Friday the 13th. Problem of the Month for October 2023

The day of the week on January 1st, determines the days of the week for the entire year. We wish to determine which choice of day for January 1 will yield a year with the maximal number of Friday the 13th's. We shall not consider leap years although the process below could easily be modified to handle those.

First observe that the 13th of the month occurs on the following days of the year:

13, 31+13, 31+28+13, etc. Now, let us consider each of these 12 numbers modulo 7:

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> 13 mod 7;
                                  6                                   (1)
> (31 + 13)mod 7;
                                  2                                   (2)
> (31 + 28 + 13) mod 7;
                                  2                                   (3)
> (31 + 28 + 31 + 13) mod 7;
                                  5                                   (4)
> (31 + 28 + 31 + 30 + 13) mod 7;
                                  0                                   (5)
> (31 + 28 + 31 + 30 + 31 + 13) mod 7;
                                  3                                   (6)
> (31 + 28 + 31 + 30 + 31 + 30 + 13) mod 7;
                                  5                                   (7)
> (31 + 28 + 31 + 30 + 31 + 30 + 31 + 13) mod 7;
                                  1                                   (8)
> (31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 13) mod 7;
                                  4                                   (9)
> (31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 13) mod 7;
                                  6                                   (10)
> (31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 13) mod 7;
                                  2                                   (11)
> (31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 13) mod 7;
                                                                      (12)
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$$4 \tag{12}$$

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There are 3 months that all have the same modulus, namely 2 mod 7, and thus these three months February, March and November all have the 13th fall on the same day of the week. This is the largest of the shared moduli. Thus, the largest number of Friday the 13th's in a given year will be three.

Now, all we need to do is figure out what day January 1st should be to generate this pattern. Clearly any month starting on a Sunday will have a Friday the 13th. (The Sundays of this month will fall on the 1st, the 8th, the 15th and so on. If the 15th is on a Sunday, the 13th will fall on Friday.) Now the maximal number of times we get a Friday the 13th, as we have seen above, occurs when the first of the Friday's the 13th falls in February. So we need the first of February to be a Sunday. This forces January 1st to be a Thursday.

Any non-leap year that begins on a Thursday will have the maximal number (i.e. 3) of Friday the 13th's. This happened in 2015.

Note that there is a Friday the 13th this month. October, 2023.