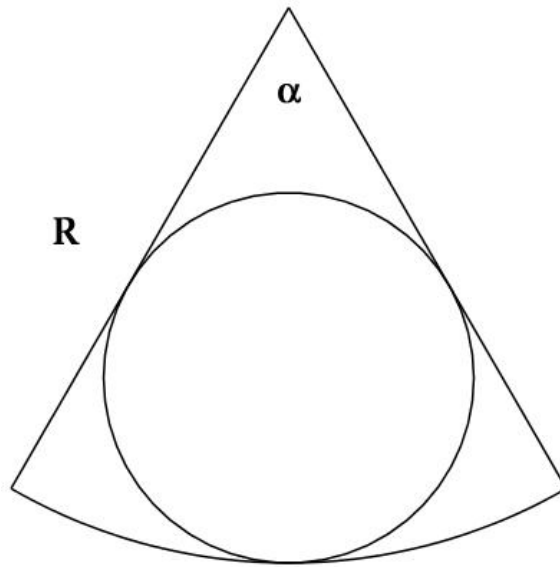


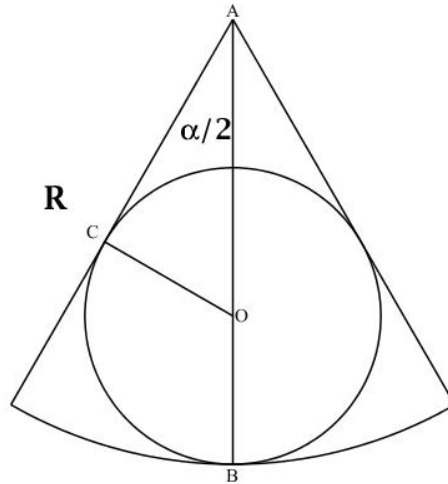
The Problem of the Month
December 2021

Imagine a sector of a circle with radius R and central angle α .
Find the radius, r , of the circle inscribed in the sector.



The Solution:

Consider the following diagram:



Draw a line segment from the apex of the sector, A, to the point, B, where the curved end of the sector is tangent to the inscribed circle. This segment will bisect the apex angle and it will pass through the center, O, of the inscribed circle. Draw segment OC from the center of the inscribed circle to a point, C, of tangency of the circle and the edge of the sector. Now $|AO| = R-r$ and $|OC| = r$. Now $\sin(\alpha/2) = r / (R-r)$.

$$\text{Thus } R \sin(\alpha/2) - r \sin(\alpha/2) = r$$

$$\text{This yields } R \sin(\alpha/2) = r(\sin(\alpha/2) + 1)$$

$$\text{Solving for } r \text{ yields the answer: } \mathbf{r = R \sin(\alpha/2) / [1 + \sin(\alpha/2)]}$$