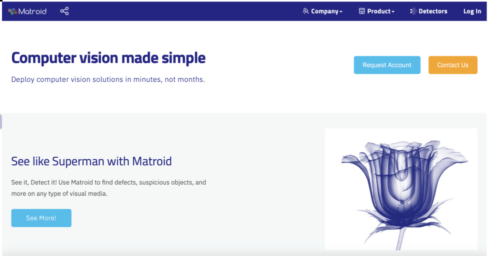

D. Tamayo


N. Yamzon

Oh, the places we've been! And the matroids you'll find!







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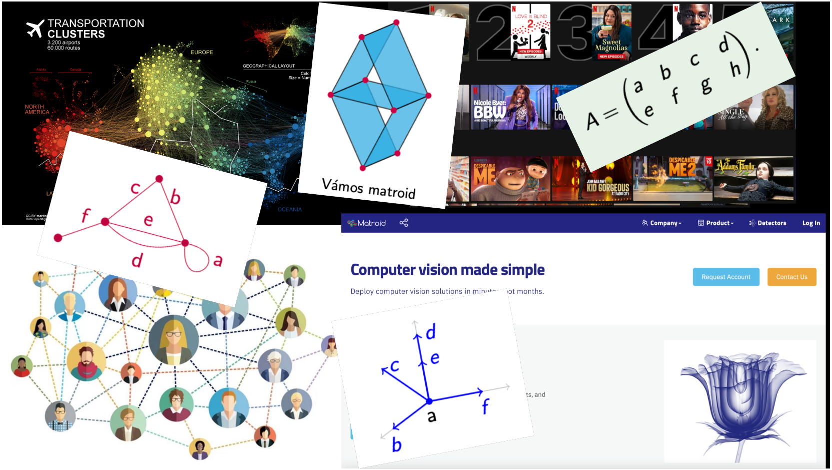

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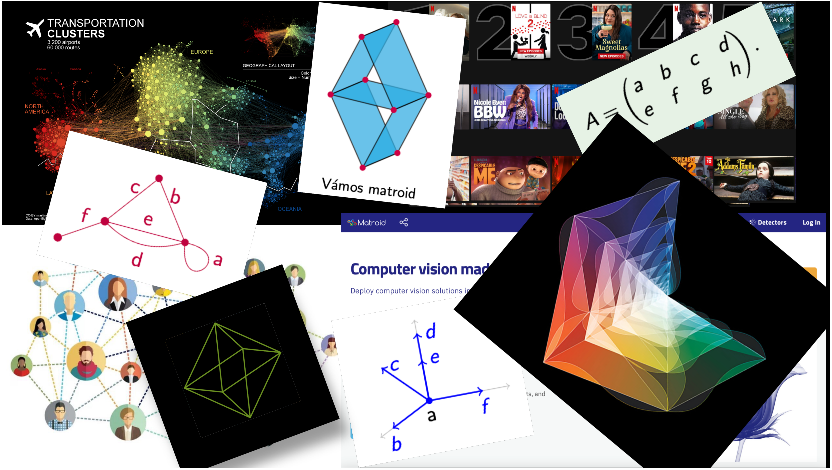

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EXIT
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STATE UNIVERSITY
Berkeley
UNIVERSITY OF CALIFORNIA
UCDAVIS
UNIVERSITY OF CALIFORNIA
MSRI
SAINT MARY'S COLLEGE OF CALIFORNIA

thank you
danke
рахмат
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
dziękuję
bedankt
спасибо
Баярлалаа
sukriya
kop khun krap
go raibh maith agat
mochchakkeram
terima kasih
grazie
arigatô
dakujem
мерси
takk
tapadh leat
감사합니다
ありがとう
ευχαριστώ
xièxie
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dank je
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mersi
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