# MAA Metro NY Problem of the Month
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## Problem

Let $n$ be a positive integer. Prove that for any set of $n$ integers, there exists a nonempty subset whose sum is divisible by $n$.

## Solution

Let $A := \{a_1, a_2, a_3, \ldots, a_n\}$ be a set of $n$ integers. Consider the sums of the form $S_k := a_1 + a_2 + a_3 + \ldots + a_k$ for $k \in \{1, 2, 3, \ldots, n\}$.

If $S_k$ is divisible by $n$ for some $k$, we choose the subset $\{a_1, a_2, a_3, \ldots, a_k\}$ and we are done.

Otherwise, suppose that $n$ does not divide $S_k$ for any $k \in \{1, 2, 3, \ldots, n\}$. In such case, the $n$ sums $S_1, S_2, S_3, \ldots, S_n$ only have $n - 1$ possible residues mod $n$. By the Pigeonhole Principle, two of them must have the same residue, say $S_i \equiv S_j \pmod{n}$ with $1 \leq i < j \leq n$. Therefore $a_1 + a_2 + a_3 + \ldots + a_i \equiv a_1 + a_2 + a_3 + \ldots + a_j \pmod{n}$.

Canceling out, we get $a_{i+1} + a_{i+2} + \ldots + a_j \equiv 0 \pmod{n}$, so the subset $\{a_{i+1}, a_{i+2}, \ldots, a_j\}$ has sum divisible by $n$, as desired. $\square$