The Problem of the Month
May 2021

The Problem:

Consider a roll of toilet paper where the radius of the hollow cardboard core is 1 inch. Suppose, further, that the radius of the full roll (the core together with all the toilet paper) is 2 inches. If we take the thickness of a single sheet of toilet paper to be 1/100 of an inch, find the length of the entire roll if it were to be un-spooled. (We take the thickness of the cardboard core to be zero.)

The Solution:

First observe that a roll of toilet paper is ***not*** a spiral. Rather it is a set of nested concentric cylinders. The first (inner most) layer has radius 1 + 1/100. The next layer has radius 1 + 2/100, and so on. Continuing in this way, we see that the total length of the un-spooled roll will be the sum of the circumferences of all of the layers. Thus:

$L = 2\pi(1+1/100) + 2\pi(1+2/100) + \ldots + 2\pi(1 + 100/100)$

$L = 2\pi\ (100 + 1/100 * (1 + 2+ 3+4+\ldots+100))$

$L = 2\pi\ (100 + 1/100 *(100*101/2))$

[Recall that $1 + 2 + 3 + \ldots + n = n*(n+1)/2$]

$L = 2\pi\ (100 + 101/2)$

$L= \pi\ (200 + 101)$

$L = \pi\ (301) = 945$ inches. This is 78.75 feet.