2008 Annual Spring Meeting of the Metropolitan New York Section of the MAA

The Courant Institute of Mathematical Sciences
New York University
Saturday, 3 May 2008

Abstracts of Contributed Papers and Posters

Research Presentations

Presider: Jerry G. Ianni, LaGuardia Community College

1:20 – 1:40 PM: ***The Spirit of Discovery: The Digital Roots Of Integers***

Eric Milou and Jay L. Schiffman, Rowan University

In numerous mathematics classes, students are being asked to learn mathematics in the spirit of discovery. In such a manner, the hope is that the intrinsic beauty of mathematics is accessible to all students and that making conjectures, forming hypotheses, and analyzing patterns can help students compute fluently and to solve problems creatively and resourcefully (NCTM, 2000). In the set of activities discussed in this paper, our students' (pre-service teachers) objectives included: examining patterns and making conjectures, using data analysis to construct line plots and tables all in the spirit of discovering mathematics. In order to place our activity in perspective and incorporate the ideas alluded to above, we consider an idea by Neil J. A. Sloane (1973) called the multiplicative digital root of an integer. All integers referred to in the paper unless otherwise qualified shall connote positive integers. Our problem is concerned with generating a possible next term in the following integer sequence: $6788 \rightarrow 2688 \rightarrow 768 \rightarrow 336 \rightarrow 54$. This problem was posed by the authors to students enrolled in a course designed for pre-service teachers at a mid size university in the Mid-Atlantic Region as well as participants engaged in in-service workshops. We thus initiate our journey to place this seemingly innocent appearing sequence in perspective and exhibit the richness of the mathematics this is ripe for discovery by students. The activity endeavors to stimulate meaningful discovery by having participants collect data, employ technology, explore patterns, form conjectures based upon the analysis of such patterns, employ counting tools, and improve computational proficiency.

1:40 – 2:00 PM: ***The Optimal Form of Distribution Networks Applied to the Kidney and Lung***

Walton R. Gutierrez, Touro College

A model is proposed to minimize the total volume of the main distribution networks of fluids in relation to the organ form. The minimization analysis shows that the overall exterior form of distribution networks is a modified ellipsoid, a geometric form that is a good approximation to the external anatomy of the kidney and lung. The variational procedure implementing this minimization is similar to the traditional isoperimetric theorems of geometry.

This paper was published at the Journal of Biological System, Vol. 15, No. 4, 419-434, 2007. (December issue) World Scientific Publishing. http://www.worldscinet.com/jbs/jbs.shtml

2:00 – 2:20 PM: ***Sabermetrics: The Math Behind Baseball,*** Allen Paltrow

Major League Baseball is the most recorded sport in history. Every pitch, catch, win, run, hit, or steal is painstakingly recorded. The amount of data lying around for the MLB this century is astronomical. What do you get when baseball fans and rogue statisticians meet? The branch of mathematics known as sabermetrics, the objective mathematical study of baseball. Practitioners of sabermetrics believed that in the piles and piles of baseball data lying around, there were objective truths of baseball to be found. If one were to look close enough, he believed, correlations, formulas, and patterns would emerge, not only allowing you to get an objective look at past games and players, also allowing to use data to actually predict future results. Why Baseball? If a quarterback successfully completes a 30 yard pass to a touchdown, is that a measurement if his skill, or the skill of the linebacker who tackled an on coming player to defend the pass? Baseball uniquely tests each player individually, allowing for a pure base of statistics. Since Lewis, statistics have become a large part of baseball. A large part of sabermetrics is combining simple statistics (ie: runs, hits, batting average, at bats) to make bigger (and hopefully more significant) ones. Among the most popular sabermetric creations (most not covered in this paper) are Base Runs (BsR), Defense Independent Pitching Statistics (DIPS), Equivalent average (EQA), Late-inning pressure situations (LIPS), On-base plus slugging (OPS), The Pythagorean expectation, and Value over replacement player (VORP).

2:20 – 2:40 PM: ***On Gray Code, Signed Iteration and Cantor Sets,*** Matthew Manilow

Signed iteration, i.e. $\pm g(\pm g(\ldots \pm g(x)\ldots))$, of the line $g(x) = ax + b$ (and its inverse $h(x) = 1/(ax+b)$) 'decodes' the binary reflected Gray code (referred to as "the" Gray code) in the following sense, e.g.: $-g(+g(-g(-g(x)))) = (-a^0 - a^1 + a^2 - a^3)b - a^4 x.$

Associating $+$ with 0, and $-$ with 1, the sign sequence on the left (from left to right) is 1011, which is the Gray code for 13, and on the right, in order of ascending powers of $a's$ in the parentheses, it is 1101, which is binary 13. A similar feedback pattern holds for the signs of b as the partial quotients of the continued fractions generated by the signed iteration of $h(x)$. The general cases are proven by induction.

Given a continuous, strictly monotonic increasing function with a single fixed point *p* and positive in the closed interval $[-p, p]$ of the upper half-plane (i.e. $0 < f(-p) < f(p)$), then the set of all signed iterates of $f(p)$ is a Cantor set in $[-p, p]$, with the points ordered by the associated Gray codewords.

Middle-alpha Cantor sets, generated by signed iteration of $ax+b$, provide an example. Strictly monotonic decreasing, and negative valued functions yield symmetric results. Gray code associations also occur with the logistic equation, and in Farey and other trees of fractions. In every association the code is all 1's at points that are in some sense minimal. Signed iteration of complex functions, generates Mandelbrot and Julia set-type fractal images (with the sign sequence considered as an embedded binary code) immediately raising many questions, e.g. Hausdorff dimensions, symmetries, further Gray code, or other binary code associations, if any, and inviting computer graphic experimentation. Different sign sequences used in the iteration of the same polynomial produce Mandelbrot and Julia set-type fractal images, each with the same respective symmetries, but different individual characteristics.

2:40 – 3:00 PM: ***Further Relationships between Chebyshev Polynomials Fibonacci and Lucas Numbers and other Functions of Interest***
Harvey J. Hindin, Emerging Technologies Group

The connection between Chebyshev Polynomials of the first and second kind (for certain real and complex arguments) to Fibonacci and Lucas numbers has been well-established in the periodical literature, books, and on the web. Many relationships, curiosities, identities, finite and infinite summation and product evaluations, and other mathematical facts have been derived, or hinted at in these sources. However, it turns out; only the surface has been scratched.

This talk provides additional mathematical facts of the types just mentioned as well as some which do not fit into these types. Some of these facts are believed to be new. On the other hand, as in all work of this type with a large literature, it is not possible to be 100 percent sure. Within the time allotted, these facts are provided along with proofs, or outlines of proofs, of their existence. Both elementary and advanced mathematical concepts are used, as needed. Background material will be provided as well as an extensive bibliography. Part of this talk will use the fact that certain special functions (e.g., the Chebyshev polynomials) are a subclass of a more general class of function, and can be expressed as hypergeometric functions. This relationship, in turn, leads to still more beautiful and often unexpected, mathematical facts of inherent interest. All of this is related to the solutions of second order difference equations of various kinds. The talk concludes with suggestions for further work for interested parties.

After listening to this presentation, the audience will have a better understanding of the many relationships mentioned and or discussed. In addition, many attendees will likely have been exposed to a kind of analysis that they have not hitherto seen. Their mathematical perspective will be broadened thereby.

3:00 – 3:20 PM: ***Unraveling the Complexity of the Metatarsal Length Pattern of the Human Foot***
Phillip H. Demp, Temple University

The primary objective of this pilot study was to generate a shape classification of the metatarsal length patterns (MLP) by using a conic curve model. A clinical objective was to determine a mathematical basis for shortening or lengthening one or more metatarsals by osteotomy in order to aid the surgeon in determining the amount of bone needed to be resected to correct or prevent a pathomechanical problem. The differences among the MLPs were quantified by using rectangular coordinates of the five metatarsal heads (from dorsoplantar, weightbearing radiographs) to determine unique conic curves, their unique eccentricities and their conic types. A comparison was made between the MLPs of nonhuman Primates and the MLPs of modern humans. It was shown that those modern humans whose MLPs exhibit an ellipse are considered to have an atavistic pattern which translates into a diagnosis of pathomechanical MLP. This approach produced a model of metatarsal geometry that yields a unique parameter (eccentricity) that one may use to quantitatively distinguish MLPs between healthy and pathological populations within modern man. This study has now been extended as a research project funded by the NIH.

NOTE: This is an example of how a combination of elementary mathematics and an anatomical pattern can produce an exciting tool for clinical decision making. Simplified models that demonstrate clinical utility fall within the scope of undergraduates.

Pedagogical Presentations

Presider: Emad Alfar, Nassau Community College

1:20 – 1:40 PM: ***Emergent Spatial and Geometric Thinking: A Comparative Study of Young American and Chinese Children's Block Play***

Chia-ling Lin, Nassau Community College & Daniel Ness, Dowling College

Given a strong association between Lego and block play and mathematical activity particularly related to geometric forms, can free play within a Lego and block environment further inform us about young children's mathematical thinking? How do young American children compare with Chinese children of the same age regarding geometric and other forms of mathematical knowledge? Few studies compare East Asian and American children's involvement in spatial and geometric thinking activities. The aim of this study is to contribute to an understanding of emergent geometric thinking through a comparison of American children and Chinese children during Lego and block play. This study employs naturalistic observation methodology. We obtained videotapes of young children's everyday behavior during Lego and block play. All the videotaped observations were coded, and statistical analyses were performed on the coded material to compare the two groups. The contributions of this investigation yield a number of implications from both a developmental and cognitive perspective. First, it is evident that children possess a powerful intellectual capacity well before they enter formal schooling, and a great deal of this competence has to do with mathematical thinking. Second, the use of a spatial and geometric coding system may afford researchers with more insight on young children's spontaneous spatial and geometric knowledge from the perspectives of age, gender, socioeconomic class, and cross-cultural comparisons. Third, the outcome of this study does not reflect Piaget's topological primacy thesis in terms of the types of spatial and geometric activities in which four- and five-year-old children are involved. Through this investigation, educators can become more cognizant of the origins of young children's geometric knowledge and its development.

1:40 – 2:00 PM: ***Mathematics and Critical Thinking Interdisciplinary Pair: Advantages and Benefits,*** Yasser Hassebo & Judit Torok, LaGuardia Community College

Students in basic math classes face many challenges. They are not only trying to grasp the material, but they often lack good study skills, and they suffer from serious math anxiety. Students in a critical thinking class often struggle with supporting their viewpoints with valid reasons and data. To tackle these challenges we worked in a paired learning community as a team of Introduction to Algebra (MAT096) and Critical Thinking (HUP102) through real-life joined assignments. The relationship between toxins, pollution and asthma in New York City was used as a framework in our assignments.

- Mathematics enhances critical thinking skills: The students' critical thinking projects were considerably enhanced. In the mathematics class they were given real-life problems, data, graphs, and they implemented the mathematical concepts into the critical thinking project. Therefore, they were able to use mathematical evidence to support their claims in debates and arguments.
- Critical thinking improves mathematical skills: A significant improvement in mathematical skills and student attitudes towards math has been observed. In critical thinking, students worked with topics related to math anxiety, problem solving skills and motivation.

This presentation explains how the students, faculty, and college benefited from this interdisciplinary pairing and instructors' collaboration. We will highlight how these two courses can best support each other.

2:00 – 2:20 PM: ***The Cauchy Functional Equation in Teaching Statistics***

Alexander Vaninsky, Hostos Community College

It is suggested to use Cauchy functional equation $f(x+y)=f(x)+f(y)$, its solution $f(x)=kx$, and other functional equations that follow from it as a theoretical base for a statistical procedure aimed at finding a rate of magnitude of an observed phenomenon: linear, polynomial, or exponential, correspondingly.

An objective is to demonstrate that mathematics, statistics, and spreadsheet data processing form a logical chain. Starting with Cauchy functional equation $f(x+y)=f(x)+f(y)$, that implies $f(x)=Cx$ for continuous functions, we expand it further to $f(xy)=f(x)f(y)$, and $f(x+y)=f(x)f(y)$, that imply $f(x)=xC$ and $f(x)=Cx$, respectively. Each of these equations represents three different rates of expansion of an observed phenomenon: slow, moderate, or avalanche-like. Examples help understanding practical importance of the right rate estimation: AIDS, bird-flue, population living in poverty etc. A spreadsheet is developed that calculates statistics of the paired *t*-test, and thus, allows for determination of an actual rate of expansion. Malthusian theory of population growth is used as an example.

2:20 – 2:40 PM: ***Asymptotes of a logarithmic function can be hazardous to your financial health***
Frank Wang, LaGuardia Community College/CUNY

Since 2006, many major financial institutions have raised monthly credit card minimum payment from 2% to 4% of the balance to respond to government's crackdown on unfair business practices. Earlier this year, Senator Dianne Feinstein of California introduced legislation requiring credit card companies to inform consumers of the costs of paying only the monthly minimum payment. From a mathematical point of view, banks are simply exploiting the asymptotic behavior of the logarithmic function to make profits. We will analyze these news items, and provide educators with materials to inform students that a credit debt could turn into a decade-long commitment, generating thousands more in interest in the meantime. In this process, we illustrate that the logarithmic function, which has a vertical asymptote, is the inverse function of the exponential function. Additionally, Senator's legislative effort offers educators a wonderful opportunity to demonstrate how mathematical knowledge can empower citizens.

2:40 – 3:00 PM: ***Mathematics Exams in the CLEP Program: Their Relevance to the College Curriculum***, Robin O'Callaghan, The College Board (NY)

This paper will discuss the test development process for the four CLEP mathematics exams (College Algebra, College Mathematics, Precalculus, and Calculus). The presenter will describe the curriculum surveys that are designed to keep the exams relevant to current classroom practices, the setting of test specifications and standards, and the work of college faculty committees in guiding and reviewing the assembly of the exams. A brief discussion of the role of the online calculator in the Precalculus exam will also be included.

3:00 – 3:20 PM: ***Wonder, Discovery and Intuition in Elementary Mathematics***
Andrew Grossfield, Ph. D., P. E., Vaughn College

A major problem today concerns educating the next generation of engineers, mathematicians and researchers. The value of our concentration on drilling and testing appears questionable. Some students perform well on tests but do not understand why the various algorithms work. Others do poorly, become overwhelmed and give up with feelings of hopelessness. Conceivably, computational ability may not reflect mathematical insight or be a reliable measure of creativity. With readily available calculators, the next generation may not be well served if young people are trained and judged on the speed and accuracy of their computations. Perhaps we should be nurturing insight, analytical judgment and the ability to recognize errors.

Unfortunately, current public discussion involves repeated testing and not the genuine joy of discovery. Areas of mathematics contain wonderful concepts and ideas, which can pique the natural curiosity of young students and enchant them into furthering their mathematical studies. The inner structure of mathematical objects, properly introduced, should suffice to captivate young minds.

My proposition is that with appropriate additional explanation, many youngsters could comprehend and take delight in visual presentations of mathematical concepts. Why not start the analytical education of our children early in elementary school? Accordingly, I have been designing sets of slide shows with the aim of providing a visual framework, which will illuminate the essence of some mathematical concepts.

I will show two examples of the slide sets, both providing visual representations of the edge between arithmetic and elementary algebra: One is called Fast Counts; the other Basic Laws.

Each of the slide shows introduces a surprising mathematical fact. In the spirit of mathematical strategy, the progression of slides builds on established facts to explore the unknown. It is hoped that students who view the slides might develop the insight, intuition and confidence needed to successfully explore other analytical situations.

Student Presentations

Presider: David N. Seppala-Holtzman, St. Joseph's College (Brooklyn)

1:20 – 1:40 PM: ***Introducing k-long Numbers,*** Anthony Delgado, Purchase College
Advisors: Michael L. Gargano, Marty Lewinter, Joseph F. Malerba

An integer is oblong if it can be written $n(n+1)$ for some positive integer n. An integer is k-long if it is of the form $n(n+k)$, where k is nonnegative. This generalizes oblong numbers, all of which are 1-long. A number can be k-long for several values of k. A k-long number is strictly k-long if it is not j-long for any j satisfying $0 \leq j < k$. We show that given any $k > 0$, there exists an integer N such that for all $n > N$, the numbers $n(n+k)$ are strictly k-long. This implies the following. Given $n > 1$, let $d(n)$ be the smallest positive difference, $|a-b|$, among all ways to write n^2 as ab. Then $d(n)$ goes to infinity as n does.

1:40 – 2:00 PM: ***Symmetries of the Five Platonic Solids***
George Kuster, Stephanie Kenny & Alicia Bridgwood, St. Joesph's College (Patchogue)
Advisor: Professor Skenderi

The purpose of this research paper is to investigate the symmetries of the five platonic solids. In order to study mathematical group symmetry, we focused on the rotational symmetries of the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. To help visualize the symmetries we have constructed models of these figures. By studying the symmetries of these groups we realized each is isomorphic to another group, either the symmetric group or alternating group of n elements. In addition, we have used Maple to gather similar information from our study of the models. Using the models we have determined the elements of each group. There are 24 rotational symmetries for the tetrahedron and cube, and 60 rotational symmetries for the isocahedron and the dodecahedron. In addition to these two pairs being bijective with other groups, the pairs themselves also form dualities. If you inscribe the cube with the tetrahedron, the axes of symmetries are the same. The same principal applies to the dodecahedron and the isocahedron. We will show that the rotational groups of symmetries, can be shown to be bijective with their corresponding symmetric groups or alternating group just by proving they are on to. Since each of the corresponding groups have the same finite number of elements, we only have to show that they are on tobecause, intuitively they are already one to one. This is true because they have the same number of elements.

2:00 – 2:20 PM: ***Phi Patterns in Nature and Beyond***
Leigh Johnson, Theresa Sampson & Heather O'Connell, St. Joseph's College (Patchogue)
Advisor: Dr. Donna Marie Pirich

While conducting research for Senior Seminar in Mathematics, three students find particular interest in, "The Distance of the Planets from the Sun and Their Atmospheric Composition," by Charles William Johnson. In this paper, Johnson postulates the existence of a Phi pattern in the distances of the planets from the Sun, if Ceres is included as a "dwarf planet" representative of the asteroid belt between Mars and Jupiter. The student authors validate Johnson's work with data retrieved from NASA databases. However, the question remains, "What happens if Ceres is not included?" The student authors answer this question using linear regression, tables, and graphs. The reduced data set shows Jupiter as an outlier, and a Phi pattern does not exist. To address the situation, the student authors postulate the existence of a "missing planet" via regression analysis techniques. The location of the "missing planet" is statistically within the domain of Ceres, and the Phi pattern is evident. The student authors apply similar techniques in searching for the existence of Phi patterns elsewhere in our solar system. In particular, they conduct research on Neptune, Uranus, and Saturn, with respect to the location of their moons. The student authors illustrate the existence of Phi patterns within the planetary systems of Neptune, Uranus, and Saturn. Additional research can be expected using this material. The student authors suggest further exploration of the impact of Kepler's Law, the possibility of the existence of other planets in our solar system, and finally, applications to other galaxies and solar systems.

2:20 – 2:40 PM: ***Prime Fractions: A Universe of Rings***
Sohil Jain, The Bronx High School of Science • Advisor: Ms. Joanne Strauss

Fractions with prime denominators, excluding 2 and 5, yield an array of patterns and theories that are that are incorporated into circular rings. These ring patterns are justified using a unique addition form and numerous inscribed polygons to derive the standard equation. The standard equation further helps to expand upon the distinctiveness of these fractions.

2:40 – 3:00 PM: ***"Shortcuts" and "Operations" on Figurate Numbers***
Kaity Tung, The Bronx High School of Science • Advisor: Ms. Joanne Strauss

Around 300CE, Diophantos, a Greek Mathematician, had made various accomplishments on figurate numbers. Figurate numbers are the number of dots that can be arranged into polygons. Thus, they are also called polygonal numbers. There are various kinds of figurate numbers, such as triangular, square, pentagonal, and hexagonal numbers. These different subsets of polygonal numbers can be found using specific formulas based on the patterns that can be seen from the figures. Moreover, different types of polygonal numbers have different relationships with each other. The adding of the two types of figurate numbers can form a different category of figurate numbers. In addition, specific types of polygonal numbers also have internal relationships with each other. After applying order of operations to figurate numbers of the same category, why they stay in shape? The sums, differences, and products will indicate these internal relationships. Figurate numbers are commonly used numbers; however, people use them without realizing it.

3:00 – 3:20 PM: ***Observing Patterns in an Array of Numbers***
Amy (Chun Yun) Hsu, The Bronx High School of Science • Advisor: Ms. Joanne Strauss

One can often observe and find patterns in an array of numbers. My paper presents the patterns I observed and found in a particular array of numbers. It also presents various formulas and other discoveries I found due to these patterns observed. These interesting discoveries and formulas involve triangle numbers.

Contributed Posters

The Effectiveness of the "Do Math" Approaches - the Bridge to Close the Cognitive Gap Between Arithmetic and Algebra, Violeta C. Menil and Olen Dias, Hostos Community College

This research was done during the Academic Year 2006-2007 with Arithmetic and Algebra students participating in the Community College Collaborative Incentive Grant at Hostos Community College of the City University of New York. The research was aimed at closing the cognitive gap between Arithmetic and Algebra through innovative techniques called the *Do Math Approaches*. The Do Math approaches included the following strategies: interactive participation in the math lab and/or computer work, tutorials, math journals, and portfolios. Real learning comes through *doing* math problems and/or activities by the students themselves. The role of the faculty is simply to motivate, facilitate and clarify concepts that needed more understanding.

Through the *Do Math Approaches*, we were able to identify and correct misconceptions, identify areas of weaknesses and gain a better understanding of the students' mathematical thinking. The most significant contribution of this research was that with the intervention - *Do Math Approaches*, the treatment group known as the research class performed better than the control group (regular class) in terms of their performance in the departmental final examinations. Overall, there was a significant difference in the average performance of the research class and the regular class, ($p \leq 0.10$). In terms of the COMPASS results, the research class exceeded the average pass rates of the regular class. Further, the COMPASS pass rates of the research class also exceeded the average pass rates of the Math department. Because of the success rates in both the final examinations and the COMPASS pass rates, we concluded that the *Do Math Approaches*, the teacher factor and the tutor factor did close the cognitive gap between arithmetic and algebra.

The Enigma of Stanislaw Ulam: Mathematical Triumph in the Face of Brain Injury
Alexander Atwood, Suffolk County Community College

In 1946, the mathematician Stanislaw Ulam was struck down by viral encephalitis. After emergency brain surgery, he underwent a recovery in which he regained many of his non-mathematical skills. However, his mathematical powers were substantially transformed. Although he was unable to concentrate on one subject for more than a few minutes and had significant difficulty in performing simple mathematical operations such as solving quadratic equations, his creative mathematical powers were substantially enlarged. In the next ten years, Ulam, in conjunction with von Neumann, Teller, Fermi and others, would create some of the outstanding achievements of applied mathematics in the twentieth century. These include the creation of the Monte Carlo Method, the formulation of the breakthrough principle which led to Thermonuclear Weapons, and seminal discoveries of non-linear dynamics in the Fermi-Pasta-Ulam problem. How was Ulam able to create these monumental mathematical achievements in the face of crippling mathematical limitations? How might have Ulam's brain compensated for his technical weaknesses? How did his ability to work with other mathematicians enable him to overcome his inability to concentrate? What can we learn from Ulam about the way in which the brain creates mathematics?

The Voynich Manuscript Revisited, Marina Vulis, University of New Haven

We will discuss the mystery of the Voynich manuscript written in some unknown artificial language. The manuscript was apparently written in the 15^{th} century. For many years, cryptographers and mathematicians tried to decipher the Voynich manuscript using but to no avail. The scientists also failed to come to an agreement on its origin. It is interesting to look at the cryptanalytic techniques used to decipher the manuscript. Is there a future?

Sequences of Rational Approximations for the Square Roots of Certain Primes
James E. Carpenter, Iona College

An approach to finding a sequence of rational numbers which quickly converges to the square root of 2 has been attributed to the Greek mathematician Eudoxus. In this paper, the Ladder of Eudoxus is proved and extended to any prime of the type n^2+1.

The Quadratic Formula is Actually a Special Case of a More General Formula
Ron Skurnick and Mohammad Javadi, Nassau Community College

Let $p(x)$ be an nth degree polynomial, where $n \geq 2$. In this presentation, we shall derive a formula for finding the roots of the $(n-2)^{nd}$ derivative of $p(x)$. We will then show that the Quadratic Formula is a special case of this formula, with $n = 2$.

Where Did $n x^{n-1}$ Come From?, George H. McCormack, LaGuardia Community College

A number of students taking Calculus I in college have either seen some of the subject before in precalculus or are taking the course for a second time. These students may be familiar with $\frac{d}{dx}\left(x^n\right) = n x^{n-1}$, but typically have no idea of its origin. As a point of discussion, it may be important for them to know that calculus did not emerge full-blown the way it is presented in classrooms and textbooks today. During the lifetime of Newton and Leibniz, the definition of the limit and even the term derivative did not exist. In fact, what we would call calculus was first presented as a set of algebraic rules for computation that could be managed by any scientist or engineer.
What follows is the story of the algebra and analytic geometry that were the precursors of derivative. Today, if you asked the question, "Where did $\frac{d}{dx}\left(x^n\right) = n x^{n-1}$ come from?" the answer would probably include a demonstration of a limit of the difference quotient on $f(x) = x^n$, but this would be a far cry from the answer a student would have received when calculus was first invented.

Solution of Equations of Projectile Motion Using Euler's Method

George Klimi, Pace University (NYC) & City Tech (Brooklyn)
Jack Lowenthal, City Tech (Brooklyn)

In general the differential equations of projectile flight are solved numerically using Runge-Kutta's methods, or other similar methods, because they contain a complicated function of resistance and the air density function that decreases with the projectile altitude. When the projectile is launched with speed less than 256 m/s the differential equations of flying projectiles can be solved in quadratures using Euler's approach that considers constant the density function. As a matter of fact the solution of the differential equations of projectiles motion with the Euler's method is obtained using Otto's tables.

The following approach to the solution of differential equations of projectile flight is based on the approximation method introduced by the great mathematician Leonard Euler. The theoretical results we have obtained solving the equations of projectile flight are illustrated with two examples that demonstrate the use of Euler's method to find the elements of projectile trajectory of trench mortars using a graphing calculator TI-83+ avoiding the use of Otto's tables.

The accuracy of outcomes based on Euler's assumption of a constant air density is exceptional.

Math Circles in the Metro NY Region

Ming Jack Po, Columbia University (CCNY)
Dmitry Sagalovskiy, Google
Japheth Wood, Bard College MAT Program

There is a growing interest among mathematicians and math educators to provide enrichment opportunities for their local communities. We will highlight the recently formed New York Math Circle, which offers programs for students and for teachers, as well as discuss recent efforts to start a Math Circle in the Mid-Hudson Valley.