Problem of the Month - February 2024 Solution

Solution below provided by Koa Okano.

Only wildcats and eagles are in contention of winning, so we will only focus on:

- Wildcats vs Bulldogs
- Wildcats vs Rams
- Eagles vs Rhinos
- Eagles vs Bulldogs

Both teams have 2 matches left, so there are nine possible combinations for wildcats and eagles each.

(W,W), (W,L), (W,D), (L,W), (L,L), (L,D), (D,W), (D,L) ,(D,D)

Where W is +3, L is +0, and D is +1. So the possible point gains are

+6, +3, +4, +3, +0, +1, +4, +1, +2

This means that probability of gaining certain amount of points for each team is

- +0 , $\frac{1}{9}$
- +1 , $\frac{2}{9}$
- +2 , $\frac{1}{9}$
- +3 , $\frac{2}{9}$
- +4 , $\frac{2}{9}$
- +6 , $\frac{1}{9}$

Consider every combination of points gained. Because eagles must earn 2 or more points to win, we can highlight the combinations where eagles win.

(W,E)						
	(0,0)	(0,1)	(0,2)	(0,3)	(0,4)	(0,6)
	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)	(1,6)
	(2,0)	(2,1)	(2,2)	(2,3)	(2,4)	(2,6)
	(3,0)	(3,1)	(3,2)	(3,3)	(3,4)	(3,6)
	(4,0)	(4,1)	(4,2)	(4,3)	(4,4)	(4,6)
	(6,0)	(6,1)	(6,2)	(6,3)	(6,4)	(6,6)

Consider same diagram but with $(P(W), P(E))$

(P(W),P(E))						
	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$
	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$
	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$
	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$
	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{2}{9})$	$(\frac{2}{9},\frac{1}{9})$
	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{2}{9})$	$(\frac{1}{9},\frac{1}{9})$

Consider the same diagram but with $P(W) \cap P(E)$

$P(W) \cap P(E)$						
	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{2}{81}$	$\frac{1}{81}$
	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{4}{81}$	$\frac{2}{81}$
	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{2}{81}$	$\frac{1}{81}$
	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{4}{81}$	$\frac{2}{81}$
	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{2}{81}$	$\frac{4}{81}$	$\frac{4}{81}$	$\frac{2}{81}$
	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{1}{81}$	$\frac{2}{81}$	$\frac{2}{81}$	$\frac{1}{81}$

Therefore, by the diagram, chance of eagles being declared champion is $\frac{1}{81}(1 + 2 + 2 + 1 + 4 + 4 + 2 + 2 + 1 + 2 + 2)$**, which is** $\frac{23}{81}$