SOLVING MATHEMATICAL MYSTERIES

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Mystery #1: It's *Magic*

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Pick two integers

Hey, students!

Go to student.desmos.com
and type in:

SZ8 HQV

Mystery #1: It's *Magic*

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Add together your first and second integer

Add together integers in 2 and 3

Add together integers in 3 and 4

⋮

Continue to the bottom

⋮

Add together integers in 8 and 9

Mystery #1: It's *Magic*

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Now add the entire column of 10 integers together and write your answer in the box.

Sum:

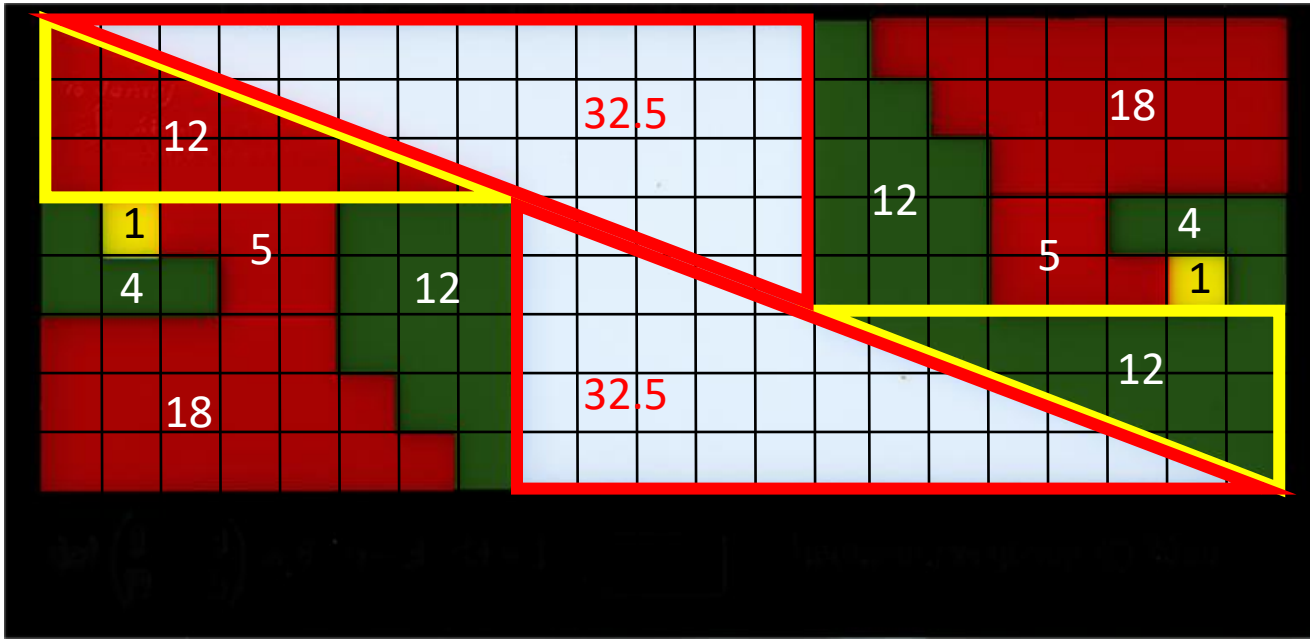
Can you figure out the "trick" before the end of this talk?

Mystery #2: The sum of its parts?

Area = length x width = 21 x 8 = 168

Area = sum of areas for each piece

= 2 (1 + 4 + 5 + 12 + 12 + 18 + 32.5) = 169

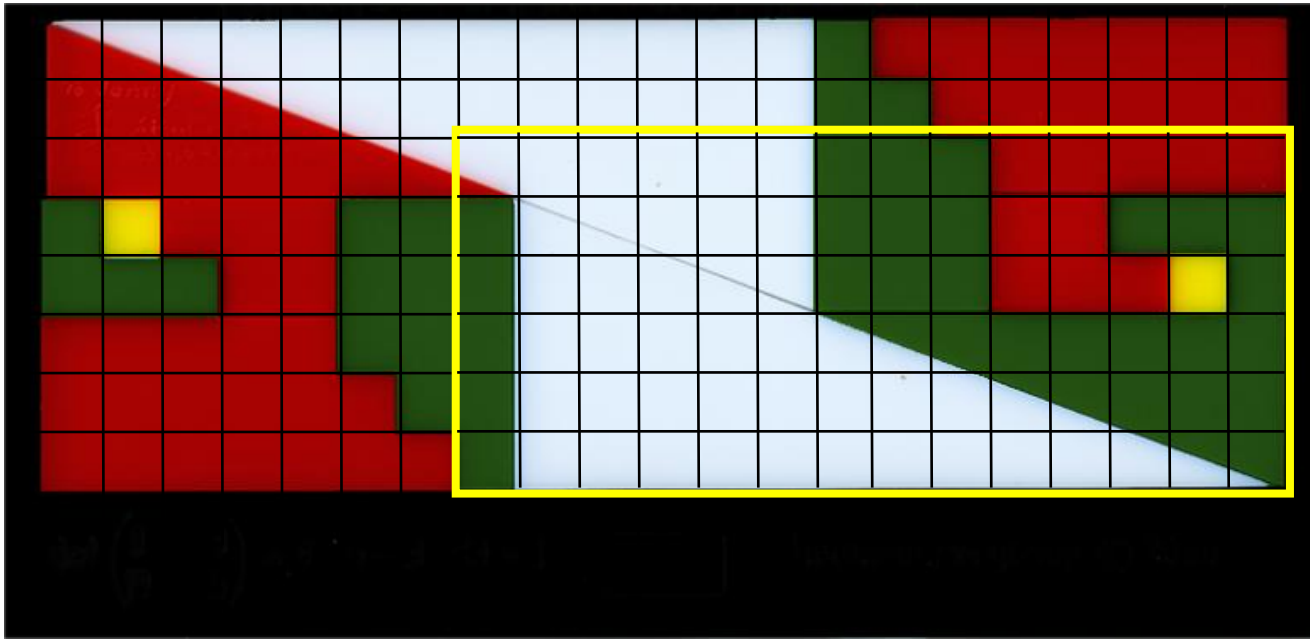


Area of triangle = ½ x base x height

Helaman Ferguson, *Fibonacci Box*

Mystery #2: The sum of its parts?

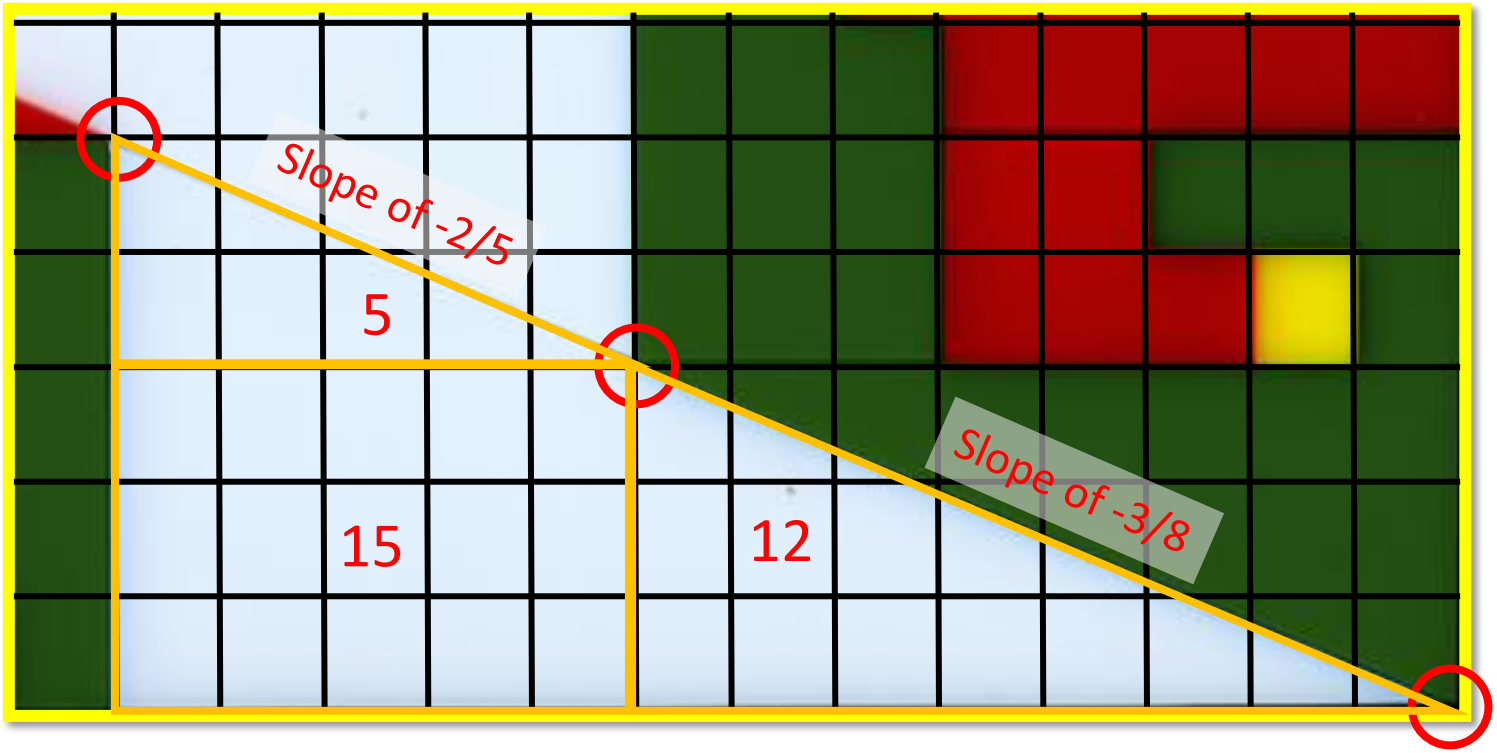
PROBLEM: 168 ≠ 169



Helaman Ferguson, *Fibonacci Box*

Mystery #2: The sum of its parts?

PROBLEM: 168 ≠ 169



Area of large white "triangle" is actually 32 NOT 32.5

Helaman Ferguson, *Fibonacci Box*

Mystery #2: The sum of its parts?

Area = length x width = 21 x 8 = 168

Area = sum of areas for eac

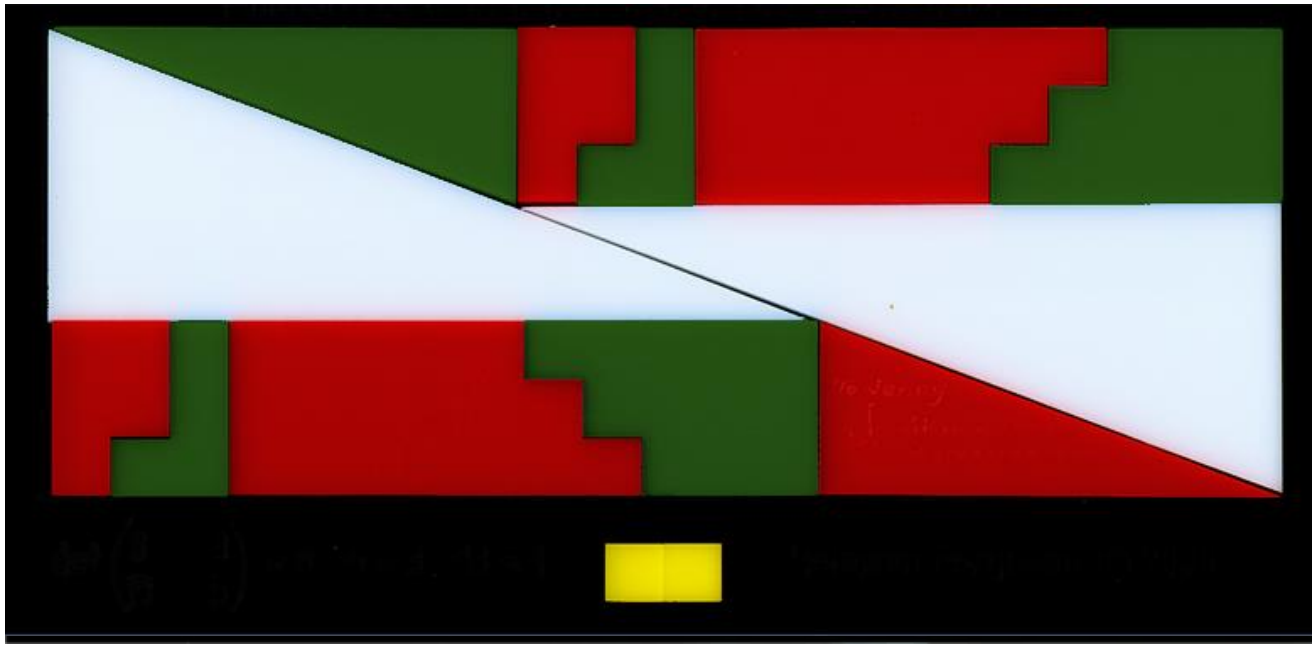
= 2 (1 + 4 + 5 +

Mystery solved

Area of triangle = ½ x base x height

Helaman Ferguson, *Fibonacci Box*

Mystery #2: More to consider



Helaman Ferguson, *Fibonacci Box*

What is “Fibonacci” about the box?

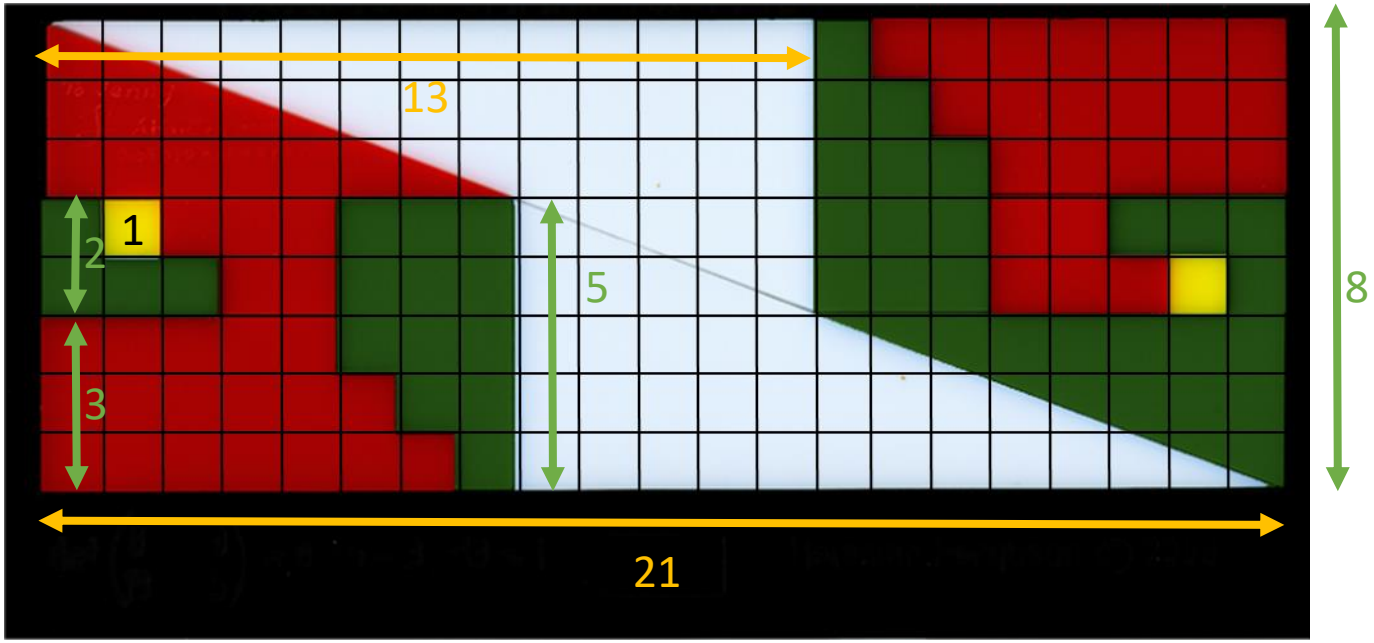
- The **Fibonacci numbers** are a sequence
  0, 1, 1 ,2 ,3 ,5 ,8 ,13 ,21 ,…
  for next integer — add the two previous

- …, 34, 55, 89, 144, 233, 377, 610,…

- Mathematically,
  $F(0) = 0, F(1) = 1$, and for $n \geq 2$
  $$F(n) = F(n-1) + F(n-2)$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

What is “Fibonacci” about the box?



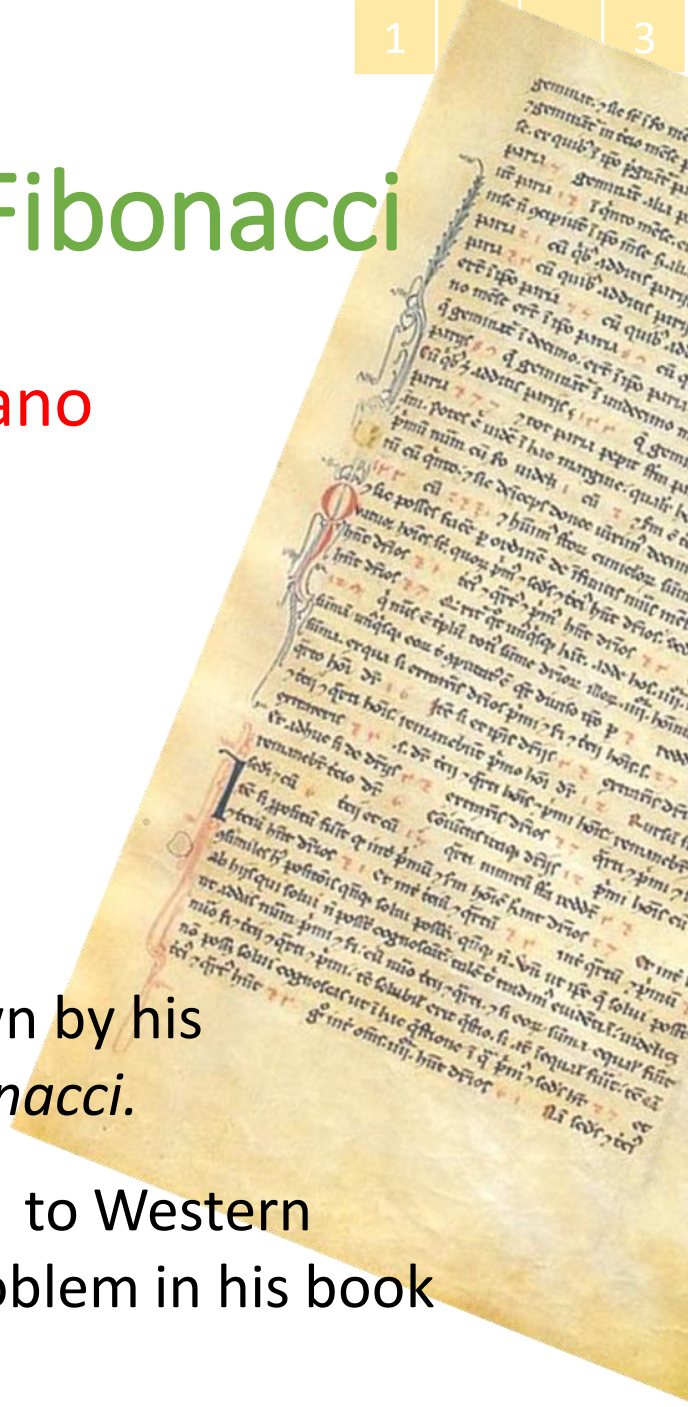
Fibonacci numbers 1, 2, 3, 5, 8, 13, 21 appear as lengths.

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

A Fascination with Fibonacci

The namesake: Leonardo Pisano

12th Century
Born: 1170
Died: 1250



Leonardo Pisano is better known by his nickname Fibonacci — *filius Bonacci.*

Fibonacci introduced sequence to Western European mathematics as a problem in his book *Liber Abaci.*

A Fascination with Fibonacci

Pingala (Indian poet in Sanskrit tradition C 450 BC-200 BC)

Virahanka

Date? : 7th -8th Century AD

Analyzed poetic meter based on long & short vowel sounds. First *authority to give rule* for formation of numbers.

# of beats	Arrangement of sounds	#	F(n)
1	s	1	F(2)
2	ss, l	2	F(3)
3	sss, sl, ls	3	F(4)
4	ssss, ssl, sls, lss, ll	5	F(5)
5	sssss, sssl, ssls, slss, lsss, sll, lsl, lls	8	F(6)

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Why this Fascination?

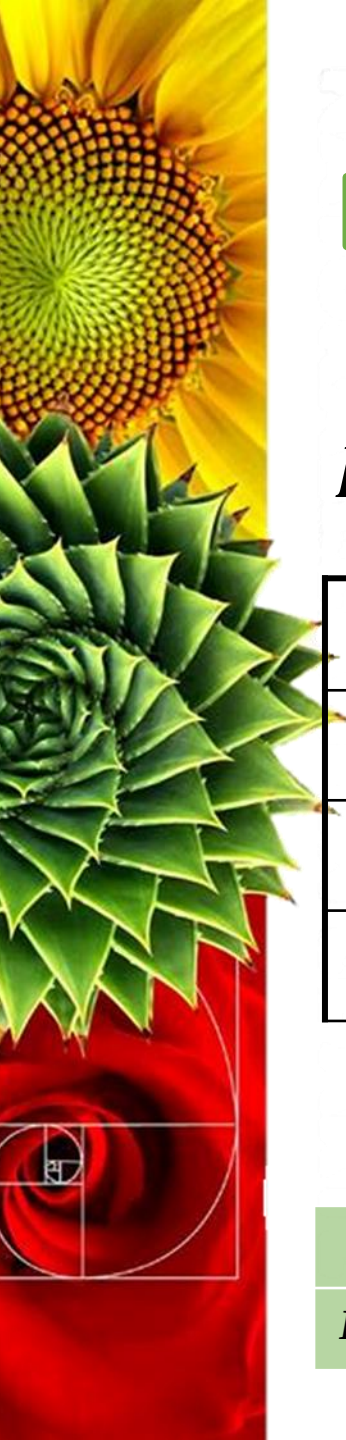
- Astonishing patterns (entire books dedicated to such)

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Experiment: Discover a Pattern

Find the sum of Fibonacci numbers from $F(0)$ to $F(n)$ for $n = 5, 6, 7, 8$.

$0 + 1 + 1 + 2 + 3 + 5 =$	
$0 + 1 + 1 + 2 + 3 + 5 + 8 =$	
$0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 =$	
$0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 =$	



n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Experiment: Discover a Pattern

Find the sum of Fibonacci numbers from $F(0)$ to $F(n)$ for $n = 5, 6, 7, 8$.

$0 + 1 + 1 + 2 + 3 + 5 =$	12
$0 + 1 + 1 + 2 + 3 + 5 + 8 =$	20
$0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 =$	33
$0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 =$	54
$F(0) + F(1) + F(2) + \cdots + F(n) =$	

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Why this Fascination?

- Astonishing patterns (entire books dedicated to such)
- Beautiful proofs

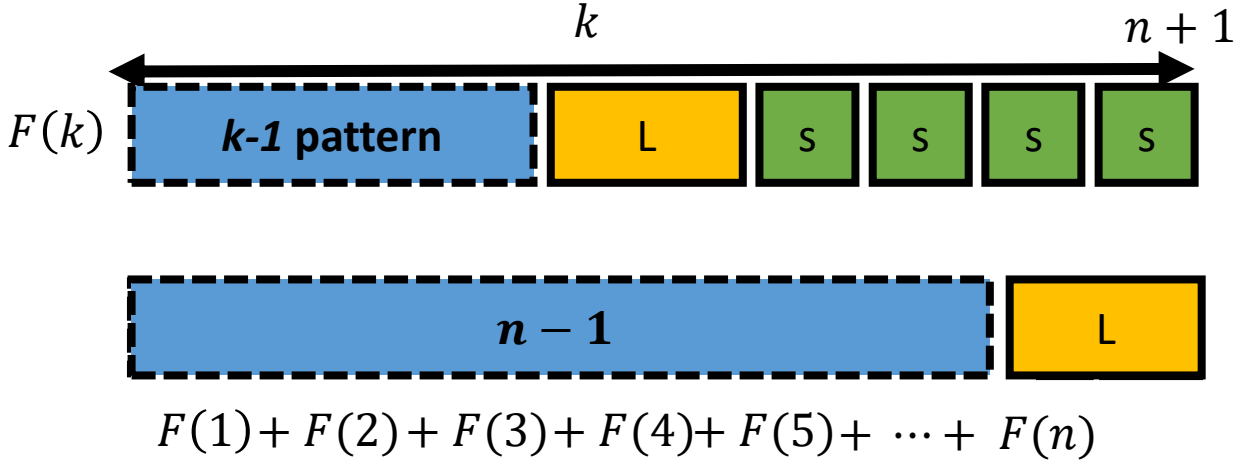
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

$$F(0) + F(1) + \cdots + F(n) = F(n+2) - 1$$

Following Virahanka, the number of patterns $\boldsymbol{n}$ **beats** long is $\boldsymbol{F(n+1)}$.

Question: How many patterns of long and short vowel sounds with $\boldsymbol{n+1}$ **beats** contain at least one long sound?

- Answer 1: $F(n+2) - 1$
- Answer 2: Consider the location of the last long vowel.



$$F(1)+F(2)+F(3)+F(4)+F(5)+\cdots+F(n)$$

Why this Fascination?

- Astonishing patterns (entire books dedicated to such)
- Beautiful proofs
- Frequent occurrences in art and nature
  (on purpose or by coincidence?)

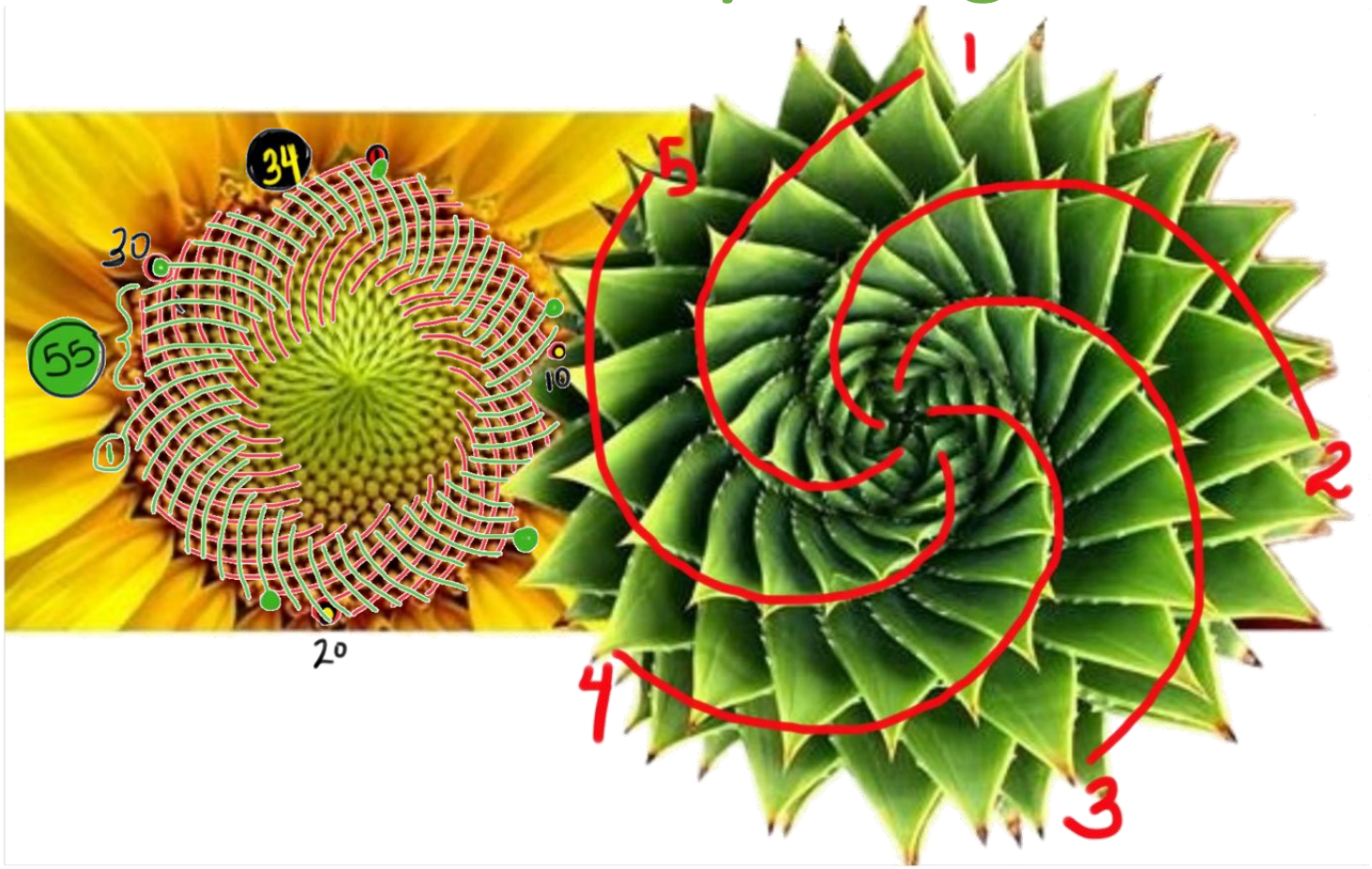
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Coincidence or by design?

- 3 petals — lily, iris
- 5 petals — buttercup, wild rose, larkspur, columbine
- 8 petals — delphiniums
- 13 petals — ragwort, corn marigold, cineraria
- 21 petals — aster, black-eyed susan, chicory
- 34 petals — plantain, pytethrum
- 55, 89 petals — michelmas daisies, the asteraceae family

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Coincidence or by design?



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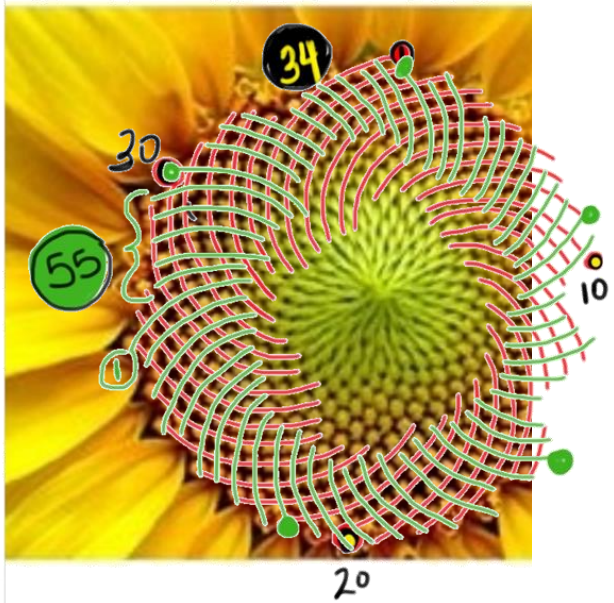

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$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Strong Law of Small Numbers

There aren't enough small numbers to meet the many demands made of them.

Attributed to Martin Gardner. Popularized by Richard Guy

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Purposeful Occurrences in Art and Architecture

Public Art Project
Smokestack in Turku, Finland

Photography Provided by Carrie Gartner

http://columbiabusinesstimes.com/2018/02/15/smokestack-art-coming-loop/

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Purposeful Occurrences in Art and Architecture

Fibonacci Fountain in Bowie, Maryland

Helaman Ferguson (2001)
Photography by Oswaldo Jimenez

Thirteen water cannons are located at Fibonacci number intervals along the x- axis in the $y = 0$ vertical plane.

http://helasculpt.com/2013/02/10/fibonacci-fountain-essential-singularity-ii/

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
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Purposeful Occurrences in Art and Architecture

THE 7 FINGERS

PROJECT FIBONACCI

International artist collaborative.
Teams have **21** days to create a show with themes structured around the first **eight** Fibonacci numbers.

Photography by Andrea Lopez (2007, Mexico city).

http://7fingers.com/shows/creations/projet-fibonacci

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Purposeful Occurrences in Art and Architecture

Uyen Nguyen

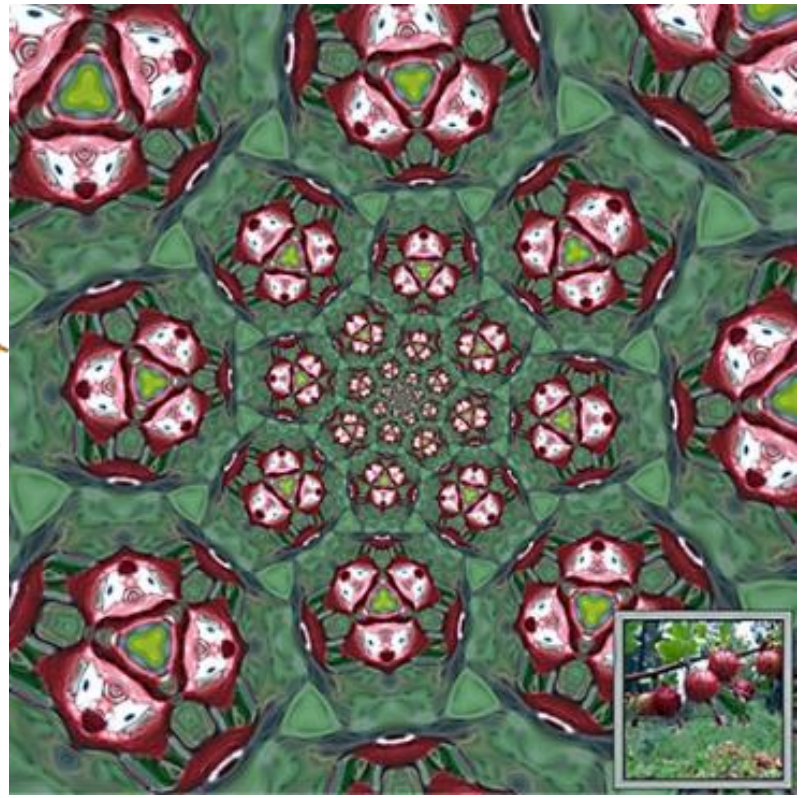
Origami artist and aspiring fashion designer.

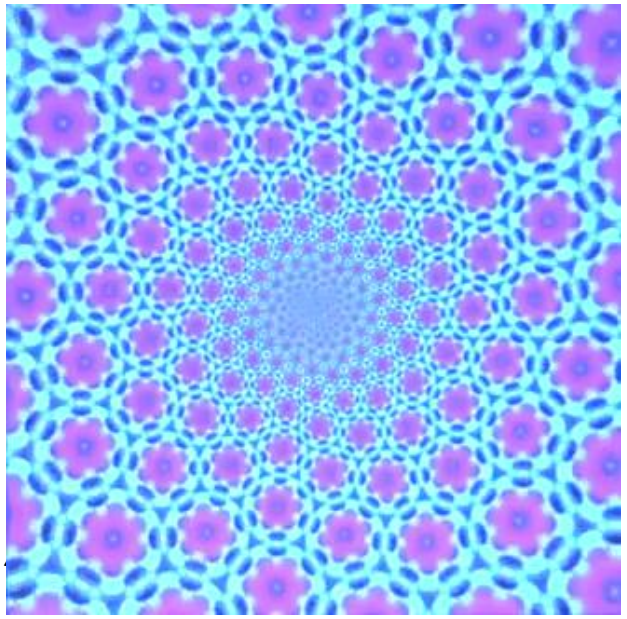
Screenshots from *The Mathematics of Origami* (exhibiting through January 5, 2020) by Museum of Mathematics (MoMath).

https://www.facebook.com/mentalflossmagazine/videos/718436465227467/

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Purposeful Occurrences in Art and Architecture





Frank Farris (2017)

http://www.ams.org/mathimagery/displayimage.php?album=48&pid=753#top_display_media

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Purposeful Occurrences in Art and Architecture

Public Art Project
Sunnyside Piazza Portland, Oregon

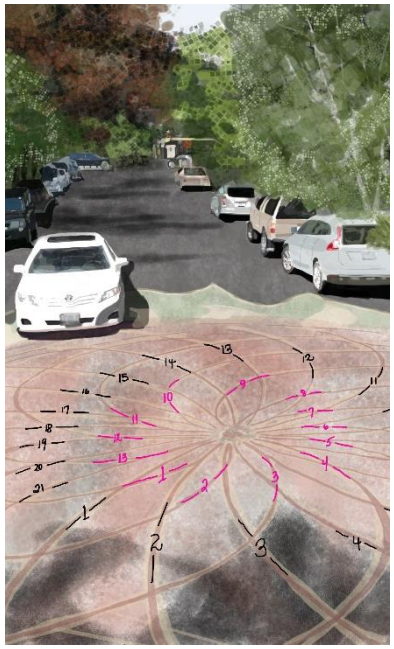
By City Repair (2001)

Photography by
Mari@gatherandgrow.org

https://gatherandgrow.org/2013/05/26/building-villages-in-the-city/

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Sunnyside Piazza


n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Mystery #3: What is this number?

1.61803398501735 7

Golden ratio????

NO. But close.

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887498\ldots$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Mystery #3: What is this number? 1.618033985017357

- What makes the Golden ratio “golden”?
- What is relationship between Fibonacci numbers and the Golden ratio?
- Should I use

  1.618033989017357

  or

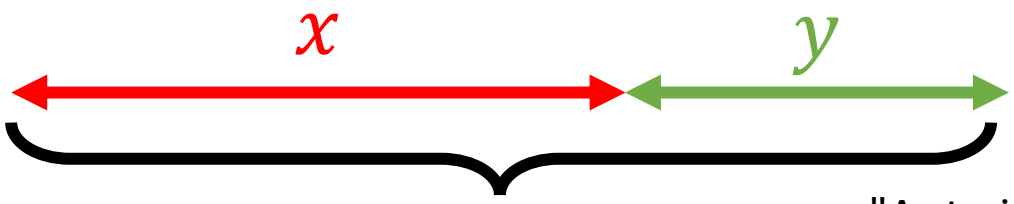
  1.618033988749894…

  in my story and artwork?

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

What makes ϕ golden?

- First recorded discussion of the golden ratio in Euclid's *Elements*. Called "extreme and mean ratio".



$$x + y$$

$$\frac{x+y}{x} = \frac{x}{y}$$

"A straight line is said to have been cut in **extreme and mean ratio** when, as the whole line is to the greater segment, so is the greater to the lesser."

Euclid (c. 325–c. 265 BC)

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

What makes ϕ golden?

A golden rectangle is any rectangle having

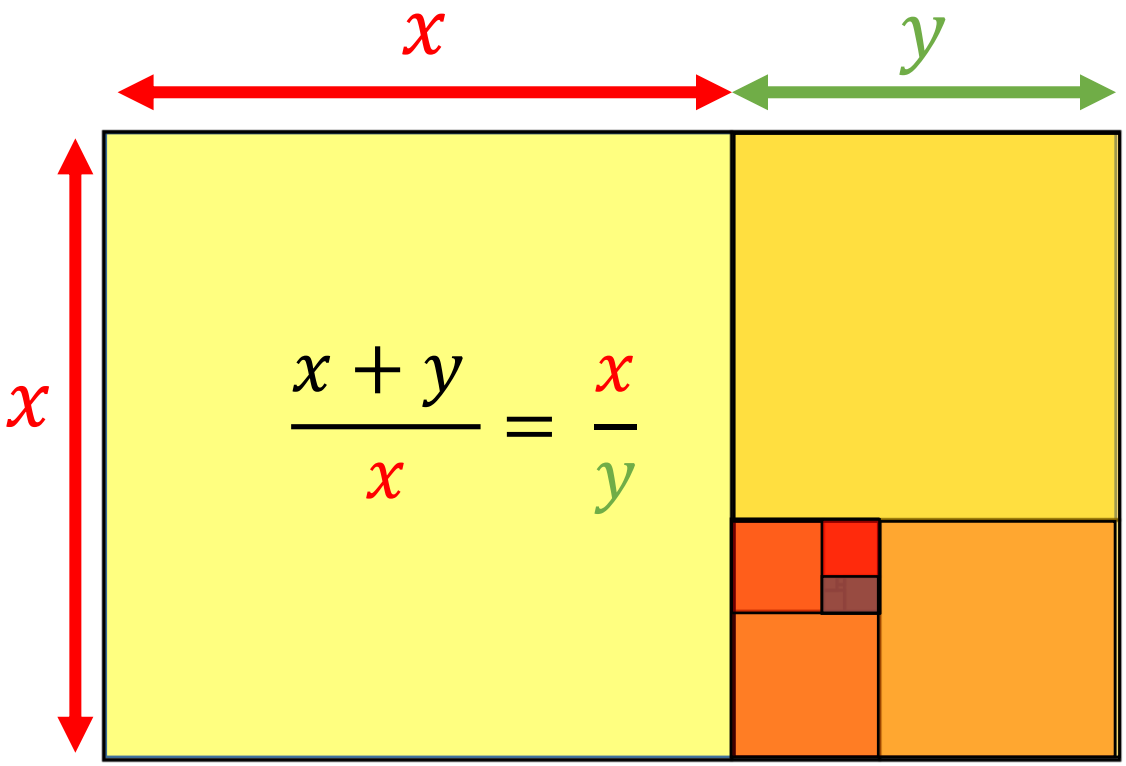
$$\frac{length}{width} = \varphi \approx 1.618$$

supposedly ***divine***.

At least according to Luca Paciolo (1445-1517) in his treatise *Divina Proportione*.

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

What makes ϕ golden?

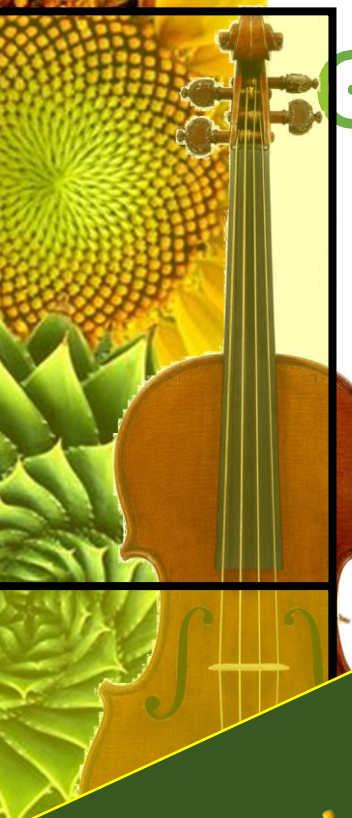


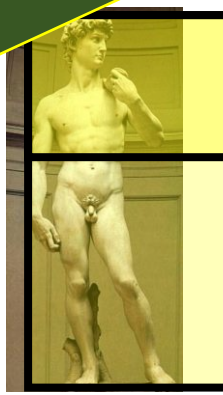
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

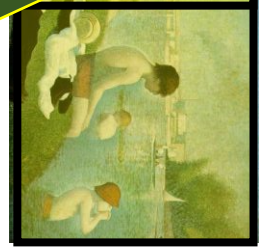
Golden ratio ….

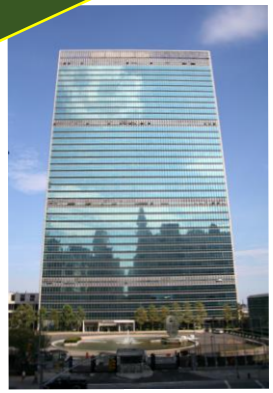
…occurs in

- Architecture ?
- Art ?
- Music ?
- Natur
-

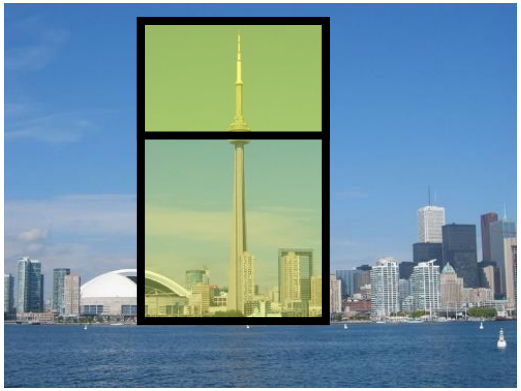








Le Corbusier's
UN Building

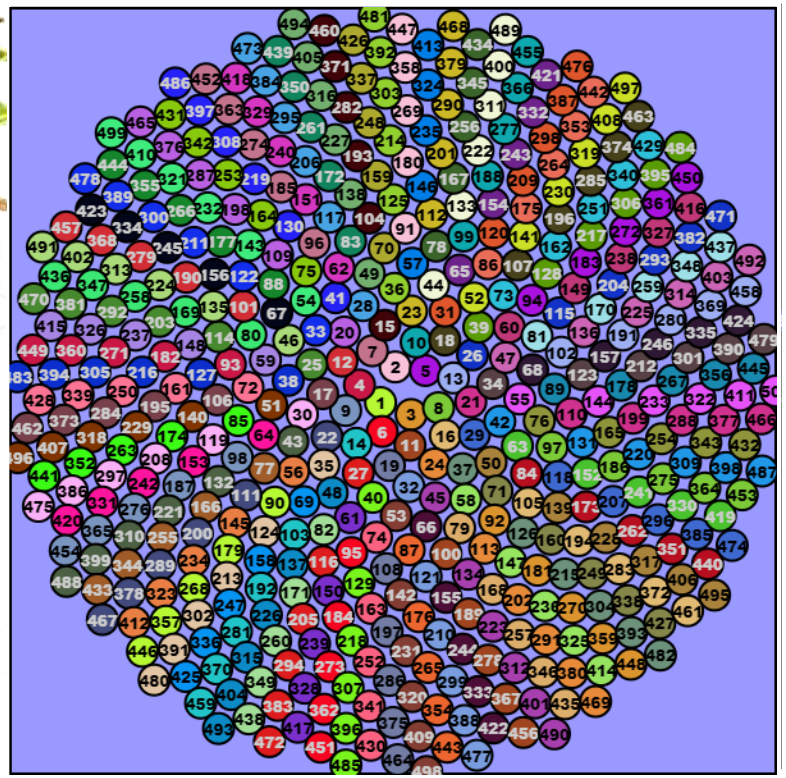


CN Tower in Toronto

Conclusion: Unless there is evidence of intentional use of the golden ratio, its likely a coincidence.

Spirals in Plants

- Growth occurs at one point; Twists as it grows
- IF new seed/leaf/branch occurs at ϕ* revolution *(137.5 degrees)*



500 seeds
89 spirals

http://merganser.math.gvsu.edu/david/seeds/

ϕ from Fibonacci

Ratios of consecutive Fibonacci numbers converge to the golden ratio ϕ

1/0=	Undefined	13/8=	1.625
1/1=	1	21/13=	1.6153846154…
2/1=	2	34/21=	1.6190476190…
3/2=	1.5	55/34=	1.6176470588…
5/3=	1.666….	89/55=	1.6181818181…
8/5=	1.6	144/89=	1.6179775280…

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Mystery #3: What is this number? 1.6180339

Mystery solved

=

nber

=

10946

1.618033985017357…

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Mystery #1: Skill not M

Mystery solved

1	A
2	B
3	
9	
10	

…mber

$$1 + 1 + 2 + \ldots + F(n) = F(n+2) - 1$$

Sum:

$$55A + 88B$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$F(n)$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377

Mystery #1: Final Touch

- Ask your volunteer to divide the 9th number into the 10th number.

- What do you give as the answer?

$$\frac{21A}{13A} < \frac{21A+34B}{13A+21B} < \frac{34B}{21B}$$

ϕ from Fibonacci

Successive ratios of consecutive Fibonacci numbers converge to the golden ratio ϕ

1/0=	Undefined	13/8=	1.625
1/1=	1	21/13=	1.6153846154…
2/1=	2	34/21=	1.6190476190…
3/2=	1.5	55/34=	1.6176470588…
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Mystery #1: Final Touch

- Ask your volunteer to divide the 9th number into the 10th number.

- What do you give as the answer? 1.62

$$\frac{21A}{13A} < \frac{21A+34B}{13A+21B} < \frac{34B}{21B}$$

Reflections

3 seemingly unrelated mysteries

1. Magic trick
2. Lost Area Puzzle
3. Significance of 1.618033989017357

Connected by

- Fibonacci numbers 0, 1, 1, 2, 3, 5, 8, 13, … or
- the Golden Ratio $\phi = 1.618033988749894\ldots$

- Lesson:
  - Understand simple ideas deeply
  - Find joy and magic of math all around you

Thank you ! Gracias !

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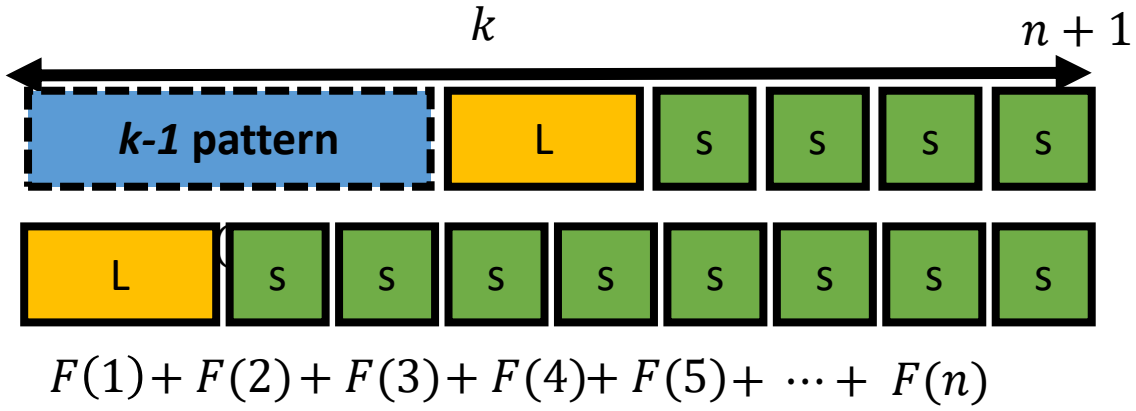

$$F(1)+F(2)+F(3)+F(4)+F(5)+\cdots+F(n)$$