Problem of the Month - March 2024 Solution

2	4	5	6	4	9	
7	10	12	10	6		

Table 1: Game board

Define the sequences $\langle a_n \rangle_{n=1}^{6}$ by $\langle 2, 4, 5, 6, 4, 9 \rangle$ and $\langle b_n \rangle_{n=1}^{5}$ by $\langle 7, 10, 12, 10, 6 \rangle$. That is, the two sequences represent the values of the top row (sequence a_n) and bottom row (sequence b_n) of the nth space on the board (where the board spaces are labeled 1 to 7 from left to right). Let v_n be the maximum value of the nth space on the board for n from 2 to 7 and define $v_1 = 0$. Clearly we have that $v_2 = 2$. Working backwards, we see that

$$v_n = \max\{v_{n-1} + a_{n-1}, v_{n-2} + b_{n-2}\}$$

for $n \geq 3$. That is, for $n \geq 3$, the maximum value at space n is either the maximum value at space $n-1$ plus the value of the top row on space $n-1$ or the maximum value at space $n-2$ plus the value of the bottom row at space $n-2$. This must be the case since these are the only two ways in which to arrive at space n. So now

$$\begin{aligned}
v_3 &= \max\{2 + 4, 0 + 7\} = 7 \\
v_4 &= \max\{7 + 5, 2 + 10\} = 12 \\
v_5 &= \max\{12 + 6, 7 + 12\} = 19 \\
v_6 &= \max\{19 + 4, 12 + 10\} = 23 \\
v_7 &= \max\{23 + 9, 19 + 6\} = 32.
\end{aligned}$$

So Dr. Math's highest possible score is 32.