### Explaining the Math of Queer Relationship Dynamics

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2 The Stable Matching Problem

**3** The (More Modern) Stable Marriage Problem

### **4** Further Investigations

# Outline

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# Preliminaries

# **Bipartite Graphs**

Consider a graph G = (V, E):



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# **Bipartite Graphs**

Consider a graph G = (V, E):



### Definition

*G* is **bipartite** if there are sets *A*, *B* that partition  $V [V = A \cup B, A \cap B = \emptyset]$  such that every edge can be written  $e = ab \in E$ , with  $a \in A$  and  $b \in B$ .

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# Matchings

### Definition

A matching in G is a subset of edges  $E' \subset E$  such that each vertex belongs to at most one edge.

### 

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# Matchings

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 $a_1$  $a_3$  $a_4$  $b_1$ hA  $b_5$  $b_{2}$ 

We will look at maximal matchings, which are not a subset of any other matching.





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Each vertex ranks its interest in being matched with another vertex:





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Each vertex ranks its interest in being matched with another vertex:



Node	Pre	feren	ice C	)rder	
A	B	F	C	E	D
B	F	A	E	D	$\boldsymbol{C}$
C	F	E	A	$\boldsymbol{B}$	D
D	B	$\boldsymbol{A}$	C		
E	F	D	C	B	
F	E	C			



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A match  $i, j \in V$ other" to their current partners.

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ning is <b>unstable</b> if there are
such that $i$ and $j$ "prefer each
to their current norther

Node	Pre	ferer	ice C	rder	
A	B	F	C	E	D
B	F	A	E	D	$\boldsymbol{C}$
C	F	E	A	$\boldsymbol{B}$	D
D	B	$\boldsymbol{A}$	C		
E	F	D	C	B	
F	E	C			



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Node	Pre	feren	ice C	rder	
A	B	F	C	E	D
B	F	A	E	D	$\boldsymbol{C}$
C	F	E	A	$\boldsymbol{B}$	D
D	B	$\boldsymbol{A}$	C		
E	F	D	C	B	
F	$\boldsymbol{E}$	C			

### Definition

A matching is **stable** if there does not exist an unstable pair of vertices.



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Node	Pre	feren	ce C	)rder	
A	B	F	C	E	Ø
B	F	A	E	$\boldsymbol{D}$	Ø
C	F	E	A	B	D
D	B	Ă	C		
E	F	D	C	B	
F	E	C			

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Node	Pre	feren	ice O	rder	
A	B	F	$\boldsymbol{C}$	E	D
B	F	A	E	$\boldsymbol{D}$	C
C	F	E	$\boldsymbol{A}$	B	D
D	B	A	C		
E	F	D	C	B	
F	E	C			
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Node	Pre	feren	ice O	rder	
A	B	F	Ø	E	D
B	F	$\boldsymbol{A}$	E	Ø	C
C	F	E	Ă	B	D
D	$\boldsymbol{B}$	A	C		
E	F	D	C	B	
F	E	C			

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Node	Pre	feren	ice O	rder	
A	B	F	C	E	D
B	F	$\boldsymbol{A}$	E	D	C
C	F	E	A	B	D
D	B	A	$\boldsymbol{C}$		
E	F	D	C	B	
F	E	C			

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### The Stable Matching Problem

• Assigns medical students to residency programs.



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- Assigns medical students to residency programs.
- Note that this is a bipartite graph.



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- Assigns medical students to residency programs.
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- The programs can accept more than one student.





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- Assigns medical students to residency programs.
- Note that this is a bipartite graph.

• The programs can accept more than one student.

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Student	Prefe	ren	ce Order								
$s_1$	$p_1$ $p_1$	$p_3$		Program	Pre	feren	ce O	rder			
$s_2$	$p_3$	$p_{2}$	$p_1$	$p_1$	$s_5$	$s_6$	$s_1$	$s_7$			
$s_3$ $s_4$	$\begin{array}{ccc} p_3 & p_3 \\ p_3 & p_3 \end{array}$	$p_1$	$p_2$	$p_2$	$s_7$	$s_1$	$s_5$	$s_2$	$s_6$	83	$s_4$
$s_5$	$p_1$ $p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	$s_6$	
$s_6$	$p_2$ $p_2$	$p_3$	$p_1$								
$s_7$	$\mid p_1 \mid p_$	$p_2$	$p_3$								



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1 The students all submit a proposal to their top program that hasn't rejected them yet (they've already sent applications).



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Student	Pre	feren	ce Order								
$s_1$	$p_1$	$p_3$		Program	Pre	feren	ce O	rder			
$s_2$	$p_3$	$p_2$	$p_1$	$p_1$	$s_5$	$s_6$	$s_1$	$s_7$			
$s_3$	$p_3$	$p_1$	<b>2</b> 2-	$p_2$	$s_7$	$s_1$	$s_5$	$s_2$	$s_6$	$s_3$	$s_4$
84 87	$p_3$	$p_1$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	$s_6$	
85 80	$p_1$	$p_3$	$p_2$								
50 87	$p_2$ $n_1$	$p_3$	$p_1$ $p_2$								
01	P1	$P^2$	$P_{2}$								

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Student | Preference Order

- The students all submit a proposal to their top program that hasn't rejected them yet (they've already sent applications).
- 2 Each program considers all of its proposals to this point, *temporarily* accepts the top ones it has space for, and rejects the rest.

$s_1$	$p_1$	$p_3$		Program	Pre	feren	ce O	rder			
$s_2$	$p_3$	$p_2$	$p_1$	$p_1$	$s_5$	$s_6$	$s_1$	$s_7$			
$s_3$	$p_3$	$p_1$		$\frac{1}{p_2}$	87	S1	$s_{5}$	59 	$S_{6}$	52	$S_A$
$s_4$	$p_3$	$p_1$	$p_2$	$r_2$	S1	SE	- 0 - So	S7	54	86	~4
$s_5$	$p_1$	$p_3$	$p_2$	P3	01	05	03	01	04	00	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								

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$s_1$	$p_1$	$p_3$		Program	Pre	feren	ce Oi	der			
$s_2$	$p_3$	$p_2$	$p_1$	$p_1$	85	$S_6$	$s_1$	87			
$s_3$	$p_3$	$p_1$		$p_2$	$s_7$	$s_1$	$s_5$	$s_2$	$s_6$	$S_3$	$s_4$
$s_4$	$p_3$	$p_1$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$\overline{s_7}$	$s_4$	$s_6$	1
$s_5$	$p_1$	$p_3$	$p_2$		_	, i	-		_	, in the second s	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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$s_1$	$p_1$	$p_3$		Program	Pre	feren	ce Oi	rder			
$s_2$	$p_3$	$p_2$	$p_1$	$n_1$	SE	Se	81	Set			
$s_3$	$p_3$	$p_1$		P1	00	00	<b>U</b>	21			
$S_A$	<b>D</b> 2	$\mathcal{D}_1$	$p_2$	$p_2$	$s_7$	$s_1$	$s_5$	$s_2$	$s_6$	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	$s_6$	
$s_6$	<b>p</b> 2	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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Student	Prefe	renc	e Order								P
$s_1$	<b>p</b> 1	$p_3$		Program	Pre	feren	ce Oi	rder			T M
$s_2$	$p_3$	$p_2$	$p_1$	$p_1$	$s_5$	$s_6$	$s_1$	87			S 
$s_3$	$p_3$	$p_1$		$p_2$	$s_7$	$s_1$	$s_5$	$s_2$	$s_6$	$S_3$	s <sub>4</sub>
$s_4$	$p_3$	$p_1$	$p_2$	$p_3$	$s_1$	$s_5$	<b>s</b> 3	$s_7$	$s_4$	$s_6$	- Ir
$s_5$	$p_1$	$p_3$	$p_2$		-						
$s_6$	<b>p</b> <sub>2</sub>	$p_3$	$p_1$								
$s_7$	<b>p</b> 1	$p_2$	$p_3$								





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Student	Pre	ferend	ce Order									Match Proble
$s_1$	<b>p</b> 1	$p_3$		Program	Pre	feren	ce O	rder				The (Moder
$s_2$	$p_3$	$p_2$	$p_1$	$\frac{1}{p_1}$	<i>s</i> <sub>5</sub>	$s_6$	<i>s</i> <sub>1</sub>	87				Marria
$s_3$	$p_3$	$p_1$		$p_2$	87	$s_1$	$s_5$	$s_2$	<u>s6</u>	$s_3$	$s_4$	Furthe
$s_4$	$p_3$	$p_1$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	$s_6$		Invest
$s_5$	$p_1$	$p_3$	$p_2$	-								
$s_6$	<b>p</b> 2	$p_3$	$p_1$									
$s_7$	$p_1$	$p_2$	$p_3$									



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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce O	rder			
$s_3$	$p_3$	$p_1$		$p_1$	$s_5$	$s_6$	$s_1$	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	<i>s</i> 7	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	$s_6$	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								

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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Oi	rder			
$s_3$	$p_3$	$p_1$		$p_1$	<i>s</i> <sub>5</sub>	$s_6$	$s_1$	84			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	<b>8</b> 7	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	<u>\$6</u>	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_{1}$	$p_2$	$p_3$								





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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	rder			
$s_3$	$p_3$	$p_1$		$p_1$	<i>s</i> <sub>5</sub>	$s_6$	$s_1$	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
85	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	86	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								





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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	$s_5$	$s_6$	<u>\$</u> 1	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	<u>\$6</u>	
$s_6$	<b>p</b> 2	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	$s_5$	$s_6$	<u>\$</u> 1	87			
$s_4$	<b>p</b> 3	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	$s_4$	<u>\$6</u>	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	$s_5$	$s_6$	<u>\$</u> 1	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	$s_1$	$s_5$	$s_3$	$s_7$	<u>8</u> 4	<u>\$6</u>	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								

THE STABLE MARRIAGE PROBLEM

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The (More Modern) Stable Marriage Problem

- **1** Any students not currently in a program submit a proposal to their top program that hasn't rejected them yet.
- 2 Each program considers all of its proposals to this point, *temporarily* accepts the top ones it has space for, and rejects the rest.

Student	Pref	erenc	e Order								
$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	<i>s</i> <sub>5</sub>	$s_6$	<i>s</i> 1	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
35	$p_1$	$p_3$	$p_2$	$p_3$	<i>s</i> <sub>1</sub>	$s_5$	$s_3$	$s_7$	<u>\$4</u>	<u>\$6</u>	
<sup>8</sup> 6	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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Stable Marriage Problem Further Investigations

- Any students not currently in a program submit a proposal to their top program that hasn't rejected them yet.
- **2** Each program considers all of its proposals to this point, temporarily accepts the top ones it has space for, and rejects the rest.

Student	Pret	ferenc	e Order								
$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	$s_5$	$s_6$	<u>\$1</u>	87			
$s_4$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	<i>s</i> <sub>1</sub>	$s_5$	$s_3$	$s_7$	<u>84</u>	<u>\$6</u>	
$s_6$	$p_2$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								





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- **1** Any students not currently in a program submit a proposal to their top program that hasn't rejected them yet.
- 2 Each program considers all of its proposals to this point, *temporarily* accepts the top ones it has space for, and rejects the rest.

Student	Pref	ferenc	e Order								
$s_1 \\ s_2$	$p_1 \\ p_3$	$p_3 \ p_2$	$p_1$	Program	Pre	feren	ce Or	der			
33	$p_3$	$p_1$		$p_1$	<i>s</i> <sub>5</sub>	$s_6$	<i>s</i> 1	87			
34	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	$s_4$
5	$p_1$	$p_3$	$p_2$	$p_3$	<i>s</i> <sub>1</sub>	$s_5$	$s_3$	$s_7$	<u>84</u>	<u>\$6</u>	
36	$p_2$	$p_3$	$p_1$								
$s_7$	$p_{1}$	$p_2$	$p_3$								



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- Any students not currently in a program submit a proposal to their top program that hasn't rejected them yet.
- **2** Each program considers all of its proposals to this point, *temporarily* accepts the top ones it has space for, and rejects the rest.

Student	Pref	ferenc	ce Order								
$s_1$	$p_1$	$p_3$									
$s_2$	$p_3$	$p_2$	$p_1$	Program	Pre	feren	ce Or	der			
$s_3$	$p_3$	$p_1$		$p_1$	<i>s</i> <sub>5</sub>	$s_6$	<u>\$</u> 1	87			
$\mathbf{s_4}$	$p_3$	$p_1$	$p_2$	$p_2$	87	$s_1$	$s_5$	$s_2$	<u>\$6</u>	$s_3$	84
$s_5$	$p_1$	$p_3$	$p_2$	$p_3$	<i>s</i> <sub>1</sub>	$s_5$	$s_3$	$s_7$	<u>\$4</u>	<u>\$6</u>	
$s_6$	$p_{2}$	$p_3$	$p_1$								
$s_7$	$p_1$	$p_2$	$p_3$								



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How well does this work?

- This algorithm always terminates.
- The output will be a stable matching.
- Since we focused on the students'

preferences, they got the best stable

matching possible!

÷1	PI	PO	
$s_2$	$p_3$	$p_2$	$p_1$
$s_3$	$p_3$	$p_1$	
$s_4$	$p_3$	$p_{1}$	$p_2$
$s_5$	$p_1$	$p_3$	$p_2$
$s_6$	$p_2$	$p_3$	$p_1$
$s_7$	$p_1$	$p_2$	$p_3$

S 1

Student | Preference Order

n no



THE STABLE MARRIAGE PROBLEM

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What Assumptions Did We Make?

• Consistently ordered preferences



![](_page_41_Picture_3.jpeg)

![](_page_41_Picture_4.jpeg)

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What Assumptions Did We Make?

![](_page_42_Picture_1.jpeg)

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Further Investigations

• Consistently ordered preferences (*This is a non-trivial assumption*!)

What Assumptions Did We Make?

![](_page_43_Picture_1.jpeg)

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Further Investigations

• Consistently ordered preferences (This is a non-trivial assumption!)

• Bipartite graph

![](_page_44_Picture_0.jpeg)

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Further Investigations

# The (More Modern) Stable Marriage Problem

Edison Hauptman – THE STABLE MARRIAGE PROBLEM

![](_page_45_Picture_0.jpeg)

![](_page_45_Figure_2.jpeg)

Node	Preference Order				
A	B	F	C	E	D
B	F	$\boldsymbol{A}$	E	D	C
C	F	E	A	B	D
D	B	A	$\boldsymbol{C}$		
E	F	D	C	B	
F	E	C			

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![](_page_46_Picture_0.jpeg)

![](_page_46_Figure_2.jpeg)

Here we found a stable matching. But can we always find one?

Node	Pre	eference Order					
A	B	F	C	E	D		
B	F	$\boldsymbol{A}$	E	D	C		
C	F	E	A	B	$\boldsymbol{D}$		
D	B	A	$\boldsymbol{C}$				
E	F	D	C	B			
F	E	C					

THE STABLE MARRIAGE PROBLEM

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![](_page_47_Picture_0.jpeg)

![](_page_47_Figure_2.jpeg)

Here we found a stable matching. But can we always find one?

Node	Preference Order					
A	B	F	C	E	D	
B	F	$\boldsymbol{A}$	E	D	C	
C	F	E	A	B	D	
D	B	A	$\boldsymbol{C}$			
$oldsymbol{E}$	$oldsymbol{C}$	D	$\boldsymbol{F}$	B		
F	$\boldsymbol{E}$	C				

In a slightly different world, maybe E prefers to be matched with C instead of F. Now what happens?

### THE STABLE MARRIAGE PROBLEM

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The (More Modern) Stable Marriage Problem

![](_page_48_Picture_0.jpeg)

![](_page_48_Figure_2.jpeg)

Node	Preference Order				
A	B	F	C	E	D
B	F	$\boldsymbol{A}$	E	D	C
C	F	E	A	B	$\boldsymbol{D}$
D	B	A	$\boldsymbol{C}$		
E	C	D	F	B	
F	E	C			

In a slightly different world, maybe E prefers to be matched with C instead of F. Now what happens?

### THE STABLE MARRIAGE PROBLEM

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![](_page_49_Picture_0.jpeg)

![](_page_49_Figure_2.jpeg)

Node	Pre	Preference Order				
A	B	F	C	E	D	
B	F	$\boldsymbol{A}$	E	D	C	
C	F	$\boldsymbol{E}$	A	B	D	
D	B	A	$\boldsymbol{C}$			
E	$\boldsymbol{C}$	D	$\boldsymbol{F}$	B		
F	E	C				

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The (More Modern) Stable Marriage Problem

![](_page_50_Picture_0.jpeg)

![](_page_50_Figure_2.jpeg)

Node	Pre	Preference Order				
A	B	F	C	E	D	
B	F	$\boldsymbol{A}$	E	D	C	
C	F	$\boldsymbol{E}$	A	B	D	
D	B	A	C			
E	$\boldsymbol{C}$	D	F	B		
F	E	C				

### THE STABLE MARRIAGE PROBLEM

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![](_page_51_Picture_0.jpeg)

![](_page_51_Figure_2.jpeg)

Node	Preference Order				
A	B	F	C	E	D
B	F	$\boldsymbol{A}$	E	D	C
C	F	$oldsymbol{E}$	A	B	D
D	B	A	C		
E	$\boldsymbol{C}$	D	F	B	
F	E	$\boldsymbol{C}$			

### THE STABLE MARRIAGE PROBLEM

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![](_page_52_Picture_0.jpeg)

![](_page_52_Figure_2.jpeg)

Node	Preference Order				
A	B	F	C	E	D
B	F	$\boldsymbol{A}$	E	D	C
C	F	E	A	B	D
D	B	A	C		
E	C	D	F	B	
F	E	$\boldsymbol{C}$			

### THE STABLE MARRIAGE PROBLEM

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![](_page_53_Picture_0.jpeg)

![](_page_53_Figure_2.jpeg)

Node	Pre	Preference Order				
A	B	F	C	E	D	
B	F	$\boldsymbol{A}$	E	D	C	
C	F	E	A	B	D	
D	B	A	C			
E	C	D	$\boldsymbol{F}$	B		
F	E	$\boldsymbol{C}$				

### THE STABLE MARRIAGE PROBLEM

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![](_page_54_Picture_0.jpeg)

![](_page_54_Figure_2.jpeg)

Preference Order Node A DR CBEA DCCFE $A \quad B$ D D BAC ECD FBF $\boldsymbol{E}$ C

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Further Investigations

And we're back to where we started...

![](_page_55_Picture_0.jpeg)

![](_page_55_Figure_2.jpeg)

And we're back to where we started...

Node	Pre	Preference Order				
A	B	F	C	E	D	
B	F	A	E	D	C	
C	$\boldsymbol{F}$	$oldsymbol{E}$	A	B	D	
D	B	A	C			
E	$\boldsymbol{C}$	D	$\boldsymbol{F}$	B		
F	$\boldsymbol{E}$	$\boldsymbol{C}$				

This cycle caused a problem. And since none of C, E, Fprefer someone else more, there can be no stable matching. THE STABLE MARRIAGE PROBLEM

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# The Stable "Roommates" Problem

![](_page_56_Picture_1.jpeg)

![](_page_56_Figure_2.jpeg)

Node	Preference Order				
A	B	F	C	E	D
B	F	A	E	D	C
C	F	$\boldsymbol{E}$	A	B	D
D	B	A	C		
E	$\boldsymbol{C}$	D	$\boldsymbol{F}$	B	
F	$\boldsymbol{E}$	$\boldsymbol{C}$			

THE STABLE MARRIAGE PROBLEM

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Preliminaries

The Stable Matching Problem

The (More Modern) Stable Marriage Problem

# The Stable "Roommates" Problem

![](_page_57_Picture_1.jpeg)

![](_page_57_Figure_2.jpeg)

Bipartite graphs have no triangles, so this kind of cycle can never happen there.

Node	Preference Order					
A	B	F	C	E	D	
B	F	A	E	D	C	
C	$\boldsymbol{F}$	$oldsymbol{E}$	A	B	D	
D	B	A	C			
E	$\boldsymbol{C}$	D	$\boldsymbol{F}$	B		
F	$oldsymbol{E}$	$\boldsymbol{C}$				

THE STABLE MARRIAGE PROBLEM

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# The Stable "Roommates" Problem

![](_page_58_Picture_1.jpeg)

# 

Bipartite graphs have no triangles, so this kind of cycle can never happen there.

Node	Preference Order				
A	B	F	C	E	D
B	F	A	E	D	C
C	$\boldsymbol{F}$	$oldsymbol{E}$	A	B	D
D	B	A	C		
E	$\boldsymbol{C}$	D	$\boldsymbol{F}$	B	
F	$oldsymbol{E}$	$\boldsymbol{C}$			

Hence, if we remove the condition that everyone has to be straight, relationships become *mathematically* harder!

THE STABLE MARRIAGE PROBLEM

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Preliminaries

The Stable Matching Problem

The (More Modern) Stable Marriage Problem

Thank You!

- MAA Maryland/DC/Virginia Region
- James Madison University Math Department
- University of Pittsburgh Math Department (especially Prof. Jeff Wheeler)
- Prof. Elizabeth Reid (Marist College)
- ...and you all, for your time and attention!

![](_page_59_Picture_7.jpeg)

THE STABLE MARRIAGE PROBLEM

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Preliminaries

The Stable Matching Problem

The (More Modern) Stable Marriage Problem

Further Investigations

April 27, 2024

![](_page_60_Picture_0.jpeg)

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The (More Modern) Stable Marriage Problem

Further Investigations

# Further Investigations

- In the Stable Marriage Problem (the boring one), can we find a stable matching that balances out each groups' preferences?
  - (Viet/Lee/Trang/Chung 2016) use simulations to approximate optimality.
- (Irving 1985): Algorithm shows when the Stable "Roommates" Problem has a stable matching on 2n vertices, and finds one if it exists.
  - (Cseh/Manlove 2016) show that allowing for "unacceptable partners" (as we've done throughout) makes the problem NP-hard.
- Can we extend this problem (and the idea of stability) to allow for "matches" of 3 or more people?
  - (Biró 2007) defines a "stable fractional matching" in a hypergraph, and proves

that one always exists.

![](_page_61_Picture_9.jpeg)

THE STABLE MARRIAGE PROBLEM

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