MAA Metro NY June 2025, Problem of the Month

Show that it is impossible to construct an equilateral triangle in the plane with vertices that are lattice points.

Computer or AI assisted/generated solutions will not be accepted.