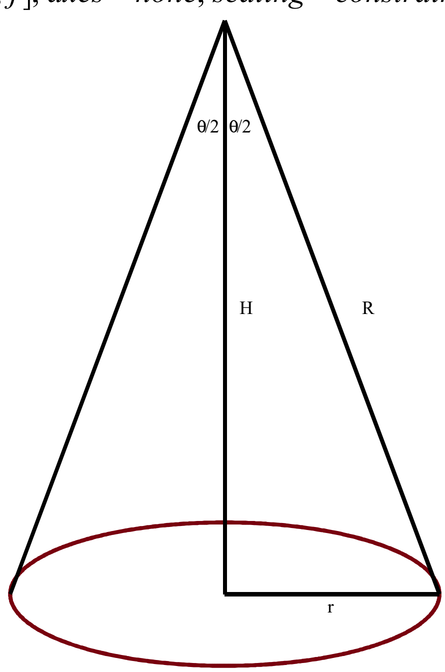
Here we investigate the relationship between the apex angle of a right circular cone and the central angle of the circular sector which one gets by slicing the cone along its slant height and unrolling it.

Let us start with a right circular cone with altitude H and slant height R. Let r be the radius of the circular base. Let the apex angle equal θ.

```
> restart : with(plots) : with(plottools) :
```

$$> \ a := implicitplot\left(\frac{x^2}{9} + \frac{y^2}{1} = 1, x = -4..4, y = -3..3\right) :$$

```
> b := line([0, 8], [-3, 0]) : c := line([0, 8], [3, 0]) : dd := line([0, 8], [0, 0]) : e := line([0,
    0], [3, 0]) :
> f := textplot([.3, 4, "H"]) : g := textplot([2, 4, "R"]) : h := textplot([1.5, -.2, "r"]) :
> i := textplot([-.22, 6.5, "θ/2"]) : j := textplot([.22, 6.5, "θ/2"]) :
>
>
>
> display([a, b, c, dd, e, f, g, h, i, j], axes = none, scaling = constrained);
```



We have the following:

$$r=\sqrt{R^2-H^2} \qquad \cos\left(\frac{\theta}{2}\right)=\frac{H}{R} \quad \Rightarrow H$$

$$=R\cos\left(\frac{\theta}{2}\right)$$

Let C be the circumference of the circular base of the cone. Then:

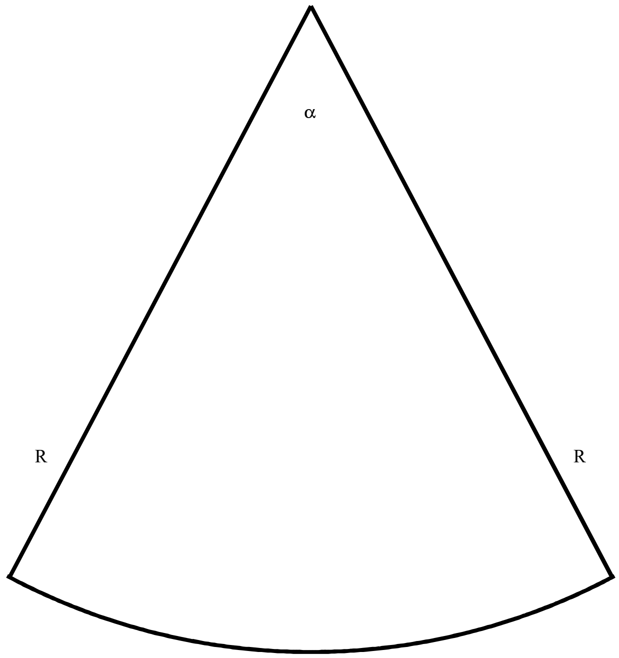
$$C=2\,\pi r=2\,\pi\sqrt{R^2-H^2} = 2\,\pi\sqrt{R^2-R^2\cdot\cos^2\left(\frac{\theta}{2}\right)}$$

$$= \; 2\,\pi\,R$$

$$\sqrt{1-\cos^2\left(\frac{\theta}{2}\right)}$$

Now let us slice this cone along its slant height and unroll it to get a circular sector:

```
> restart : with(plots) : with(plottools) :
> a := arc([0, 6], 6, 3·π/2 − .49 .. 3·π/2 + .49) :
> b := line([0, 6], [2.8, 0.7]) : c := line([0, 6], [-2.8, 0.7]) :
> dd := textplot([2.5, 1.8, "R"]) : e := textplot([-2.5, 1.8, "R"]) :
> f := textplot([0, 5, "α"]) :
>
>
> display([a, b, c, dd, e, f], scaling = constrained, axes = none);
```



Let α be the apex angle of this sector. Note that the length of the arc of the circular sector is the circumference of the base of the cone which we called C. Now we have:

$$\frac{\alpha}{C} = \frac{2\pi}{2\pi R} = \frac{1}{R} \Rightarrow \alpha = \frac{C}{R} = \frac{2\pi R \sqrt{1 - \cos^2\left(\frac{\theta}{2}\right)}}{R}$$

$$= 2\pi \sqrt{1 - \cos^2\left(\frac{\theta}{2}\right)}$$

Thus, we have α as a function of θ. Note that it is independent of H and R.

$$\alpha = 2\pi \sqrt{1 - \cos^2\left(\frac{\theta}{2}\right)} = 2\pi$$

$$\sqrt{\sin^2\left(\frac{\theta}{2}\right)} = 2\,\pi\,\sin\left(\frac{\theta}{2}\right)$$

$$\text{Thus } \boldsymbol{\alpha = 2\,\pi\,\sin\left(\frac{\theta}{2}\right)}$$

Note that when $\theta = \pi$, the cone becomes a disk and $\alpha = 2\pi$. The other extreme occurs when $\theta = 0$ and the cone becomes a vertical line. Here $\alpha = 0$.

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