# MetroMath 





| Bronx | Brooklyn | Columbia | Dutchess |
| :--- | :---: | :---: | :---: |
| Greene | Manhattan | Nassau | Orange |
| Putnam | Queens | Richmond | Rockland |
| Suffolk | Sullivan | Ulster | Westchester |

# ANNUAL MEETING 

Saturday, 29 April 2017
8:30 AM - 5:15 PM
Hostos Community College (CUNY)
Bronx, NY
(More Information Contained Within)

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## TABLE OF CONTENTS

List of Section Officers ..... 2
Table of Contents ..... 3
Map and Membership Count of the Metro NY Section ..... 3
Message from the Chair, by Elena Goloubeva ..... 4
Message from the Chair-Elect, by Janet Liou-Mark ..... 4
Message from the Governor, by Abe Mantell ..... 5
Treasurer's Report, by Armen Baderian ..... 6
25 and 50 Year Members ..... 6
Hostos Community College Campus Map ..... 6
2017 Annual Meeting Program ..... 7
Abstracts and Brief Speaker Biographies ..... 8
Featured Article: A Note on Monovariants and Minimal Fibonacci Representations, by Minerva Catral, Pari Ford, Pamela Harris, Steven Miller, Dawn Nelson ..... 11
Book Review: Algebra in Context, by Amy Shell-Gellasch and J. B. Thoo, reviewed by David Seppala-Holtzman ..... 13
MAA Student Chapters and Departmental Membership, by David Seppala-Holtzman ..... 14
Call For Participants and Involvement ..... 15
Math in the News from the MAA ..... 15
Events Calendar (Local and National) ..... 17
Paid Advertizing ..... 18
Annual Spring Meeting Announcement Poster (please post in your department) ..... 19
Metro NExT Flyer ..... 20


## MESSAGE FROM THE SECTION CHAIR

I would like to begin this message by thanking the team of section officers, committee chairs, and active members. The section has a great team of people working together to make sure that it is active, and that the programming remains diverse, meaningful, and rewarding for the entire membership. It's a real pleasure and honor to work for them and with them.

Our 2016 Section Meeting at Vaughn College was extremely successful. Vaughn College did a fantastic job hosting the event, and I want to thank all people that were involved in planning and putting the event together.

Our next meeting will take place on Saturday, April $29^{\text {th }}, 2017$ at Eugenio María de Hostos Community College in The Bronx. Details of this meeting (including registration information) are in this newsletter. Our invited speakers are absolutely amazing. Carl Simon from the University of Michigan will present Mathematics is contagious and contagion can be mathematical, which will address the analyses of mathematical models of the spread of contagious infections and applications of these models to the analyses of smoking and crime. Rosa Orellana from Dartmouth College will discuss symmetric functions. The focus of her presentation will be symmetry and coloring. Also, we will conduct a panel discussion on teaching. It should be an exciting event! Please visit the section website for updates.

Young faculty, please come and join Section NExT (Metro NExT). This program is active again. It is aimed at supporting young faculty, new and rising Ph.D.'s in mathematics or mathematics education.

Please use our Speakers Bureau. We will be happy to serve local schools and communities by providing interesting speakers on a wide variety of mathematical topics.

Dear colleagues, please contact me or Section officers if you would like to become actively involved, if you seek more information or want to share ideas. Step up to become a Section officer. You can start with hosting a meeting. Let us know if your institution would be willing to host a future meeting. Visit the MAA NY Metro website to see what is happening. As always, I welcome your input and feedback on all Section matters.

I hope to see many of you at the April meeting! Mark your calendars with this important date, come, bring your colleagues, and inspire college and high school students to join!

Elena Goloubeva, Webb Institute

## MESSAGE FROM THE SECTION CHAIR-ELECT

Warmest greetings to all the MAA Metro New York Section members!
In The Mathematical Sciences in 2025, the National Research Council (2013) recommended that mathematics departments enable students of all social, ethnic, and economic backgrounds to succeed in their pursuit of a mathematics degree and to be supported through best practices.

Retaining underrepresented groups (minorities and women) in the mathematical sciences at all levels continues to be a national challenge. Many of these students face significant barriers which prevents them to reach their academic potential. This year's annual meeting at Hostos Community College will provide engaging presentations and discussions, not only in the mathematical sciences, but also in promoting a diverse and supportive mathematical climate for all students. Our prominent guest speakers will expand your mathematical and pedagogical knowledge and our Section NExT program
will welcome untenured faculty into an engaging and supportive community. I look forward to meeting you at the April 29th meeting!

Janet Liou-Mark, New York City College of Technology (CUNY)
National Research Council (2013). The Mathematical Sciences in 2025. Washington, DC: The National Academies Press.

## MESSAGE FROM THE SECTION GOVERNOR

The following are my reports from the two most recent Board of Governors (BoG) Meetings of the MAA. The first was held at MathFest on 3 August 2016 in Columbus, OH; and the second at the JMM on 3 January 2017 in Atlanta, GA. A personal emergency prevented me from attending the JMM. Thus, that report is my summary from the official minutes. I include only those items most relevant, or of interest, to MAA members in the Metro NY Section.

## BoG at MathFest 2016

The US team placed first at the 2016 International Mathematical Olympiad for the second consecutive year, and for the first time all six members of the team won gold medals.

Treasurer Jim Daniels addressed the continuing deficit, and was hopeful that the MAA expects to gain revenue from the publishing partnership that is in development.

## BoG at JMM 2017

The MAA's second century campaign just concluded. Even without a final tally, it is clear that the campaign's goal of raising 7.5 million dollars has been exceeded. The MAA's bid to host the International Mathematical Olympiad in 2021 was unanimously accepted and the Association is in a position to raise the necessary funds. In fact, the MAA (especially the competitions program) is attracting much interest from corporate sponsors.

Current membership numbers are:
Core Members - 9352, Departmental Members - 224, Students - 8158 (of which 7618 are through Departmental Memberships), K-12 Teachers - 5582 (teachers who register teams for AMC 8, 10 or 12).

Michael Pearson, Executive Director, noted that we have more than 23,000 members and should think about from where our members should come. He said that $95 \%$ of those who major in mathematics end up in non-academic positions; annually there are a 1000 Ph.D.'s in mathematics and 500 go to academia. The MAA needs to expand its audience as we capitalize on the growing recognition of the value of mathematics across society. We should look at different types of membership such as corporate membership as well as consider what benefits current memberships should carry; for example a new feature of Departmental Membership is a reduction in the price of Maplesoft.

Pearson continued by pointing to revenue trends for membership, journals, and books, all of which are valuable to the Association, but the MAA cannot handle all aspects of the publishing. The MAA has been acting as contractor with partners for printing and composition; but are now considering a partnership in which the partner will serve as the contractor; this will help to maintain our publications program and puts us on a path for long term success.

The 2017 budget report was presented, and it was noted that without the Simons injection of \$250,000 per year, the MAA has been running a \$760,000 deficit each year. The proposed 2017 budget shows a deficit of $\$ 851,000$, but the publishing partnership could reduce that by $\$ 350,000$ and we are hopeful for a renewal from Simons to give the Association \$250,000.

Abe Mantell, Nassau Community College (SUNY)

## TREASURER'S REPORT

(as of 3/06/17)

| Business Checking | $\$ 8,540.21$ |
| :--- | :--- |
| Business Money Market | $\$ 15,078.24$ |
| 6-Month Business CD | $\$ 425.00$ |
| Total | $\$ 24,043.45$ |

All accounts are with J.P. Morgan Chase Bank. Further details will be provided at the annual meeting.
Armen Baderian, Nassau Community College (SUNY)

## 25 and 50 Year Members

The following members will be recognized during the Awards Ceremony at our May meeting. The 25 year members are offered free registration, the 50 year members free registration and lunch (who said there's no such thing as a free lunch?!!!).

25 Years: Joan Birman (Columbia University), George Leibman III, Elliott Landowne, Carole Sirovich

50 Years: none

# Hostos Community College (CUNY) 

500 Grand Concourse Bronx, NY 10451


# 2017 ANNUAL MEETING PROGRAM <br> Saturday, April 29 Hostos Community College (CUNY), Bronx, NY 

| 8:30-12:15 PM | Registration | $3^{\text {rd }}$ Floor <br> Hallway |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { 8:30-12 Noon } \\ & \text { 3:15-5:15 PM } \end{aligned}$ | Refreshments | Café 311 |
| 8:30-3:30 PM | Book Exhibits | Bridge $3^{\text {rd }}$ Floor |
| 9:15-9:35 AM | Welcome <br> Dr. Christine Mangino, Provost, Hostos Community College <br> Dr. William Baker, Chairman, Department of Mathematics, Hostos CC <br> Dr. Elena Goloubeva, Chair of the Metropolitan NY Section of the MAA, Webb Institute | Café 311 |
| 9:35-10:35 AM | Invited Speaker <br> Mathematics is Contagious and Contagion can be Mathematical Dr. Carl Simon, University of Michigan | Café 311 |
| 10:35-10:45 AM | Break |  |
| 10:45-12:20 PM | Strategies and Research in Teaching <br> Moderator: Dr. David Seppala-Holtzman, St. Joseph's College <br> Teaching via Guided Discovery <br> Dr. Rosa Orellana, Dartmouth College <br> Helping Students in Lower Division Mathematics Courses <br> Dr. Danté Tawfeeq, John Jay College of Criminal Justice <br> The Creativity of Teaching-Research Methodology TR/NYCity Model <br> Dr. Bronislaw Czarnocha, Hostos Community College <br> Formative Summative Assessments: Some of My Favorite Exam Questions <br> Dr. Ron Buckmire, National Science Foundation | Café 311 |
| 12:20-1:30 PM | Lunch | $\text { Gym } 3^{\text {rd }}$ <br> Floor |
| 1:30-1:55 PM | Business Meeting and Awards Ceremony | Café 311 |
| 2:00-3:00 PM | Invited Speaker <br> Symmetry and Coloring <br> Dr. Rosa Orellana, Dartmouth College | Café 311 |
| 3:15-5:15 PM | Metro NExT - New Experiences in Teaching <br> Moderators: Dr. Johanna Franklin, Hofstra University Dr. Mutiara Sondjaja, New York University | Café 311 |
| 3:15-4:00 PM | Metro NExT Invited Speaker <br> Funding Opportunities at the National Science Foundation in Mathematical Sciences Dr. Ron Buckmire, National Science Foundation | Café 311 |
| 4:10-5:15 PM | Metro NExT Panel Discussion on Diversity in the Mathematics Classrooms Panelists: Dr. Trace Jordan, New York University <br> Dr. Janet Liou-Mark, New York City College of Technology <br> Dr. Danté Tawfeeq, John Jay College of Criminal Justice | Café 311 |
| 3:15-5:15 PM | Contributed Paper Sessions <br> Research Session - C 356, Pedagogy Session - C 451 <br> Miscellaneous Session - C 352, Faculty/Student Session - C 466 <br> Student Session I-C 359, Student Session II - C 355 |  |
| 3:15-4:15 PM | Contributed Poster Session | Atrium and Bridge $3^{\text {rd }}$ Floor |
| 4:10-4:55 PM | Pearson for Mathematics and MyMathLab <br> See pages 8-10 for Abstracts and brief Speaker Biographies | C 351 |

## INVITED SPEAKERS

MATHEMATICS IS CONTAGIOUS AND CONTAGION CAN BE MATHEMATICAL Dr. Carl Simon, The University of Michigan




#### Abstract

Analyses of mathematical models of the spread of contagious infections have suggested effective interventions to stop or slow disease spread - interventions such as vaccination regimes (who and when), prophylactic treatments, hospital procedures, and behavioral recommendations. A key component in these analyses has been the basic reproduction number as a threshold between endemic disease and no disease. This lecture will present some background on these analyses and describe recent efforts to extend them to smoking and crime. Why do teenagers start smoking? What difference does it make? What are the advantages and disadvantages of different interventions in the fight against crime, interventions such as harsher or gentler prison sentences, increased or decreased police presence, more or less intense social programs? What data do we need to evaluate these interventions and how can we find such data?


#### Abstract

Biography: Carl P. Simon is Professor of Mathematics, Economics, Complex Systems and Public Policy at The University of Michigan. He was the founding Director of the UM Center for the Study of Complex Systems (1999-2009). He has served as the Associate Director for Social Science and Policy of the Michigan Energy Institute and as Director of the U-M Science and Technology Policy Program. His research interests center around the theory and applications of dynamical systems. He has applied dynamic modeling to the spread of AIDS (in particular, the role of primary infection), staph infection, malaria and gonorrhea, to smoking initiation, the spread of crime, and the evolution of ecological and economic systems. His research team won the 1995 Howard M. Temin Award in Epidemiology for Scientific Excellence in the Fight against HIV/AIDS and the 2005 Kenneth Rothman Epidemiology Prize for paper of the year in Epidemiology, He was named the U-M LS\&A Distinguished Senior Lecturer for 2007 and received the U-M Distinguished Faculty Achievement Award in 2012.


## SYMMETRY AND COLORING Dr. Rosa C. Orellana, Dartmouth College



Abstract: The theory of symmetric functions has many applications to enumerative combinatorics, group theory, Lie Theory and algebraic geometry. This talk is a short introduction to the theory of symmetric functions and their applications. We'll discuss a particular case of these functions - symmetric chromatic functions, and the connection between the symmetric chromatic function and graph coloring.

Biography: Rosa Orellana received her Ph.D. from UCSD in 1999 under the guidance of Hans Wenzl. After graduation, she won a University of California President's Postdoctoral Fellowship at UC San Diego. In 2000, she joined the department of mathematics at Dartmouth College. She is currently Full Professor at Dartmouth where she is lucky enough to teach some of the best students in the country. Rosa has mentored many students on research projects and during the summer of 2013, she led a group of eighteen minority students for MSRI-UP. In 2006, she received the John M. Manley Huntington Memorial Award at Dartmouth for outstanding research, teaching and mentoring. Rosa is very interested in making students feel welcome, so at Dartmouth she has served as the advisor for the math club. In addition, Rosa co-founded a chapter of the Association for Women in Mathematics in an effort to increase the number of women taking and majoring in mathematics at Dartmouth. She has also organized Sonia Kovalevsky Math days to encourage middle and high school girls in our community to study mathematics. Her area of research is algebraic combinatorics. Algebraic combinatorics is an area of mathematics that studies objects that have combinatorial and algebraic properties. An example of such object is the ring of symmetric functions. In algebraic combinatorics, algebraic methods are used to answer combinatorial questions, and conversely, combinatorial techniques are applied to problems in algebra. Recently her work has focused on the Kronecker product of two irreducible representations of the symmetric group.

## METRO NExT INVITED SPEAKER

## FUNDING OPPORTUNITIES AT THE NATIONAL SCIENCE FOUNDATION IN MATHEMATICAL SCIENCES <br> Dr. Ron Buckmire, National Science Foundation



Abstract: We shall present examples of funding opportunities at the National Science Foundation to support mathematical sciences. We will discuss programs that reside in the Division of Mathematical Sciences (DMS), in the Directorate for Mathematical and Physical Sciences (MPS), in the Division of Undergraduate Education (DUE), and in the Directorate for Education and Human Resources (EHR). Examples of currently funded projects will be included and questions and inquiries encouraged.

Biography: Ron Buckmire is the Lead Program Director of the Scholarships for Science, Technology, Engineering and Mathematics (S-STEM) program housed in the Division of Undergraduate Education (DUE) at the National Science Foundation (NSF). Before coming to NSF in May 2016 as a permanent program director responsible for mathematics education, he had served as a rotator (temporary NSF Program Director) in DUE from 2011-2013. He has been a faculty member at Occidental College in Los Angeles since 1994, serving as chair of the Mathematics department (2005-2010, 2015-2016) and achieving the rank of Full Professor in 2014 after beginning his academic career as a Minority Postdoctoral Scholar-in-Residence. Ron holds Mathematics degrees (Ph.D., M.Sc. and B.Sc.) from Rensselaer Polytechnic Institute. He was the Principal Investigator on an S-STEM project: DUE-1457943, Creating Opportunities in Science and Mathematics for Occidental Students (COSMOS). He has published peer-reviewed articles in an eclectic collection of journals such as Numerical Methods for Partial Differential Equations, IMA Journal of Management Mathematics, Works and Days and the Albany Law Review. His primary areas of research interest include mathematical modeling, applied mathematics, numerical analysis (specifically nonstandard finite-difference approximations of ordinary and partial differential equations), mathematics education and the scholarship of teaching and learning. He is an avid tennis fan, often active on social media (on Twitter @madprofessah and on Instagram @ronbuckmire) and has regularly updated the blog The Mad Professah Lectures since 2005.

# STRATEGIES AND RESEARCH IN TEACHING PANELISTS 

## TEACHING VIA GUIDED DISCOVERY <br> Dr. Rosa C. Orellana, Dartmouth College


#### Abstract

The philosophy behind guided discovery is that students can discover ideas and methods by themselves. The book for the course consists of carefully designed problems to lead the students to discover and prove the main ideas. There is considerable evidence that this leads to deeper learning and more understanding.


## FORMATIVE SUMMATIVE ASSESSMENTS: SOME OF MY FAVORITE EXAM QUESTIONS Dr. Ron Buckmire, National Science Foundation


#### Abstract

Assessment is a key component of all teaching and learning. These are often categorized into two parts: formative and summative. Formative assessments are evaluation instruments used before the end of the learning period to provide students and teachers with feedback to assist in the learning process. Summative assessments are evaluation instruments that are used at the end of a learning period to provide information about the level of learning that occurred compared to some benchmark or standard. In this talk I will present some examples of some summative assessments from undergraduate mathematics (primarily calculus) that I believe had a formative effect (i.e. profound and long-lasting) on both students and myself.


# THE CREATIVITY OF TEACHING-RESEARCH METHODOLOGY TR/NYCITY MODEL Dr. Bronislaw Czarnocha, Hostos Community College 



Abstract: Dr.Czarnocha will discuss the creativity of TR/NYCity methodology of teaching and doing research simultaneously. The methodology has been developed and practiced in the community colleges of the Bronx leading to deep understanding of learning pathways of its student population while at the same contributing to the general knowledge in the field. Central in grasping creativity of the TR/NYCity model is the concept of bisociation introduced by Koestler (1964). He defined bisociation "as a spontaneous flash of insight, which...connects previously unconnected frames of reference and makes us experience reality at several planes at once". Manifestations of student mathematical creativity will be discussed.

Biography: Dr. Broni Czarnocha is a quantum physicist PhD turned mathematics teacher-researcher. Faculty member in the Mathematics Department of Hostos CC since Sept. 1993, and in 2010 he was granted full professorship; one of the founding members of the RUMEC (Research in Mathematics Education Community), the nationally based research group which widely introduced the APOS theory of mathematical conceptual development into Math Ed profession. He was the Principal Investigator (with Vrunda Prabhu) on the 2002-2006 NSF/ROLE Grant \# 0126141 ( $\$ 400,000$ ), 3 years for the Teaching Experiment Introducing Indivisibles into Calculus Instruction, and successful grant writer and coordinator of mathematics teaching-research for the 2005-2008 Comenius 2.1, European Commission (476,000euro) international grant Professional Development of Teacher-Researchers anchored in Poland with participation of Hungary, Italy, Spain and Portugal. He published 42 papers and edited two Handbooks of teachingresearch. Originally interested in the Teaching-Research methodology, since the local CUNY grant received in 2010 he focused his investigations on the mathematical creativity of Aha! Moment proposing bisociation of Arthur Koestler as the new definition of creativity. He is also currently investigating the Mathematics of Fairy Tales and the application of mathematics to the detailed analysis of social events. His hobby is glider pilot.

## HELPING STUDENTS IN LOWER DIVISION MATHEMATICS COURSES TO BECOME BETTER MANAGERS OF PROBLEMS THAT THEY PERCEIVE TO HAVE NO APPARENT SOLUTIONS Dr. Danté A. Tawfeeq, John Jay College of Criminal Justice



Abstract: In this discussion I will engage issues related to college level at-risks students' success in lower division mathematics course. This discussion will be grounded in issues related to efficacy and the possible repercussions on the learning of mathematics during post-secondary education do to assessment policies at the secondary level. Also, I speak about some of the curriculum and policy initiatives that were implemented in the Math Foundations Quantitative Reasoning program that helped in reducing the failure rates of at-risk students in college algebra.

Biography: Dr. Danté A. Tawfeeq is an associate professor of mathematics at John Jay College of the City University of New York. He is also the director of the Math Foundations Quantitative Reasoning program. As a director, he develops curriculum and policies that support students' learning of mathematics at the college. His professional portfolio includes 9 years of teaching and program assessment at middle and secondary schools. Additionally, Dr. Tawfeeq has 14 years of experience teaching mathematics and mathematics teacher education courses at both majority and minority serving institutions of higher education. Dr. Tawfeeq was named a Massachusetts Institute for College and Career Readiness (MICCR) Senior Research Fellow. He will be in this role from 2015 to 2018. Dr. Tawfeeq also received a Fulbright Award in January 2015 to work with the Namibia University of Science \& Technology (NUST). Dr. Tawfeeq's research interest includes the analysis of Black and Hispanic students' performance on mathematical assessments; PD for in-service teachers of mathematics that work with large at-risk populations; the instruction of mathematics to underprepared college students; problem centered learning in algebra and calculus, the design and validity of mathematics assessments; and the intellectual identity of urban at risk male students. The results from several research and service funded projects that Dr. Tawfeeq directed has led to published work.

## FEATURED ARTICLE

A Note on Monovariants and Minimal Fibonacci Representations Minerva Catral, Pari L. Ford, Pamela E. Harris, Steven J. Miller, and Dawn Nelson ${ }^{1}$


#### Abstract

We describe an interesting application of monovariants. In particular, we prove that among all decompositions of a positive integer as a sum of Fibonacci numbers, the Zeckendorf decomposition has the fewest number of summands.


An invariant is a quantity that remains unchanged under a specified set of transformations. In comparison, a monovariant is a quantity that may vary, but only in one direction. The monovariants that we describe below decrease. Like most monovariants ours are non-negative integer valued and hence we know that our monovariants will not decrease indefinitely.

Édouard Zeckendorf proved that every natural number can be expressed uniquely as a sum of nonconsecutive Fibonacci numbers (with $F_{1}=1, F_{2}=2$ and $F_{n+1}=F_{n}+F_{n-1}$ ) [7]. Moreover, the Fibonacci sequence is the only sequence with this property [4]. The purpose of this note is to provide a short proof of a result that may or may not be known in the literature, namely that among all decompositions of a positive integer as a sum of Fibonacci numbers the Zeckendorf decomposition has the fewest number of summands. Previous authors showed that the Zeckendorf decomposition is minimal among binary decompositions, where adjacent terms are allowed but all multiplicities are at most 1 ; see [1, 2, 5, 6].

Consider the example of 64 . Its Zeckendorf decomposition is: $64=55+8+1$. The results here show that any other decomposition of 64 , using Fibonacci numbers, uses three or more summands (e.g., $64=21+21+21+1)$.

Our proof proceeds by defining a monovariant that decreases as we transform any decomposition into the Zeckendorf decomposition. Recently, the authors of [3] extended the use of decreasing monovariants to prove a set of necessary and sufficient conditions under which positive linear recurrence sequences, which generalize the Fibonacci numbers, have minimal Generalized Zeckendorf Decompositions.

Let $D(m)$ be a given decomposition of $m$ as a sum of Fibonacci numbers:

$$
m=a_{1} F_{1}+a_{2} F_{2}+\cdots+a_{n} F_{n}, \quad a_{i} \in\{0,1,2, \ldots\} .
$$

We define the index of the decomposition by

$$
\operatorname{index}(D(m)):=a_{1} \cdot 1+a_{2} \cdot 2+\cdots+a_{n} \cdot n
$$

and the number of summands by

$$
\# \operatorname{summands}(D(m)):=a_{1}+a_{2}+\cdots+a_{n} .
$$

Thus if we write

$$
64=5 F_{1}+7 F_{2}+3 F_{5}+1 F_{7}=5 \cdot 1+7 \cdot 2+3 \cdot 8+1 \cdot 21,
$$

its index is 41 and the number of summands is 16 . We denote the Zeckendorf decomposition of $m$ by $Z(m)$.
(continued)

[^0]Theorem. If $D(m)$ is any decomposition of $m$ as a sum of Fibonacci numbers, then

$$
\# \operatorname{summands}(Z(m)) \leq \# \operatorname{summands}(D(m))
$$

Proof. The proof follows by showing that we can transform any $D(m)$ into $Z(m)$ without increasing the number of summands by doing two moves.

1) Replace $F_{n-1}+F_{n}$ with $F_{n+1}$ if $n \geq 2$. In other words, if we have two adjacent terms, use the recurrence relation.
2) Replace $2 F_{n}$ with $F_{n-2}+F_{n+1}$ if $n \geq 3$. This is permissible as

$$
2 F_{n}=F_{n}+F_{n}=F_{n-2}+F_{n-1}+F_{n}=F_{n-2}+F_{n+1} .
$$

If $n \leq 2$, replace $2 F_{2}$ with $F_{1}+F_{3}$ and replace $2 F_{1}$ with $F_{2}$.
Note that in all cases and all moves the number of summands decreases or is unchanged.
Let's look at what happens to the index (our monovariant) when we apply the transformations. After a type (1) move the index decreases by $(2 n-1)-(n+1)=n-2$, except when $n=2$ in which case the number of summands that are $F_{1}$ and $F_{2}$ decreases. After a type (2) move, the index decreases by $(2 n)-(2 n-1)=1$, except if we're in the last two cases of $2 F_{2}$ or $2 F_{1}$. In those situations the index remains unchanged; however, the number of summands that are $F_{1}$ or $F_{2}$ decreases. Thus, either the index decreases or we decrease the number of $F_{1}{ }^{\prime} s$ and $F_{2}{ }^{\prime} s$ and then iterate.

Since the index cannot decrease indefinitely, and there are only finitely many decompositions whose index is at most a given amount, we can only perform finitely many moves and thus the process must terminate. Once we reach a decomposition where we cannot apply either move, then we do not have any multiple summands and we do not have any adjacent terms - thus we must have reached the Zeckendorf decomposition!

## References

[1] J. L. Brown, Jr., Zeckendorf's Theorem and Some Applications, The Fibonacci Quarterly, Vol. 2, No. 3 (Oct. 1964), pages 163-168.
[2] J. L. Brown, Jr., A New Characteristic of the Fibonacci Numbers, The Fibonacci Quarterly, Vol. 3, No. 1 (Feb. 1965), pages 1-8.
[3] K. Cordwell, M. Hlavacek, C. Huynh, S. J. Miller, C. Peterson, and Y. N. Truong Vu, On Summand Minimality of Generalized Zeckendorf Decompositions, https://arxiv.org/abs/1608.08764, preprint, 2016.
[4] D. E. Daykin, Representation of Natural Numbers as Sums of Generalized Fibonacci Numbers, J. London Mathematical Society, Vol. 35 (1960), 143-160.
[5] H. H. Ferns, On the Representation of Integers as Sums of Distinct Fibonacci Numbers, The Fibonacci Quarterly, Vol. 3, No. 1 (Feb. 1965), pages 21-29.
[6] P. Monteiro and R. W. Newcomb, Minimal and Maximal Fibonacci Representations: Boolean Generation, The Fibonacci Quarterly, Vol. 14, No. 1 (Feb. 1976), pages 9-11.
[7] E. Zeckendorf, Représentation des nombres naturels par une somme des nombres de Fibonacci ou de nombres de Lucas, Bulletin de la Société Royale des Sciences de Liége, Vol. 41 (1972), pages 179-182.

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## BOOK REVIEW

Algebra in Context, by Amy Shell-Gellasch and J. B. Thoo Reviewed by Walter Meyer, Emeritus Professor, Adelphi University

This unusual and interesting book is a textbook for students whose mathematical education has stopped short of calculus but who need some additional mathematics, for general education or for studies in education for example. Its main and important innovation is the generous addition of historical information. It has been clear for some time that students in these sorts of courses often lose interest in a course with purely mathematical content, and the idea of perking it up with history will likely be effective with many students. Students will see that mathematical ideas do not come on tablets from on high, but are devised by human beings embedded in various cultures.

Here is how the book breaks down along mathematical lines ${ }^{1}$ : About 135 pages on numeration systems and arithmetic (including methods devised by the Greeks, Babylonians, Indians, Chinese, Mayans, Native North Americans); 42 pages on Logic, Set Theory and Rational, Irrational and Real Numbers; 56 pages on Number Theory; 250 pages of conventional college algebra including: linear and quadratic equations, a bit about the Theory of Equations (e.g., Descartes Rule of Signs), Logarithms, and the Rule of Three (proportions).

The authors often show how different sources or authors from long ago have handled the same problem. For example, in the matter of extracting square roots, we find methods due to Heron of Alexandria, to Theon of Alexandria, to the Bakhshali Manuscript of India and finally to the Frenchman Chuquet. These discoveries took place over a millennium and in diverse places, testifying that the appeal of mathematics is durable and universal. The exposition of a method is typically peppered with "Now You Try It" challenges to the reader to repeat a computation and more general "Think About It" questions that are open-ended and can lead a student, if he or she is successful, to having the feeling of actually doing creative mathematics. Maps and photographs are plentiful, emphasizing that both the mathematics and the historical details are real.

In the chapter on Linear Problems about $2 / 3$ of the page count consists of pages which are heavily historical. In doing this count, I found it hard to separate the history from the mathematics. Historical material is not simply sprinkled on like croutons in your soup, where it is easy to eat around them. For example students are asked not only to repeat old style calculations but to compare how the calculation would go using different methods. The chapters vary somewhat in the intensity of historical material. For example the chapters on foundations have less than the chapter on Linear Problems.

In regard to Babylonian mathematics, the authors point out that this is an active area of research because of the ongoing work in archeology and translation.

Word problems from historical sources are often mentioned and, in the grand old tradition, they are often more playful than real. However, in Chapter 23 (The Rule of Three) and Chapter 24 (Logarithms), there are problems (e.g., compound interest) that carry the whiff of real present-day usefulness. There is some material I have not seen before devoted to the Richter scale for earthquakes as applied to the San Francisco earthquake of 1906.

This is a book well worth trying. You don't need to know anything about history to begin with, but you will learn a lot about it and so will your students.

[^1]
## MAA Student Chapters and Departmental Membership

David Seppala-Holtzman, St. Joseph's College
We professional mathematicians understand the benefits of "citizenship" in the mathematical community. Here we keep abreast of mathematical news, receive intellectual stimulation and find likeminded colleagues with whom to interact. For the most part, the entire concept of a mathematical community is foreign to our students. In order to make students aware of the existence of such a community and to help them come to appreciate its value, the MAA established the Student Chapter Program. Unfortunately, this required students to pay for membership out of their own pockets, albeit at highly discounted rates. Students who had not yet come to see membership as having any value at all would be disinclined to join. This inhibited the growth of the program.

Now, with the restructuring of the entire benefits fee structure, a solution may be at hand. The MAA has created Departmental Membership with the following benefits: One faculty member (the Departmental Membership Administrator) gets full membership privileges:
1 Membership in the local MAA Section
( Online subscriptions to:

- The American Mathematical Monthly (10 issues per year)
- The College Mathematics Journal (5 issues per year)
- Mathematics Magazine (5 issues per year)
- Math Horizons (4 issues per year)
- MAA FOCUS (6 issues per year)
(1) The Membership Administrator will receive a print subscription to MAA FOCUS, MAA's newsmagazine (published 6 times per year).
All faculty members in the department receive \$100 off every hosted WeBWorK course.
Administrator and Student Nominees will also receive MAA Math Alert, the MAA's monthly enewsletter, filled with the latest news and happenings from across our mathematical community.
(1) Student and Administrator Members receive discounts on MAA books (typically 20\% off list price), and discounted registration for MAA MathFest and the Joint Mathematics Meetings (JMM).
(1) Student and Administrator Members may join any of MAA's Special Interest Groups (SIGMAA's).

The Departmental Membership Administrator may nominate any number of mathematics students (undergraduate and graduate) for MAA membership at no additional cost. The students you nominate become MAA members with these benefits:
( Online access to all MAA journals: The American Mathematical Monthly, Mathematics Magazine, and the College Mathematics Journal
(1) Online access to Math Horizons
(1) Online access to MAA FOCUS
(1) Online access to archives for all MAA journals.
(1) Access to the online Member Library, featuring a selection of MAA's outstanding books

Discounts on meeting registration fees at both MAA MathFest and the Joint Mathematics Meetings
(1) Discounts on purchases of MAA books

The fee to become a Departmental Member is based upon the size of the academic institution in question as well as whether or not it is a Ph.D. granting institution. For more information and how to apply, visit www.maa.org. With this new procedure in place, it is now quite inexpensive and easy to bring large numbers of your students into the mathematical community. Once they become members, they are quite likely to see the benefits of having joined. There are, of course, the tangible benefits, listed above, that include access to journals, archived materials and discounts. But, just as importantly, they will come to feel a part of something larger, a community of like-minded people. Once they come to appreciate that, your students are apt to remain members for the rest of their lives.

## CALL FOR PARTICIPANTS AND INVOLVEMENT

## Go Back to School, Join The Mathematics Speakers Bureau!!!

Do you have a talk which would be suitable for local area students or their faculty? We are seeking mathematicians interested in sharing their knowledge, enthusiasm, and love of mathematics. Now in its $54^{\text {th }}$ year, the Mathematics Speakers Bureau (MSB) is composed of dedicated mathematicians who volunteer to speak to students and faculty of regional middle schools, high schools, colleges and universities on topics reaching beyond the traditional mathematics curriculum.
The primary goals of the MSB are to stimulate the interests of local youth in mathematics, to provide opportunities for students to meet active and enthusiastic mathematicians, to motivate students towards careers in the mathematical sciences, and to encourage cooperation between corporate and academic institutions in the mathematical education of area youth. Volunteers provide information about talks they are willing to give and the Bureau, in turn, advertises these talks to the faculty of local area schools. Schools contact speaker volunteers directly to make specific arrangements for a visit. Volunteers determine the number of presentations they give in any given academic year and always maintain the right to decline any invitation to speak. The Bureau web-page (sections.maa.org/metrony/speakers.html) contains an up-to-date listing of available speakers and their proposed talks. Additional information regarding the goals, history and operation of the Bureau can also be found at this site. If you wish to volunteer with the MSB, please contact Bureau Chair Dan King at dking@sarahlawrence.edu.

## MetroMath Needs You!!!

Consider submitting a short announcement, commentary, article, study, experience, or other newsworthy item in the next issue of MetroMath. Contact the editor, Abe Mantell, via e-mail: mantell@ncc.edu.

## MATH IN THE NEWS FROM THE MAA

(much more can be found at: http://www.maa.org/news and http://mathdl.maa.org)

## U.S. Team Wins First Place at the European Girls' Mathematical Olympiad

The U.S. team took first place at the European Girls' Mathematical Olympiad (EGMO) held April 6-12, 2017 in Zürich, Switzerland. In addition to the top team honors, each member of the four-person team was awarded a gold medal for their individual performance on the challenging mathematics exam.

The EGMO is an international mathematics competition for female high school students. This year's competition had 168 students from 44 teams. The U.S. team is organized by the Mathematical Association of America and has consistently placed in the top four teams with a second place team finish in 2016.

The 2017 EGMO U.S. team members are Angela Deng, Siye Zhu, Qi Qi, and Wanlin Li. Sherry Gong served as team leader and Jenny Iglesias as deputy leader.

The exam is made up of six proof-style problems given over two days. The U.S. team score was 148 out of a possible 168 points. Qi was one of only two overall contestants to receive a perfect score of 42 points on the exam.
(continued)
"We are proud of the students on our team who have worked very hard to prepare for this competition," said Sherry Gong, leader of the U.S. EGMO team. "This was a challenging exam so we are especially honored to have all four students take home individual gold medals."


#### Abstract

"The Mathematical Association of America is excited to sponsor this team of America's top math students and on behalf of the mathematical community we congratulate you on your first place finish," said Michael Pearson, the executive director of the Mathematical Association of America. "The problem-solving skills you demonstrated at this year's EGMO point to a bright future for you and mathematics."


For complete scores, visit https://www.egmo.org/egmos/egmo6/scoreboard/

## U.S. Wins First Place at International Mathematics Competition in Hong Kong

For the second year in a row, the U.S. team won first place at the 57th International Mathematical Olympiad (IMO) in Hong Kong, July 6-16.

The IMO is the World Championship Mathematics Competition for High School students, where the brightest mathematics students from more than 100 countries compete. The winning U.S. team score was 214 out of a possible 252, ahead of the Republic of Korea (207) and China (204).
"We are very excited to bring home another first-place IMO award, which serves as a recognition for the the high standard of mathematical creativity and problem-solving capabilities we have in our country,"said Po-Shen Loh, lead coach for the U.S. team and associate professor of mathematics at Carnegie Mellon University.

The six U.S. team members were selected through a series of competitions organized by the Mathematical Association of America (MAA), culminating with the USA Mathematical Olympiad. The six team members joined 70 of their peers at Carnegie Mellon University in June to immerse themselves in problem solving for three weeks at MAA's Mathematical Olympiad Summer Program.
"We have been running the U.S. Olympiad training program with a focus on the long-term development of our country's talent, and it's great to see that reflected in the continued team success a second year in a row," said MAA Executive Director Michael Pearson.

Members of the winning 2016 U.S. team were Ankan Bhattacharya, Michael Kural, Allen Liu, Junyao Peng, Ashwin Sah, and Yuan Yao, all of whom were awarded gold medals for their individual scores. Team members Liu and Yao each earned perfect test scores. The team was accompanied by Loh and deputy coach Razvan Gelca, professor of mathematics and statistics at Texas Tech University.

IMO scores are based on the number of points scored by individual team members on six problems. On each day of the two-day competition, the teams have 4.5 hours to work on three problems. Liu and Kural are the only returning team members from last year's winning U.S. team.

## MAA Selected for 'Best Writing on Mathematics'

We are happy to announce that eight articles and essays from MAA publications have been selected in The Best Writing on Mathematics 2016, a Princeton University Press collection.

This year editor Mircea Pitici included published pieces from MAA publications Math Horizons, The College Mathematics Journal, and I, Mathematician. Congratulations to all the authors who have been recognized for this prestigious selection.

MAA articles selected:
Mathematics and Teaching, by Hyman Bass. Originally published in I, Mathematician, edited by Peter Casazza, Steven G. Krantz, and Randi D. Ruden. Washington, DC: Mathematical Association of America, 2015, pp. 129-139.

The Way the Billiard Ball Bounces, by Joshua Bowman. Originally published in Math Horizons 22.3(2015): 18-22.

The Intersection Game, by Burkard Polster. Originally published in Math Horizons 22.4(2005) 8-11.
Tonight! Epic Math Battles: Counting vs. Matching, by Jennifer J. Quinn. Originally published in Math Horizons 22.3(2005): 5-9.

The Pioneering Role of the Sierpinsky Gasket, by Tanya Khovanova, Eric Nie, and Alok Puranik. Originally published in Math Horizons 23.1(2015): 5-9.

Fractals as Photographs, by Marc Frantz. Originally published in Math Horizons 23.1(2015): 18-21.
Circular Reasoning: who First Proved that C Divided by d Is a Constant?, by David Richeson. Originally published in College Mathematics Journal 46.3(2015): 162-171.

The Spirograph and Mathematical Models from Nineteenth Century Germany, by Amy Shell-Gellasch. Originally published in Math Horizons 22.4(2015): 22-25.

## EVENTS CALENDAR

Metropolitan New York Section Meeting • April 29, 2017, Hostos Community College (CUNY), Bronx, NY For more information see this newsletter, or visit: sections.maa.org/metrony/

AMS Spring Eastern Sectional Meeting • May 6-7, 2017, Hunter College (CUNY), New York, NY For more information visit: www.ams.org/meetings/sectional/2242_program.html

AMTNYS Summer Conference • July 9-12, 2017, Siena College, Loudenville, NY For more information visit: www.amtnys.org/Sub\ Pages/Conferences/Summer\ Conference.html

SIAM Annual Meeting • July 10-14, 2017, Pittsburgh, PA
For more information visit: www.siam.org/meetings/an17/
MathFest • July 26-29, 2017, Chicago, IL • For more information visit: www.maa.org/meetings/mathfest
Seaway Section Meeting • October 20-21, 2017, SUNY Broome CC, Binghamton, NY For more information visit: people.rit.edu/maacway/
AMTNYS 67 ${ }^{\text {th }}$ Annual Fall Conference • November 3-4, 2017, Buffalo, NY For more information visit: www.amtnys.org/Sub\ Pages/Conferences/Fall\ Annual\ Conference.html

AMATYC $43{ }^{\text {rd }}$ Annual Conference • November 9-12, 2017, San Diego, CA For more information visit: amatyc.site-ym.com/?2017ConfHome
NCTM Regional Conference and Exposition • November 29 - December 1, 2017, Chicago, IL For more information visit: www.nctm.org/chicago/

MAA-AMS Joint Mathematics Meeting • January 10-13, 2018, San Diego, CA
For more information visit: www.maa.org/node/87/

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[^2]

ANNUAL MEETING SATURDAY, 29 APRIL 2017

## Hostos Community College (CUNY) <br> Bronx, NY

INVITED SPEAKERS
Carl Simon, University of Michigan
Mathematics is Contagious and Contagion can be Mathematical
Rosa Orellana, Dartmouth College Symmetry and Coloring

Ron Buckmire, National Science Foundation
Funding Opportunities at the National Science Foundation in Mathematical Sciences

## STRATEGIES AND RESEARCH in TEACHING

Rosa Orellana - Teaching via Guided Discovery
Danté Tawfeeq - Helping Students in Lower Division Mathematics Courses
Bronislaw Czarnocha - The Creativity of Teaching-Research Methodology TR/NYCity Model
Ron Buckmire - Formative Summative Assessments: Some of My Favorite Exam Questions

CONTRIBUTED PAPER AND POSTER SESSIONS
Research, Pedagogical, and Student Presentations

## FOR MORE INFORMATION PLEASE VISIT OUR WEBSITE AT <br> sections.maa.org/metrony

## Join Metro NExT!

Metro NExT (New Experiences in Teaching) is a local version of Project NExT, a professional development program for new or recent Ph.D.s in mathematics. Our goal is to build a community of new faculty and graduate students in the Metro New York MAA Section to help each other develop effective strategies for all parts of our professional lives from teaching to research to service, all the while enjoying socializing and getting to know each other.

## Eligibility and Application

Metro NExT is open to all Metropolitan New York MAA Section faculty members holding a Ph.D. in mathematics or mathematics education and having full-time, pre-tenure status at a post-secondary institution. Metro NExT is also open to all graduate mathematics or mathematics education students in their final two years of doctoral study.

To become a 2017 Metro NExT fellow, please fill out the following application form by Friday, March 31, 2017.

## http://tinyurl.com/metronext2017

All Metro NExT fellows are expected to attend Metro NExT sessions during the annual section meeting (Saturday, April 29, 2017 at Hostos Community College). Fellows' annual meeting registration fee will be waived.

## Upcoming Metro NExT Events

## Welcome Brunch: March 12, 2017 (tentative, location TBA)

A casual brunch and a Pi Day themed celebration to meet other Metro NExT fellows!

## Annual Meeting of the Metro NY Section: Saturday, Aprit 29, Hostos Community College

 Metro NExT Sessions at the Annual Meeting (3:15-5:15pm):- Funding Opportunities at the NSF in Mathematical Sciences (Ron Buckmire, NSF)
- Panel Discussion on Diversity in the Mathematics Classrooms
(Panelists: Trace Jordan, NYU; Janet Liou-Mark, City Tech; Dante Tawfeeq, John Jay College)


## Questions?

For more information, please contact any of the organizers of the Metro NExT program:
Johanna Franklin (lohanna.N.Franklin@hofstra.edu), Tia Sondjaja (sondjaja@nyu.edu), or Johann Thiel (ithiel@citytech.cuny.edu)


[^0]:    ${ }^{1}$ This work began in a conversation the fourth named author had after one of his talks at HCSSiM with Patrick Dynes, and was completed by the authors at the United States Military Academy with travel support provided by the Center for Leadership and Diversity in STEM, whom we thank for their support. P. E. Harris was partially supported by a National Research Council Research Associateship Award at USMA/ARL, and S. J. Miller by NSF grants DMS1265673 and DMS1561945. We thank Peter Anderson, Curtis Cooper, Patrick Dynes, Clark Kimberling and Florian Luca for helpful discussions.

[^1]:    ${ }^{1}$ In each topic, our rough page count includes historical material.

[^2]:    * MetroMath accepts advertising at $\$ 50$ for a half-page ad and $\$ 100$ for a full-page.

