MAA Metro NY Problem of the Month

February 2026

Find all rational numbers x such that $y = \sqrt{x^2 + x + 2026}$ is rational. For example $x = 45$ will give us $y = 64$.

Computer or AI assisted/generated solutions will not be accepted.

An outline of a solution.

Let a line with rational slope r pass through $(45, 64)$. Its equation is

$$y = r(x - 45) + 64.$$

To find another intersection point with

$$y = \sqrt{x^2 + x + 2026},$$

we solve

$$r(x - 45) + 64 = \sqrt{x^2 + x + 2026}.$$

Squaring,

$$\big(r(x - 45) + 64\big)^2 = x^2 + x + 2026.$$

This yields a quadratic equation in x. Since the line already passes through $(45, 64)$, the value $x = 45$ is one root. Hence the quadratic must factor as

$$(x - 45)\big((r^2 - 1)x - (45r^2 - 128r + 46)\big) = 0.$$

Therefore x has the shape,

$$x = \frac{45r^2 - 128r + 46}{r^2 - 1}.$$

which generates all rational x. For this to be defined we require $r^2 \neq 1$, i.e. $r \neq \pm 1$. Restricting to the principal square root (so that $y \geq 0$) forces $r \in (-1, 1)$. Otherwise, defining

$$y = |r(x - 45) + 64|$$

allows any rational $r \neq \pm 1$.