MAA Metro NY September 2025 Problem of the Month

Proposed by Dr. Asher Roberts, Saint Joseph University

The Problem:

Imagine a tetrahedron with each edge being of length 1, i.e., a unit tetrahedron. Circumscribe this tetrahedron with a sphere. That is to say, draw the smallest sphere that contains this tetrahedron. Now, circumscribe the sphere with a tetrahedron. In other words, place the sphere (with the unit tetrahedron inside) inside the smallest tetrahedron that could contain it. (See the figure below.) Find the volume of the region contained inside the larger circumscribing tetrahedron that is *not* contained inside the sphere (shaded with diagonal lines in the figure). Put another way, if the sphere were solid, how much air would there be inside the circumscribing tetrahedron?