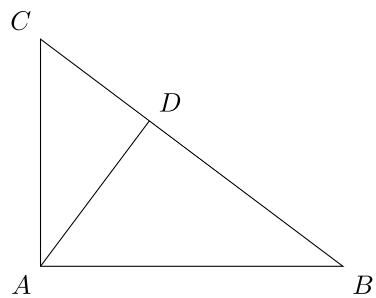
We thank all the solvers for their solutions to the March 2023 MAA Metro NY Problem of the month. We feature a solution provided by Dr. Alexander Rozenblyum of New York City College of Technology, CUNY.

In the triangle below, let $\angle BAC = \angle ADC = \frac{\pi}{2}$, $AB = a$, $AC = b$, $AD = d$ and $BC = c$.
For the solution to the first part, we begin by finding the area of the triangle ABC in two ways and equating them.

$$\frac{1}{2} \cdot BC \cdot AD = \frac{1}{2} \cdot AB \cdot AC$$



This will give us $c = \frac{ab}{d}$. Substituting for c into the Pythagorean identity, $a^2 + b^2 = c^2$ and simplifying gives the desired expression, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{d^2}$.

For the solution to the second part, we begin with $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{d^2}$ and solve for b to get $b = \frac{ad}{\sqrt{a^2-d^2}}$, where $\boxed{a > d}$. If $a = b$, in this expression, we get that $d = a\sqrt{2}$, which is not integer-valued and we must have that $a \neq b$. Let $BD = m$ and $CD = n$. Without loss of generality, let us assume that $1 < a < b$, and we can conclude that $m < n$, since $m = \sqrt{a^2 - d^2}$ and $n = \sqrt{b^2 - d^2}$. In addition, $\triangle ABD$ is similar to $\triangle CAD$, and it readily follows that $mn = d^2$, and this coupled with $m < n$ allows us to conclude that $m < d$ or $\sqrt{a^2 - d^2} < d$. The latter inequality gives us $\boxed{a < d\sqrt{2}}$.

From the boxed expressions above, we have shown $d < a < d\sqrt{2}$ and we have the added requirement that $b = \frac{ad}{\sqrt{a^2-d^2}}$ is a positive integer. For our problem $d = 120$, and $121 \leq a \leq 169$. A quick calculator check for the 49 values of a shows that $a \in \{130, 136, 150\}$. We have that $(a, b) \in \{(130, 312), (136, 225), (150, 200)\}$

which result in three distinct triangles. You can switch the order of a and b to get three additional algebraic solutions.

Editor's note: If we assume that $1 < a < b$, it follows that $\frac{1}{b^2} < \frac{1}{a^2}$ or equivalently, $\frac{1}{a^2} + \frac{1}{b^2} < \frac{1}{a^2} + \frac{1}{a^2} = \frac{2}{a^2}$ and we get $\frac{1}{d^2} < \frac{2}{a^2}$ or $a < d\sqrt{2}$. This shortens the above proof for an upper bound of a.