Rethinking Precalculus

Spencer Hamblen

McDaniel College

April 27, 2024

MAT 1107

McDaniel's MAT 1107 – College Algebra and Trigonometry

- Prerequisite for MAT 1117 – Calculus I

MAT 1107

McDaniel's MAT 1107 – College Algebra and Trigonometry

- Prerequisite for MAT 1117 – Calculus I
- Students are CS, Kinesiology, Chemistry, Health Science majors

MAT 1107

McDaniel's MAT 1107 – College Algebra and Trigonometry

- Prerequisite for MAT 1117 – Calculus I
- Students are CS, Kinesiology, Chemistry, Health Science majors
- Single course transition from developmental math to Calculus

MAT 1107

McDaniel's MAT 1107 – College Algebra and Trigonometry

- Prerequisite for MAT 1117 – Calculus I
- Students are CS, Kinesiology, Chemistry, Health Science majors
- Single course transition from developmental math to Calculus

Context:

- Unfortunately a gatekeeper – 25% DFW rate (much worse than all other MAT classes)

MAT 1107

McDaniel's MAT 1107 – College Algebra and Trigonometry

- Prerequisite for MAT 1117 – Calculus I
- Students are CS, Kinesiology, Chemistry, Health Science majors
- Single course transition from developmental math to Calculus

Context:

- Unfortunately a gatekeeper – 25% DFW rate (much worse than all other MAT classes)
- Better than 27% national average!
- Need to ensure all students acquire skills and can continue in major

Precursor: MAT 1100

Recently (2020) replaced all our developmental Math courses with MAT 1100 – Mathematical Fundamentals.

- Prerequisite for Accounting, Chemistry, Stats, Computer Science, Quantitative Reasoning courses
- Emphasizes communication, process basic skills

Precursor: MAT 1100

Recently (2020) replaced all our developmental Math courses with MAT 1100 – Mathematical Fundamentals.

- Prerequisite for Accounting, Chemistry, Stats, Computer Science, Quantitative Reasoning courses
- Emphasizes communication, process basic skills
- *Much* better reception from faculty and students

MAT 1100 Methods

Improve basic math skills

- Emphasize process & communication
- Project-based learning
- Unlimited Homework attempts

Reduce math anxiety

- No tests or quizzes
- Reflection assignments and community building
- Unlimited Homework attempts

Meaningful Classroom Experience

- Minimize time lecturing

Adapting MAT 1100 methods to MAT 1107

Kept:

- Online homework, unlimited attempts
- Group projects
- Emphasis on process and reflection

Adapting MAT 1100 methods to MAT 1107

Kept:

- Online homework, unlimited attempts
- Group projects
- Emphasis on process and reflection

Added:

- Extensive in-class activities/worksheets
- Project revision assignments
- Midterm Exam and Final (of a sort)

Online Homework

Weekly problem sets of 12 questions in WebWork (MAA)

- Unlimited attempts
- Credit only given for reaching thresholds (50%/90%/100%)
- Problems sets reopened at midterm and at end of semester

Online Homework

Weekly problem sets of 12 questions in WebWork (MAA)

- Unlimited attempts
- Credit only given for reaching thresholds (50%/90%/100%)
- Problems sets reopened at midterm and at end of semester

Benefits:

- Encourages persistence/revisiting problems
- Competence at all skills

Group Projects

2 Projects due most weeks; material necessary for projects introduced Monday and Wednesday.

- Emphasis on on writing and explaining process and concepts
- Context for mathematics
- Groups give students support network for homework/in-class work.

Project Objectives.

- Solve equations involving exponential functions and use those functions to solve problems in exponential growth.
- Translate between verbal, algebraic, graphical and numerical representations of functions.
- Demonstrate an understanding of mathematical modeling by using algebra to solve applied problems.

2 Food Safety

Newton's Law of Cooling/Heating states that the temperature T of an object at time t is given by

$$T(t) = ae^{kt} + T_s,$$

where a and k are constants determined by the object, and T_s is the *ambient temperature*, the temperature of the surrounding air.

2.1. Over a long period of time, we expect that the temperature of the turkey will approach the ambient temperature. What does that tell us about the constant k?

2.2. If we are cooking a turkey that starts at a temperature of 32°, what is the constant a in Newton's Law? (Your answer will be a function of T_s.)

2.3. We put the turkey (starting at a temperature of 32°) into the oven at 400°. After 30 minutes, the turkey has reached a temperature of 60°. Use this information to find the value of the constant k.

2.4. Given the values of the constants a and k you calculated in **2.2** and **2.3**, how long will it take for the turkey to reach 165°, the recommended minimum safe temperature[1]?

[1] https://www.fsis.usda.gov/food-safety/safe-food-handling-and-preparation/poultry/turkey-basics-safe-cooking.

Even more in-class work

Goal: Minimize time spent lecturing

- Class notes handed out each day, mixing activities with definitions and new concepts.
- Completed in groups
- Answer questions, encourage conversation

Even more in-class work

Goal: Eliminate time spent lecturing

- Class notes handed out each day, mixing activities with definitions and new concepts.
- Completed in groups
- Answer questions, encourage conversation

Class work examples

10/18 – Algebraic Properties of Logarithms

Properties of the logarithm we've seen so far:

- $\log_b(1) = 0$ and $\log_b(b) = 1$
- $\log_b(b^x) = b^{\log\_b(x)} = x$
- $\log_b(c) = d \leftrightarrow c = b^d$

Additional properties of logarithms:

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b(x/y) = \log_b(x) - \log_b(y)$
- $\log_b(c) = \ln(c)/\ln(b)$

1. Fill in the blanks in the following table of logarithmic expressions

Compressed Notation	Expanded Notation
$\ln(2x)$	$\ln(2) + \ln(x)$
$\log_{10}(2b^2/c)$	$\log_{10}(2) + 2\log_{10}(b) - \log_{10}(c)$
$\log_5(5x^3)$	
	$\log_5(4) + \log_5(9) - 2\log_5(6)$
	$3\ln(x) - 2\ln(x+1)$
$\log_2(x^2y^4/3z^{1/2})$	

2. Solve for x in the following logarithmic equations. All of the answers should be whole numbers.

   a. $6 - \log_{10}(x) = 4$

   b. $\log_3(9) = 2\log_3(x)$

   c. $\ln(x) + \ln(x - 1) = \ln 6$

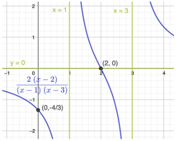
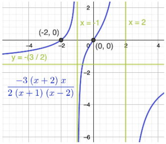
   d. $2\log_4(x - 3) = \log_4(3x - 11)$

   e. $\log_3(x) = \log_9(2x)$

Class work examples

10/2 - Graphing Rational Functions pt. 2

Below are the graphs of some rational functions, along notable points and lines associated with the functions.

Rational Function	f(x) = $\frac{2(x-2)}{(x-1)(x-3)}$	g(x) = $\frac{-3(x+2)x}{2(x+1)(x-2)}$
Graph		
Roots/ x-intercepts	(2,0)	(-2,0) and (0,0)
y-intercept	(0,-4/3)	(0,0)
Horizontal Asymptotes	y = 0	y = -3/2
Vertical Asymptotes	x = 1 and x = 3	x = -1 and x = 2

1. What aspects of the functions determine the roots?
2. What aspects of the functions determine the y-intercept?
3. What aspects of the functions determine the vertical asymptotes?
4. What aspects of the functions determine the horizontal asymptotes?
5. Why can't the functions have multiple y-intercepts?

Project Revision assignments

Individual assignment to revisit projects

Step 2. Complete a revision of the Part you have chosen, fixing an mathematical errors and adding any explanation or reasoning necessary. If an individual problem in the Part was complete and correct, and you feel that additional explanation or context is not necessary, you do not need to rewrite that problem.

Step 3. For each problem in the Part that you have corrected or added to:

- explain what was incorrect or missing from the original problem;
- explain any mistakes that you made in your original solution; and
- explain how what we've learned this semester helped you to be able to correctly complete the problem.

Midterm and Final*

Goals for exams:

- *Not* to memorize and regurgitate facts
- Demonstrate mathematical thinking
- Use accumulated knowledge to problem-solve

Midterm and Final*

Goals for exams:

- *Not* to memorize and regurgitate facts
- Demonstrate mathematical thinking
- Use accumulated knowledge to problem-solve

Implementation:

- Any class handouts/worksheets could be consulted during the exam
- Any non-graphing calculator could be used, but all steps needed to be documented
- All solutions must be accompanied by full explanation and annotation of process
- Only 2 problems on mid-term, 4 problems on final

Final Exam

For all of the questions on the exam, be sure to annotate your solutions in the column to the right of the questions. In other words, include all calculations used in finding the solutions, and explain (in words) where all of your numbers and calculations are coming from.

1. The parabola given by the function $f(x)$ has a vertex at $(1, -1)$ and a y-intercept of 3. Write the polynomial $f(x)$ in vertex form, general form, and factored form.

Explanation/annotation here

4. Sketch the graphs of the functions $y = \cos(2\theta)$ and $y = \cos(\theta)$ on the interval $[0, 2\pi)$ on the same graph. Find the exact value (in radians) of all angles where the two graphs intersect.

Explanation/annotation here

5. Go back and review the statements of the problems and your methods of solution for these four problems. How did you know what approach to take for these problems, and what in the statement of the problems indicated that you should take that approach? What specific ideas did you use in the process of solving these problems?

Did it work?

- Lesson 1: *Buy-in is the most important thing.*
  - Need to spend more time at beginning explaining big picture
  - Need to explain role/uses of class worksheets
  - Need to explain why lecturing is bad (in this context)

Did it work?

- Lesson 1: *Buy-in is the most important thing.*
  - Need to spend more time at beginning explaining big picture
  - Need to explain role/uses of class worksheets
  - Need to explain why lecturing is bad (in this context)
- Lesson 2: This class, taught this way, can achieve goals
  - Only one (non-academic) W, three DF out of 25 students (16%)
  - Had a good picture of each student's mathematical thinking and understanding
  - Covered same content as previous MAT 1107 sections

Future Plans

New class improvements:

- 2 minute skill-checks at the beginning of each class
- Improved examples on worksheets
- Directed mini-lectures

Thank you!